# A METHOD OF BEAM ENERGY SPREAD AND SYNCHROTRON TUNE MEASURMENT BASED ON DECOHERENCE SIGNAL ANALYSIS

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*Abstract*

A method of beam energy spread and synchrotron tune measurements based on the analysis of transverse decoherence\recoherence signal of kicked beam is presented. As an illustration the beam energy spread was extracted for the SLS storage ring.

## INTRODUCTION

Charged particles beam in storage ring kicked transversely in some azimuthal position starts performing betatron oscillations around the closed orbit. The beam centroid can be observed in subsequent turns by beam position monitors (BPM). If all the particles have the same betatron tune the oscillations of the beam are coherent, and the beam centroid motion is harmonic. However, if the beam contains a spread of tunes, the motion will decohere as the individual betatron phases of the particles disperse. The tune spread of individual particles in the beam may be caused both by intrinsic betatron tune spread due to transverse nonlinearity and the chromaticity that couples the particle energy spread and the tune shift [1,2,3]. The knowledge about the contribution of the transverse nonlinearity and the chromaticity to the BPM data processing is an important issue for fine energy spread and synchrotron tune measurements and nonlinear beam dynamics study.

The results which are the continuation of the study presented in PAC'09 [3], i.e. the study of decoherence of kicked beam transverse oscillations taking into account the amplitude dependent tune shifts and the 1st and 2nd order chromaticities, are presented in this paper. A procedure is developed to analyse turn by turn BPM data and extract the beam energy spread and the synchrotron tune. That procedure is applied on either TRACY [4] simulation data or experimental data taken at the SLS storage ring.

## ANALYTICAL TREATMENT

As it was mentioned in [3] in the analytical model it is assumed that the transverse distribution is Gaussian, i.e. the distortion of phase space trajectories due to nonlinearity is small. It is also assumed that the tune shift with oscillations amplitude is a quadratic function. For the case of decoherence due to chromaticity, it is assumed that the synchrotron motion is linear and the energy distribution is Gaussian. It is also assumed that the energy distribution is uncorrelated with the transverse distribution. Under these assumptions the decoherence due to chromaticity is completely independent of the transverse distribution. Furthermore, transverse coupling is neglected. All these assumptions are well justified in a well-corrected storage ring.

The centroid motion is considered in vertical phase space where there is no dispersion.

A particle in longitudinal phase space propagates in time by

$$\begin{pmatrix} \delta(n) \\ \tau(n) \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_s n & \sin 2\pi\nu_s n \\ -\sin 2\pi\nu_s n & \cos 2\pi\nu_s n \end{pmatrix} \begin{pmatrix} \delta_0 \\ \tau_0 \end{pmatrix}, \quad (1)$$

where $\delta = \Delta E / E\,\sigma_e$, $\tau = \Delta s/\sigma_s$ are normalized longitudinal coordinates, $\nu_s$ is the synchrotron tune and time is measured by turn numbers.

For a single particle the transverse displacement time evolution is given by the betatron phase

$$y(n) = \sigma_y\, r_y \cos\varphi_y(n). \quad (2)$$

where the amplitude is scaled to the rms beam size $\sigma_y$, $r_y = \sqrt{2J_y\beta_y}/\sigma_y$, $\beta_y$ is the betatron function, $2J_y$ is the Courant-Snyder invariant, $\varphi_y(n)$ is the betatron phase at the $n$-th turn. In the presence of transverse nonlinearity and nonzero value of the first and second order chromaticities the electrons with different energies and betatron amplitudes execute betatron oscillations with different tunes, and the tune shift $\Delta\nu_y = \nu_y(n) - \nu_{y0}$ with respect to the nominal tune $\nu_{y0}$ in turn $n$ is given as

$$\Delta\nu_y(n) = \mu_y r_y^2 + \mu_{yx} r_x^2 + \xi_{y1}\sigma_e\delta(n) + \xi_{y2}\sigma_e^2\delta^2(n), \quad (3)$$

where $\xi_{y1}$ and $\xi_{y2}$ are the first and the second order chromaticities respectively and

$$\mu_y = \frac{\varepsilon_y}{2}\frac{\partial\nu_y}{\partial J_y}, \quad \mu_{yx} = \frac{\varepsilon_y}{2}\frac{\partial\nu_y}{\partial J_x}, \quad (4)$$

where $\varepsilon_x$ and $\varepsilon_y$ are horizontal and vertical emittances respectively.

