BEAM POLARIZATION THEORY AND ITS APPLICATION TO HLS STORAGE RING*

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Abstract

A brief, but clear, review of beam polarization theory is given in the paper. Particularly, the algorithm of spin linear transfer matrix (SLIM) is applied to remark the situation of beam in storage ring, specific to HLS (Hefei Light Source). Theoretical analysis indicates that the beam in HLS, working at 800MeV and 2.58/3.58 transverse tunes, could keep away from a variety of spin resonances, and should be able to build up high polarization.

INTRODUCTION

It was first predicted by Ternov and rigorously justified by him and Sokolov (1962) that electrons cycling at high energy in a storage ring would become naturally polarized through the emission of spin-flip synchrotron radiation. The radiative self-polarization effect is now called Sokolov-Ternov effect. A quantitative solution for motion in a homogeneous field, by solving the Dirac equation, was given by Sokolov and Ternov (1964) [1]. Later, Derbenev and Kondratenko gave a detailed formula for the equilibrium degree of polarization [2]. S. R. Mane coming after them used semiclassical QED method to rederive the formula of equilibrium polarization [3]. Additionally, an excellent review of the electrodynamics of spin-flip synchrotron radiation has been given by Jackson [4]. A detailed presentation about these processes and a useful simulation algorithm using linear transfer matrix method are contributed by A. W. Chao [5, 6].

The polarization of a beam is actually an equilibrium physical status with spin-flip radiative polarizing effect and depolarizing effect of quantum diffusion working simultaneously. In order to obtain specific value of equilibrium polarization of a beam, one needs to calculate the spin-flip transition rates as well as the quantum diffusion rate of spin orientation. Extraordinarily, a special situation is that the spin diffusion rate becomes rather large when so-called depolarization resonances are encountered. This extends out two important projects: first is that, for an objective to achieve or maintain a high polarized beam one should study on how to cross the depolarization resonances while the beam is accelerated or decelerated; second is that, intentionally take place the depolarization resonance so as to use it to calibrate energy of particle beam accurately. Anyhow, both of them need to study on polarization dynamics and require a polarized beam. So some numerical simulations of polarization seem very necessary. Presently, some simulation codes are available such as SLIM [6], SMILE [7], and SITROS [8] and so forth. We use SLIM developed by A. W. Chao to study the situation of beam in HLS storage ring. Before this, the theory of polarization dynamics is reviewed briefly so as to understand the algorithm with less difficulty. In the end, the simulated result is remarked and the conclusion is given.

* Work supported by the National Natural Science Foundation of China under Grant No. 10875118

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POLARIZATION DYNAMICS

Single-Particle Spin Dynamics

The Hamiltonian for a relativistic electron moving in an electromagnetic field, up to the linear order in $\vec{A}$, is

$$H=(m^2c^4+c^2\vec{p}^2)^{1/2}+e\Phi-e\vec{\beta}\cdot\vec{A}+\vec{\Omega}\cdot\vec{S}=H_f+H_{int} \quad (1)$$

where m is the proper mass of electron with canonical momentum $\vec{p}$ and $e<0$ is its charge; $\vec{\beta}$ is the electron velocity in units of light velocity c and $\vec{S}$ is the spin; Φ and $\vec{A}$ are the electromagnetic potentials. The interactive Hamiltonian is separated into two terms

$$H_{int}=H_{ext}+H_{rad} \quad (2)$$

while radiation filed is treated as a perturbation. Here the subscripts signify external field and radiation field respectively. The term, H_f+H_{ext}, without interacting with radiation field is called unperturbed term. Both H_{ext} and H_{rad} have the form $e(\Phi-\vec{\beta}\cdot\vec{A})+\vec{\Omega}\cdot\vec{S}$, where

$$\vec{\Omega}=-\frac{e}{m\gamma c}\left[(a\gamma+1)\vec{B}-\frac{a\gamma^2}{\gamma+1}\vec{\beta}\cdot\vec{B}\vec{\beta}-(\frac{\gamma}{\gamma+1}+a\gamma)\vec{\beta}\times\vec{E}\right] \quad (3)$$

is the Thomas precession vector mentioned in reference [9], with γ the Lorentz factor and a the anomalous magnetic moment of electron. Besides, $\vec{E}$, $\vec{B}$ are the electromagnetic fields in the accelerator. The distinction between two interactive Hamiltonian is indicated by adding subscripts to corresponding terms with "ext" and "rad". The term (3) rotates the spin of electron, causing its variation with time (Thomas-BMT equation):

$$\frac{d\vec{S}}{dt}=\vec{\Omega}\times\vec{S} \quad (4)$$

Equations (3) and (4) lay a foundation of single-particle spin dynamics in classical field theory.

On the other hand, the perturbation theory says that the probability amplitude for an electron, initially in the state $|i\rangle$, to be found in the state $|f\rangle$ is given by

$$p_{fi}=\frac{1}{i\hbar}\int_{-\infty}^{\infty}dt\langle f(t)|H_{rad}(t)|i(t)\rangle \quad (5)$$

where the interactive Hamiltonian with radiation field has been given ahead with subscript "rad". Since the radiation power is in proportion to the probability $|p_{fi}|^2$, we can derive the instantaneous radiation power based on the specific form of interactive Hamiltonian H_{rad}.

