LATTICE CALIBRATION WITH TURN-BY-TURN BPM DATA*

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Abstract

Turn-by-turn beam position monitor (BPM) data from multiple BPMs are fitted with a tracking code to calibrate magnet strengths in a manner similar to the well known LOCO code. Simulation shows that this turn-by-turn method can be a quick and efficient way for optics calibration. The method is applicable to both linacs and ring accelerators. Experimental results for a section of the SPEAR3 ring is also shown.

INTRODUCTION

Lattice calibration of an accelerator is the process of adjusting magnet settings to compensate optics errors due to magnet imperfections or misalignment. By restoring the ideal optics with beam-based lattice calibration methods, one can often achieve better injection efficiency, beam lifetime and reliability.

The most widely used lattice calibration technique is LOCO [1], in which quadrupole strengths, BPM gains and corrector gains are fitted to match the model orbit response matrix to the measured one. Turn-by-turn BPM data have been used for optics correction by first measuring beta functions and phase advances with the linear betatron components [2, 3] in the observed beam motion and then fitting the measurements to the model.

In this study we fit turn-by-turn BPM data directly to the lattice model by comparing it to tracking data, without extracting the linear betatron motion. In our view this is the natural approach for lattice calibration with turn- by-turn data because there is no loss of information in this process. Skew quadrupoles and sextupoles can be included as fitting parameters since all linear coupling and nonlinear motion are preserved in the raw data. BPM gains can be fitted independent of the correlation with the beta functions.

The transverse phase space coordinates at one location are needed to generate tracking data. We first describe the method for phase space coordinate measurements and its application to transfer matrix measurement. The fitting scheme is explained and then demonstrated with simulation and experimental results.

MEASUREMENT OF PHASE SPACE VARIABLES AND TRANSFER MATRICES

Coordinates x' and y' can be measured with two BPMs that are separated by only a drift space,

$$x'_{1,2} = (x_2 - x_1)/L, \qquad y'_{1,2} = (y_2 - y_1)/L, \quad (1)$$

where $x_{1,2}$ and $y_{1,2}$ are horizontal and vertical positions measured at the two BPMs, with BPM 1 located upstream of BPM 2.

When turn-by-turn phase space coordinates at two locations are known, the transfer matrix between them can be readily obtained. Let $\mathbf{X}_i$ be the $4 \times N$ matrix containing turn-by-turn phase space coordinates at location i for N turns with row 1 to 4 being x,x',y,y', respectively. Then the transfer matrix from location 1 to 2, $\mathbf{M}_{21}$, satisfies $\mathbf{M}_{21}\mathbf{X}_1 = \mathbf{X}_2$. A least-square solution for $\mathbf{M}_{21}$ is $\mathbf{M}_{21} = \mathbf{X}_2\mathbf{X}_1^T(\mathbf{X}_1\mathbf{X}_1^T)^{-1}$. But the resulting matrix is usually non-symplectic because of errors in the data.

The symplectic transfer matrix can be obtained by a fitting method that imposes symplecticity. Adopting the parameterization scheme of Ref. [4] for the coupled transfer matrix, the transfer matrix is constructed from 10 free parameters p_i,$i = 1, 2, \cdots, 10$ by

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} p_1 & p_2 \\ \frac{p_1p_3-1}{p_2} & p_3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} p_4 & p_5 \\ \frac{p_4p_6-1}{p_5} & p_6 \end{pmatrix}, \\ \mathbf{C} &= \begin{pmatrix} p_7 & p_8 \\ p_9 & p_{10} \end{pmatrix}, \quad \mathbf{C}^+ = \begin{pmatrix} p_{10} & -p_8 \\ -p_9 & p_7 \end{pmatrix}, \end{aligned} \quad (2)$$

and using Eq. (2-4) of Ref. [4]. The merit function may be defined by

$$\chi^2(p) = \sum_{i=1}^{4}\sum_{n=1}^{N} \frac{(\tilde{X}_2(i,n) - X_2(i,n))^2}{\sigma_i^2}, \quad (3)$$
$$\text{with} \quad \tilde{\mathbf{X}}_2 = \mathbf{M}_{21}\mathbf{X}_1,$$

where $X(i,n)$ is the $(i,\ n)$ element of matrix $\mathbf{X}$, $\tilde{\mathbf{X}}$ represents the predicted coordinates, σ_i, the rms noise level for coordinates x,x',y and y', respectively. The non-symplectic matrix can be used to generate the initial parameters required for the fitting.

This method can be used to obtain the one-turn transfer matrix. In this case phase space coordinate measurements at only one location is needed. The data matrix for turns $n = 2, 3, \cdots, N$ and that for $n = 1, 2, \cdots, N-1$ serve as $\mathbf{X}_2$ and $\mathbf{X}_1$, respectively. It is worth noting that this method for transfer matrix measurement is completely model independent.

