LATTICE MODELING FOR SPEAR3

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Abstract

We use measured or simulated magnetic fields for dipoles and quadrupoles to build a lattice model for SPEAR3. In a non-symplectic approach the phase space coordinate mapping on the fields is based on numerical integration of the equation of motion. In a symplectic approach we approximate the fields with proper fringe field models. Complication of the use of rectangular gradient dipoles in SPEAR3 is considered. Results of the model is compared to measurements on the real machine.

INTRODUCTION

A lattice model is an indispensable tool for the study of a synchrotron or storage ring accelerator. Typically a machine is built according to the model laid out in the design phase. The magnets are designed, manufactured, tested and installed to realize the design lattice. With today's technology, magnet field measurement and alignment can be achieved with high precision. However, rarely a new machine is turned on to reproduce the ideal model in reality. For example, the betatron tunes and chromaticities would be different. Quadrupole and sextupole magnets have to be adjusted to bring the machine to the desired condition. Furthermore, nonlinear behavior of the model, such as tune dependence on amplitude and high order chromaticities, usually disagree with the real machine to some extent. This could make model-based optimization of nonlinear dynamics useless for operations. Such discrepancies may come from alignment errors, magnet imperfections, cross talking of the magnetic fields of nearby magnets, and fringe field of the magnets.

For the SPEAR3 storage ring, we observed differences in tunes, chromaticities and tune dependence on amplitude between the lattice model and experiments [1]. It was realized that a potential major source of the differences would be the gradient dipoles. The SPEAR3 gradient dipoles are rectangular magnet with straight axis. The bending magnetic field and defocusing strength on the reference trajectory vary with the s-coordinates. Yet the magnets are modeled as gradient sector dipoles with constant bending field and defocusing strength.

In this study we attempt to model the dipoles properly, including its fringe field effect. Our approach is to first make an exact model by tracing particle through the actual magnetic fields. This model is non-symplectic and would be too slow for tracking. But if it reproduces the measured quantities, it can serve as a reference for the development of a faster, symplectic model.

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We will first describe the numerical solver of the Lorentz equation for a general magnetic field. Then the modeling of the SPEAR3 magnets is presented.

AN AT PASSMETHOD FOR GENERAL MAGNETIC FIELDS

For a particle with given magnetic rigidity, position and angle at the entrance face of a magnet, its position and angle at the exit face can be computed by solving the Lorentz equation with the Runge-Kutta method. We implemented an AT [2] passmethod for a general 3-dimensional magnetic field based on this approach. The magnetic field $\mathbf{B}$ of a magnet is described by its three components (B_x,B_y,B_z) as function of the Cartesian coordinates (X,Y,Z). The integration is done on the phase space coordinates (X,$\dot{X}$,Y,$\dot{Y}$,Z,$\dot{Z}$) between the two faces of the magnet. At the entrance and exit faces, these coordinates are transformed to the usual AT coordinates (x,p_x,y,p_y,$\delta p/p$,$c\tau$). Since the entrance and exit faces are usually at an angle with the boundary of the rectangular field area, special treatment is needed for dipole magnets. At the entrance face, the particles are first propagated in drift space to the edge of the field area. Then a rotation transformation is taken for particle coordinates. A reversed procedure is performed at the exit face. For an off-momentum particle, the magnetic rigidity is changed for the integration. The travel time or path length is used to calculate the phase coordinate $c\tau$.

This passmethod is tested using models with known analytical solutions, such as dipoles and quadrupoles with hard edges.

MODELING OF SPEAR3 DIPOLE MAGNETS

SPEAR3 has two types of dipole magnets, the standard bend and the 3/4 bend, which are used in the standard cells and the matching cells, respectively. The two types of magnets have identical design except the latter is shorter to make a reduced bending angle. The dipoles are rectangular gradient dipoles with a straight axis. The reference trajectory is the symmetric trajectory about the center that produces the correct bending angle. Since the reference trajectory curves through a varying bending field, the bend radius is not a constant.

The reference trajectory has been studied to determine the correct alignment requirement and procedure [3]. The measured longitudinal profile was used in that study. The ratio of the dipole component to the quadrupole component, or the position of the virtual center, was assumed to