From (1) and (3) for the betatron phase advance after $N$ turns one obtains

$$\Delta\varphi_y(N) = 2\pi\int_0^N \nu_y(n)\,dn. \quad (5)$$

In longitudinal phase space the distribution of particles is invariant in time and has the following form:

$$\rho_s(\delta,\tau) = \frac{1}{2\pi} e^{-\frac{1}{2}\left(\delta^2+\tau^2\right)}. \quad (6)$$

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In transverse phase space the situation is different. Before the kick in normalized amplitude-phase coordinates $(r,\varphi)$ the distributions of particles are the following

$$\rho_x(r_x,\varphi_x)=\frac{1}{2\pi}r_x e^{-\frac{r_x^2}{2}} \quad , \quad \rho_y(r_y,\varphi_y)=\frac{1}{2\pi}r_y e^{-\frac{r_y^2}{2}} \quad . \tag{7}$$

After application of $\Delta y'$ the horizontal phase space distribution remains unchanged while the vertical one becomes

$$\bar{\rho}_y(r_y,\varphi_y)\Big|_{n=0}=\frac{1}{2\pi}r_y e^{-\frac{1}{2}\left(r_y^2+z_y^2+2r_yz_y\sin\varphi_y\right)}. \tag{8}$$

Here $z_y=\beta_y\Delta y'/\sigma_y$ is the normalized kick. For the kicked beam after $N$ turns one gets

$$\bar{\rho}_y(r_y,\varphi_y)\Big|_{n=N}=\frac{r_y e^{-\frac{1}{2}\left(r_y^2+z_y^2\right)}}{4\pi^2}\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} r_x e^{-\frac{1}{2}\left(r_x^2+\delta_0^2+\tau_0^2+2r_yz_y\sin(\varphi_y-\Delta\varphi_y(N))\right)}dr_x d\delta_0 d\tau_0 \,. \tag{9}$$

From this distribution for the centroid displacement one gets (see also [3])

$$< y(N) >=\sigma_y\int_0^{+\infty}\int_0^{2\pi} r_y\cos\varphi_y\cdot\bar{\rho}_y(r_y,\varphi_y)\Big|_{n=N}d\varphi_y dr_y =$$
$$=\frac{\beta_y\Delta y' A(N)H(N)\sin\left(P(N)+Q(N)+2\pi\nu_{y0}N\right)}{e^{M(N)}}. \tag{10}$$

Here the functions $A(N)$, $H(N)$, and Exp[-$M(N)$] are shaping the envelope of the beam signal:

$$A(N)=\frac{1}{\sqrt{1+(4\pi\mu_{yx}N)^2}\left(1+(4\pi\mu_yN)^2\right)}e^{-\frac{z_y^2}{2}\cdot\frac{(4\pi\mu_yN)^2}{1+(4\pi\mu_yN)^2}}$$
$$M(N)=\frac{K_1^2(1-\cos T)}{1+K_2^2(T+\sin T)^2} \tag{11}$$
$$H(N)=\left[1+2K_2^2\left(T^2+\sin^2T\right)+K_2^4\left(T^2-\sin^2T\right)^2\right]^{-\frac{1}{4}},$$

with $K_1=\sigma_e\xi_{y1}/\nu_s$ , $K_2=\sigma_e^2\xi_{y2}/\nu_s$ and $T=2\pi\nu_sN$ .

Functions $A(N)$ and $H(N)$ describe a monotonous pseudo-damping of the beam centroid signal due to tune shifts with amplitude and the second order chromaticity. The exponential function Exp[-$M(N)$] describes a modulation of the centroid displacement due to the decoherence and recoherence driven by linear chromaticity, with the modulation depths slowly decreasing due to nonlinear chromaticity.

Functions $P(N)$ and $Q(N)$ describe a slow phase modulation of the fast betatron oscillation driven by the second order chromaticity and tune shift with amplitude. They are not presented here since they are not needed in the discussions of this work (see [2,3]).

Figure 1 shows the decoherence signal according to our theoretical model with typical parameters of the SLS storage ring and kick angle $\Delta y'$=200 $\mu rad$:

Table 1: Typical parameters of the SLS storage ring

| | |
|---|---|
| $\varepsilon_x$ | 5.7 nmrad |
| $\varepsilon_y$ | 0.1% of $\varepsilon_x$ |
| $\nu_{y0}$ | 8.737 |
| $\nu_s$ | $6.25*10^{-3}$ |
| $\sigma_e$ | $8.6*10^{-4}$ |
| $\xi_{y1}$ | 4.8 |
| $\xi_{y2}$ | 77 |
| $\partial\nu_y/\partial J_y$ | -606 $m^{-1}$ |
| $\partial\nu_y/\partial J_x$ | 673 $m^{-1}$ |

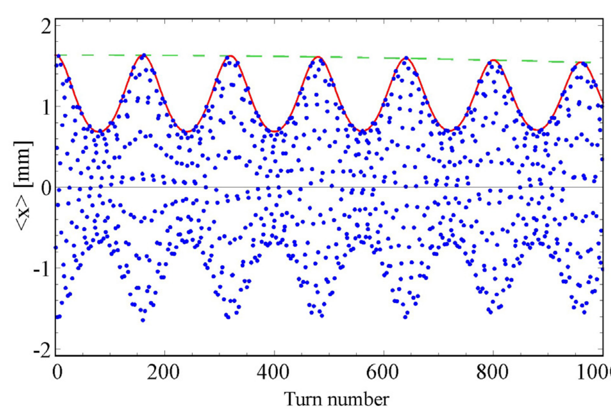


Figure 1: The decoherence signal according to the theoretical model.