LATTICE CALIBRATION WITH TURN-BY-TURN BPM DATA

With the initial transverse phase space coordinates at the entrance of an accelerator section and a lattice model, the beam positions at downstream BPMs can be predicted by tracking. The lattice model can be calibrated by comparing the tracking results to measurements. This idea

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can be illustrated with a simple case depicted in Figure 1, in which the accelerator section consists of one thin quadrupole and two drift spaces. Transverse phase space coordinates (x_1,x_1',y_1,y_1') at BPM 1 are related to readings of BPM 2 (x_2, y_2) through elements of the transfer matrix between BPM 1 and 2. Applying the least-square method to multiple-pass data, one can derive that

$$[KdL] = \frac{\sum_n (x_2 - x_1 - (L_1+L_2)x_1')(x_1 + L_1 x_1')}{L_2 \sum_n (x_1 + L_1 x_1')^2} \quad (4)$$

where $[KdL] = \frac{1}{B\rho}\int \frac{dB_y}{dx} ds$ is the integrated gradient of the quadrupole; the summation over n is for many passes.

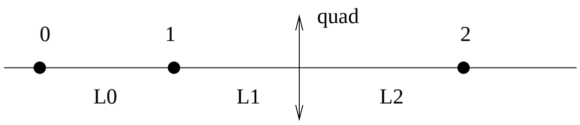

Figure 1: Calibration of one quadrupole with three BPMs.

For a general accelerator section that consists of multiple magnets and downstream BPMs, an explicit solution may not be available. However, fitting techniques can be employed to obtain the magnet strengths by adjusting their values in the model to minimize the differences between the measured and predicted beam positions.

We assume the 4-dimensional phase space coordinate at the entrance of an accelerator section can be obtained through two BPMs and that these two BPMs are very well characterized and calibrated so that there are no roll or gain errors. The other BPMs, however, may have roll or gain errors. Given the initial phase space coordinates $\mathbf{X}_1 = (x_1,x_1',y_1,y_1')$, the predicted observations at downstream BPMs are obtained by first tracking such a particle and then applying BPM gains and rolls. Suppose the accelerator section has M BPMs and P magnets to be calibrated. The target function to be minimized is

$$\begin{aligned}\chi^2 &= \sum_{n=1}^{N}\sum_{i=2}^{M+1}\left(\frac{x_i(n) - \tilde{x}_i(\mathbf{p};\mathbf{X_1(n)})}{\sigma_{xi}}\right)^2 \\ &\quad + \left(\frac{y_i(n) - \tilde{y}_i(\mathbf{p};\mathbf{X_1(n)})}{\sigma_{yi}}\right)^2, \qquad (5)\end{aligned}$$

where N is the number of passes or turns, $i = 2, \cdots, M+1$ is for the BPMs, $\mathbf{p}$ is a vector of the fitting parameters, $\sigma_{xi,yi}$ are horizontal and vertical noise level for BPM i, $\tilde{x}$, $\tilde{y}$ are predicted BPM readings and x,y are actual observed coordinates. The fitting parameters include the strengths of the magnets to be calibrated and BPM roll and gains. Therefore the fitting problem is to look for $P + 3M$ parameters from $M \times N$ data points. There is considerable redundancy in the turn-by-turn data since they represent the same optics. Data from each BPM provide additional sampling of the optics. The number of BPMs needs to be equal to or larger than the number of magnet parameters to have sufficient constraints. This fitting problem can be solved by standard nonlinear least-square algorithms, e.g., the Levenberg-Marquadt method. This fitting scheme is directly applicable to a circular accelerator because it can be seen as a transport line.

The predicted beam trajectory in Eq. (5) is based on measured initial coordinates which inevitably contain noise. This noise is propagated downstream and causes errors in the predicted trajectory. However, since the noise is random, its relative importance decreases with a large number of samples.

EXPERIMENTS ON SPEAR3

We conducted experiment on the SPEAR3 storage ring. In the experiment we connected 8 BPMs in and around a standard cell to Echotek electronics to achieve turn-by-turn capability. The configuration is shown in Figure 2. There is no insertion device in the first straight section. The wiggler between BPMs [11,6] and [12,1] was fully open during the experiment.

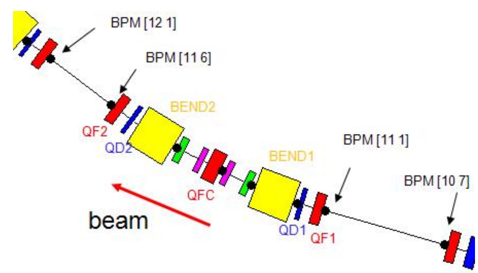

Figure 2: The SPEAR3 cell for the experiment.

Horizontal motion was excited with an injection kicker. Vertical motion was resonantly driven by a sinusoidal signal on a stripline which was stopped when the kicker was fired [5]. Free motion on both planes were then recorded. The one-turn matrix at BPM [12,1] was measured and used to calculate the transformation between the raw coordinates and the normal mode coordinates. The raw and normal mode phase space coordinates at the BPM are shown in Figure 3.