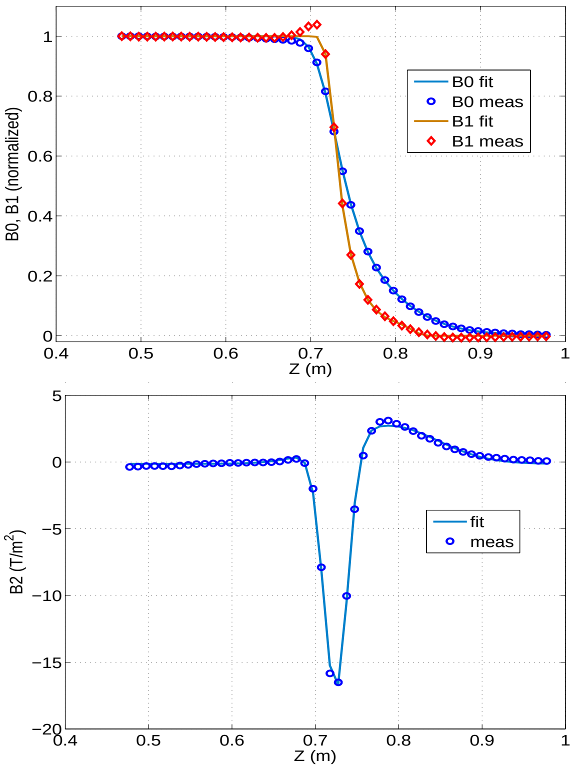

Figure 1: The dipole (B_0), quadrupole (B_1) and sextupole (B_2) components in a standard SPEAR3 dipole.

be a constant along the Z-axis, which was determined by the pole profile. However, coil measurement shows that the ratio of the integrated dipole component to quadrupole component is larger than the value, which indicates the quadrupole component in the end fields is weaker than assumed.

Field measurements

In 2007, a 2-dimensional Hall probe scan was done on the mid-plane at one end of a dipole, which provided details of the magnetic field distribution. Figure 1 shows the dipole, quadrupole and sextupole components along the Z-direction, derived from the measurement by polynomial fittings. The Z-dependence of the B_0 and B_1 components are fitted to the Enge function. The B_2 data are also fitted to a model. The fitting results are also shown. The field integral obtained from the Hall probe measurement agrees with the coil measurement.

The field model

To study the linear and nonlinear dynamics effect of the varying bending radius and the end fields, we made an analytical field model based on the measurement. According to the theory in Ref. [4], the vector potential inside the magnet is expanded to multipoles with longitudinal variations. We keep the first three multipole components. For example, the dipole vector potential is

$$
\begin{aligned}
A_X &= \frac{X^2-Y^2}{4}B_0\Theta_0'(Z), \quad A_Y = \frac{XY}{2}B_0\Theta_0'(Z), \\
A_Z &= -B_0 X\left(\Theta_0(Z) - \frac{1}{8}\Theta_0''(Z)(X^2+Y^2)\right), \quad (1)
\end{aligned}
$$

where B_0 is the maximum value and $\Theta_0(Z)$ is the Z-dependence of the B_y component on the center line. The fitted Enge function is supplied for $\Theta_0(Z)$. The quadrupole component has different Z-dependence which is described by an Enge function $\Theta_1(Z)$. We noted that the vector potential Eq. (1) produces sextupole components in the end fields, which in reality is mostly canceled by the end design of the magnet. In the model we properly choose the $\Theta_2(Z)$ function so that the total sextupole field agrees with the measurement.

The quadrupole component is in a known fixed ratio to the dipole component as defined by the pole profile and verified by measurements. But the sextupole field is not clearly related to the dipole component. The Hall probe measurement did not cover the entire dipole body. Therefore in the model we add a small constant sextupole field component to make the ratio of the integrated sextupole component to the dipole component in the model agree with the coil measurement. Overall, the model yields a magnetic field profile that agrees with measurements on both the X and Z directions.

The absolute value of dipole field was determined by the alignment criterion which specifies the distance of the magnet centerline to the vertex. This field value is found by adjusting it until a reference particle launched at the entrance face with the correct position and angle traces out a symmetric trajectory. The required field integral on the centerline for a 3 GeV beam is 1.86615 T-m. The measured field integral for the operating current is 1.86413 T-m, which was set to according to the early calculation. The calculated field integral would be 1.86420 T-m if we assume a constant B_0/B_1 ratio. Since the operating field integral is lower than the value required for a 3 GeV beam, we believe the actual beam energy in SPEAR3 is 0.1% lower. This is to be tested with beam energy measurement.

MODELING OF SPEAR3 QUADRUPOLE MAGNETS

There are four types of quadrupole magnets in SPEAR3. The field profiles were obtained by simulation with the code Opera3D [5]. The four types of magnets differ only in their iron core length. Their longitudinal fringe profiles are identical. Similar to the dipoles, we make an analytical field model for each type of quadrupoles. The longitudinal profile of the QFC magnet is shown in Figure 2 as an example.

Effects of the fringe field of a quadrupole is studied with this analytical field model by comparing the transfer matrix and second order map to an equivalent hard edge model.