The position of vertical pinger ($\beta_{y0}$=6.96m) is considered as a kick point and the position of the first BPM ($\beta_y$=9.58m) - the data registration point. Note that for the kick and observation positions spacing by the betatron phase advance $\Delta\varphi$ in the ring the formula (10) must be slightly modified:

$$< y(N) >=\sqrt{\beta_{0y}\beta_y}\,\Delta y' A(N)H(N)Exp[-M(N)]\times$$
$$\times\sin\left(P(N)+Q(N)+2\pi\nu_{y0}N+\Delta\varphi\right). \tag{12}$$

The envelope is shown as solid red line. The dashed green line plots damping factor. Note that the formula (12) does not include the radiation damping, which is slow in comparison to the processes we consider here.

## ENERGY SPREAD AND SYNCHROTRON TUNE

As one can see from (11) and (12) the distance between the neighbor maximums of the signal is equal to $1/\nu_s$. The synchrotron tune can be measured using that fact. As concerns the energy spread extraction, more detailed analysis of (11) is required. When the 2nd order chromaticity is less than 200, as it is for the SLS storage ring, one can act in the following way: from (11) it is clear that if $\xi_{y2}$, $\mu_y$, $\mu_{yx}$ are zero, then $A(N)\equiv 1$, $H(N)\equiv 1$ (there is no damping) and

$$\sigma_e = \frac{\nu_s}{\xi_{y1}}\sqrt{\ln\sqrt{\frac{\max}{\min}}} \qquad (13)$$

no matter which maximum and minimum is taken (see [5]). When $\xi_{y2}$, $\mu_y$, $\mu_{yx}$ are not equal to zero (there is damping) the value of the first maximum remains unchanged. Changes of value of the first minimum (according to (11)) depending on $\xi_{y2}$ for three different values of $\xi_{y1}$, for the kick 300 $\mu rad$ and parameters from Table 1 are given in Figure 2.

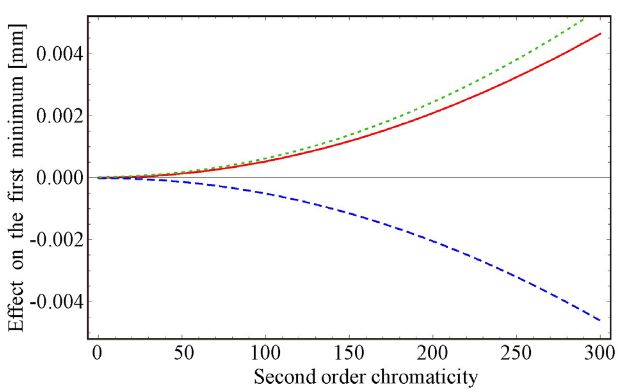


Figure 2: Dashed blue line - $\xi_{y1}$=2.8, solid red line - $\xi_{y1}$=4.8, dashed green line - $\xi_{y1}$=7.8.

For $\xi_{y2}$=77, as it is for the SLS storage ring, the effect of second order chromaticity and amplitude dependent tune shift on the value of the first minimum is about 0.4 microns (for kicks smaller than 300 μrad that difference is even smaller) which is too small compared with 20 microns of BPM resolution error. So formula (13) can be used putting in it values of the first maximum and the first minimum.

Extracting the beam energy spread and the synchrotron tune from each BPM signal obtained by TRACY simulation and averaging over all 73 BPMs we obtained 0.000851 and 0.006254 respectively. The mean square deviations (MSD) of these parameters extracted from 73 BMP signals from obtained average values are 1.3% and 0.66% respectively. By the same way the beam energy spread was extracted from the experimental data taken at the SLS storage ring. In Table 2 average values of the beam energy spread and the MSD from them for 3 different values of $\xi_{y1}$ and kick are presented.

Table 2: Average beam energy spread and MSD

| $\Delta y'$ \ $\xi_{y1}$ | | 2.8 | 4.8 | 7.8 |
|---|---|---|---|---|
| 57.51 $\mu rad$ | Av. $\sigma_e$ | 0.000973 | 0.000872 | 0.000763 |
| | MSD | 9.1% | 6.1% | 9.3% |
| 121.2 $\mu rad$ | Av. $\sigma_e$ | 0.000905 | 0.000872 | 0.000827 |
| | MSD | 6.3% | 4.1% | 5.9% |
| 185.7 $\mu rad$ | Av. $\sigma_e$ | 0.000876 | 0.000853 | 0.000845 |
| | MSD | 4.6% | 3.5% | 4.9% |

## CONCLUSIONS AND OUTLOOK

A procedure is developed to measure the beam energy spread and the synchrotron tune relying on an analytical model of transversely kicked beam decoherence signals, which includes amplitude dependant tune shifts and the 1st and 2nd order chromaticities. The estimation of precision of this method is in progress.

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