The lattice model was fitted with the turn-by-turn data according to Eq. (5). The fitting parameters were 5 quadrupoles, horizontal and vertical gains and rolls of 6 BPMs. BPM data of 200 turns for both planes were used in fitting. The rms noise of BPM readings are estimated to be about 0.020 mm for both planes. To test the ability to recover quadrupole errors, offsets are added to the initial quadrupole values. The merit function χ^2/DF drops from roughly 150 to 20 when it converges. The fitted quadrupole strengths and statistical errors from 5 data sets are listed in Table 1 along with their design and LOCO values. The correlation exists between neighboring QF and QD magnets. Consequently there may be systematic errors in the fitted solution. This is especially true for the second pair of QD and QF magnets as only two BPMs are downstream of them. For these two magnets the initial values were set to the design values without offsets.

Table 1: Quadrupole parameters

Quad	design	LOCO	fitted	rms	initial
QF1	1.823	1.824	1.810	0.001	1.883
QD1	-1.920	-1.922	-1.911	0.001	-1.890
QFC	1.683	1.683	1.694	0.002	1.653
QD2	-1.347	-1.331	-1.344	0.003	-1.347
QF2	1.691	1.686	1.691	0.002	1.691

The fitting results agree with LOCO reasonably well. The LOCO result may be more reliable in this case since it utilizes 60 BPMs (including all turn-by-turn BPMs in this experiment) around the whole ring. Each quadrupole parameter is constrained by more data samples at different locations. However, if turn-by-turn data from the same number of BPMs are used in a global fit, the result should be more or less equally reliable. This is tested in simulation in the next section.

SIMULATION WITH A FULL RING

In the simulation we use the 57 operational BPMs to generate turn-by-turn data. BPMs [10,7] and [11,1] are used to derive 4-dimensional phase space coordinates.

The model parameters to be fitted are the same as the standard LOCO setup. Magnets that share a power supply are treated as one parameter. There are a total of 72 quadrupole parameters and 13 skew quadrupole parameters. Rolls and gains of the two initial BPMs are not fitted. There are $85 + 55 \times 3 = 250$ fitting parameters in total.

Simulated data are obtained by launching a particle with initial horizontal and vertical offsets which is then tracked for a number of turns with the code AT [6]. A set of gain (2% rms) and roll (12 mrad rms) errors are created and applied for all BPMs except the two initial BPMs. Gaussian noise (50 μm rms) is added to horizontal and vertical coordinates for all BPMs.

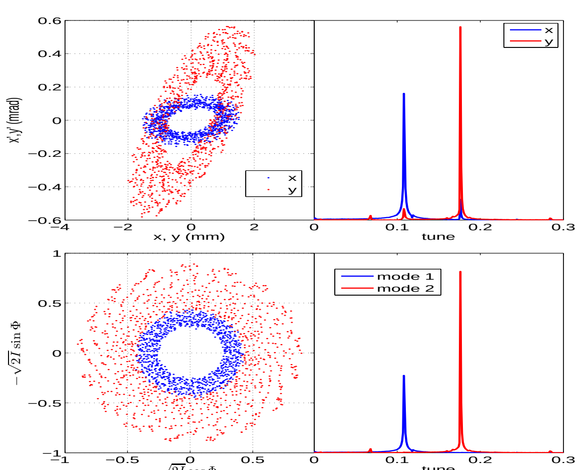

Figure 3: The phase space plots for the raw data (top) and decoupled modes (bottom) a BPM [12,1].

To test the algorithm we generated an uncalibrated lattice by adding errors to strengths of some magnets. The initial horizontal and vertical offsets are both 2 mm at a location with moderate beta functions. Tracking data of 200 turns are used in fitting. The fitting algorithm starts with the standard lattice and recovers all artificial errors. All BPM gain and roll errors are found with high precision. The results are shown in Figure 4. The fitted values and their error bars are the averages and standard deviations of fitting results from 10 random BPM noise seeds. The fitted parameters are found to agree with the expected values very well except large error bars for the QD parameters, which is inherent to the lattice model and is also observed in LOCO results [7].

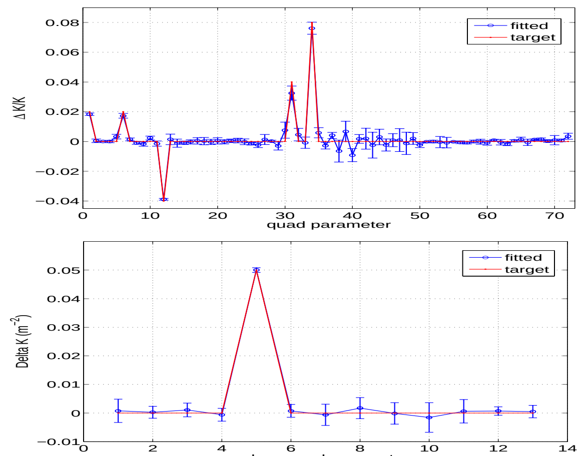

Figure 4: The fitted quadrupole (top) and skew quadrupole parameters (bottom) are compared to the expected values.

CONCLUSION

We describe a method to calibrate the lattice of a ring or a transport line using the single-pass BPM data. The method is demonstrated with simulation and experiment using the SPEAR3 storage ring.

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