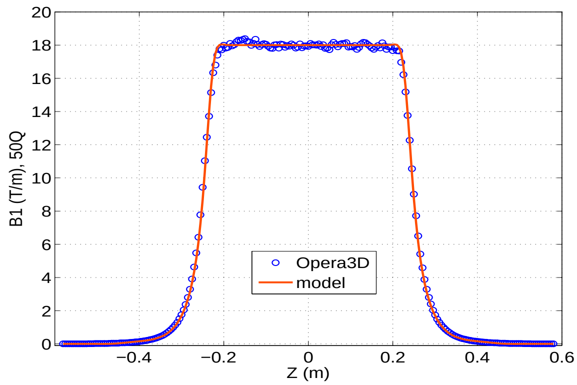

Figure 2: The quadrupole (B_1) component of the QFC magnet at 72 A, calculated with Opera3D or the analytical model.

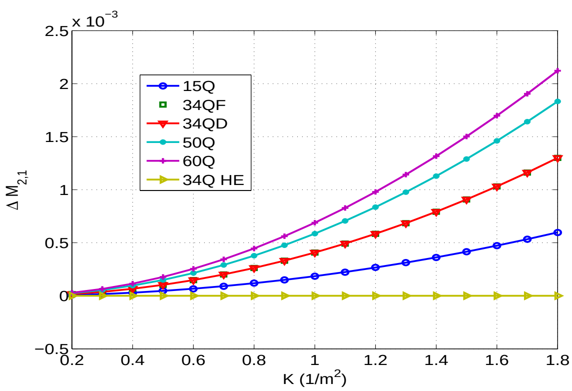

Figure 3: Differences of the transfer matrix element $M_{2,1}$ for the four types of quadrupoles due to fringe field.

Figure 3 shows the differences of the transfer matrix element $M_{2,1}$ as an example between the hard edge models and the field models. The difference between the analytical solution and the hard edge model tracking is zero.

THE SPEAR3 MODEL

The dipole and quadrupole field models are applied to the SPEAR3 lattice model using the field passmethod. The length of the field maps are longer than the present hard edge models. Negative drift spaces are inserted on both ends of these magnets to make the total length of the elements agree with reality: the total path length of the reference particle is equal to the circumference determined by the measured rf frequency and all magnet centers are not shifted. The gradient of the quadrupole magnets are calculated from the operating currents and the calibration curves derived from coil measurements.

The betatron tunes from this lattice model are 14.249 and 6.144 for the horizontal and vertical planes, respectively, while the measurement gives 14.106 and 6.177. The model chromaticities are -0.3 and 0.9, compared to the measured values 1.6 (H) and 2.0 (V) for the two planes. The original lattice model would yield tunes of 14.190 and 6.431. The tune differences between the new and old models come mostly from the dipole model. The vertical tune of the more detailed model is closer to the measurement than the old model because the integrated gradient strength of the dipoles is now more realistic and the edge focusing effect is now properly accounted for. If quadrupole fringe field effects are not included, the tunes are 14.215 and 6.180. A large discrepancy still exists between the horizontal tunes. Possible explanations include alignment errors, especially the feeddown effect of the sextupoles and quadrupole calibration errors.

Using the lattice with the new dipole model but without quadrupole fringe field, we calculated the tune dependence on amplitude after correcting the tunes and chromaticities to the measured values. The result is listed in Table 1 along with results from the old model and measurement [1]. The new model is in much better agreement to the measurement than the old model.

Table 1: Tune dependence on amplitude (1/m) by measurement and tracking with the new and old models.

	$d\nu_x/d\epsilon_x$	$d\nu_x/d\epsilon_y$	$d\nu_y/d\epsilon_x$	$d\nu_y/d\epsilon_y$
meas	1590	2460	2200	2740
new	1760	2335	2257	2862
old	1803	2145	2058	2131

CONCLUSION

We are in the progress of building a lattice model from bottom up to explain our measurement of linear and nonlinear beam dynamics. Analytical field models are built based on measured or simulated field profiles, which are then used for particle tracking. The tunes and tune dependence on amplitude of the new model agree with measurement better than the previous model. Work is in progress to understand the remaining discrepancy and to convert the new dipole model to an equivalent symplectic model for nonlinear dynamics studies.

REFERENCES

[1] J. Safranek and X. Huang, Proceedings of PAC09, Vancouver, Canada, (2009)

[2] A. Terebilo, Proceedings of PAC01, Chicago, (2001)

[3] M. Yoon, et al, Nucl. Instrum. Methods A **523** (2004), 9-18.

[4] M. Bassetti and C. Biscari, Particle Accelerators **52**, 221 (1996).

[5] OPERA-3d Users Guide, Cobham Technical Services, Vector Fields Software, U.K., 2009