Come next, the classic instantaneous radiation power formula needs two aspects of modification when quantum processes are considered. First is the recoil effect of photon emission; the other is the consideration of the electron's spin. Particularly the second aspect of quantum modification is somewhat complicated that one should distinguish two cases: whether the spin of electron flips or not after emitting photon. In the case of no spin-flip, the quantum-modified expression has the form [5]:

$$\wp_{modified} = \wp_{classic} - \wp_{classic}\left(\frac{55}{16\sqrt{3}} + \frac{1}{2}\hat{n}\cdot\hat{y}\right)\frac{\hbar\omega_c}{E} \tag{6}$$

Here $\wp_{classic}$ is the classical term what is familiar to us. Other two terms refer to the two quantum modifications mentioned above respectively, while $\hat{n}$ stands for the unit vector in the direction of electron's spin, $\hat{y}$ the unit vector in the direction of prescribed bending field, and ω_c the critical frequency of radiation. The more significant thing here should be the radiation power with spin-flip:

$$\wp_{spin-flip} = \hbar\omega \mathrm{W} \tag{7}$$

where $\hbar\omega$ is the energy of emitted photon, and

$$\mathrm{W} = \frac{5\sqrt{3}}{16}\frac{e^2\gamma^5\hbar}{m^2c^2\rho^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{8}{5\sqrt{3}}\hat{n}\cdot\hat{y}\right] \tag{8}$$

is the instantaneous transition rate of spin-flip [5], with ρ the instantaneous bending radius and $\hat{v}$ the unit vector of the electron velocity. It's easy to see, the spin-flip radiation is weak and is hard to detect. But there is a phenomenon that involves spin-flip does more easily to observe. That is the "spin-flip radiative polarization".

Since the spin possesses only two possible values along any direction, a quantization axis pointing towards $\vec{\Omega}$ needs to be chosen to diagonalize the unperturbed Hamiltonian and get the stationary spin eigenstates of the system. In the case of a planar ring, the spin quantization axis is the same for all the particles in the vertical direction, that is $\hat{n} = \pm\hat{y}$ for the above expressions and we can see obviously the spin-flip is asymmetric what causes polarization of the beam. Normally, the quantization axes are not vertical but symmetrically distributed about the vertical axis of equilibrium closed orbit when imperfect ring is considered. Derbenev and Kondratenko [2] introduced a variable vector $\hat{n}(\vec{r},\vec{p})$ as the spin quantization axis. Since $\vec{\Omega}$ depends on the specific orbital trajectory of the particle so does the quantization axis. Here $\vec{r}$ and $\vec{p}$ describe the electron's motion state with respect to ideal one. They form a six-dimensional phase space of the electron motion. The vector $\hat{n}(\vec{r},\vec{p})$ has great significance in calculation about polarization.

Multi-Particles Spin Polarization Dynamics

In order to apply results of single-particle spin dynamics onto electron beam, polarization density matrix is needed. The polarization density matrix of a beam with N electrons can be specified by a three-component vector

$$\vec{P} \equiv P\hat{P} = \frac{2}{\hbar}\cdot\frac{1}{N}\sum_{i=1}^{N}\vec{S}_i = \frac{1}{N}\sum_{i=1}^{N}\left\langle\hat{S}\cdot\hat{n}_i\right\rangle\hat{n}_i = \left\langle\left\langle\hat{S}\cdot\hat{n}\right\rangle\hat{n}\right\rangle \tag{9}$$

where $\vec{S}_i$ represents the classical spin vector, while $\hat{S}\cdot\hat{n}_i$ denotes the spin projection operator of individual electrons. The term $\left\langle\hat{S}\cdot\hat{n}_i\right\rangle$ can be understood as the spin expectation value of each electron, and the outer angular brackets denote an average over the wholeness of electrons. The polarization vector characterizes the "relative orientation" of spins of the electrons in a beam.

Now restrict ourselves to equilibrium situation, what is more interested, we get the equilibrium polarization

$$\vec{P}_{eq} = \left\langle\left\langle\hat{S}\cdot\hat{n}\right\rangle\right\rangle\left\langle\hat{n}\right\rangle = \left\langle\left\langle\hat{S}\cdot\hat{n}\right\rangle\right\rangle\left\langle\hat{n}\cdot\hat{P}_{eq}\right\rangle\hat{P}_{eq} \tag{10}$$

where $\hat{P}_{eq}$ is the unit vector in the direction of $\langle\hat{n}\rangle$, that is the direction of polarization. Make use of the dynamic equilibrium condition of spin-flip process, one would eventually obtain the reasonable approximation:

$$\vec{P}_{eq} = \frac{\left\langle \mathrm{W}_{\downarrow\uparrow} - \mathrm{W}_{\uparrow\downarrow}\right\rangle}{\left\langle \mathrm{W}_{\downarrow\uparrow} + \mathrm{W}_{\uparrow\downarrow}\right\rangle}\hat{P}_{eq} \tag{11}$$

Here $\mathrm{W}_{\uparrow\downarrow}$ means the transition rate from spin up to spin down along the quantization axis $\hat{n}$ and vice versa.

Try back to the thinking of the recoil of photon emission, the electron's energy would change $\delta E = -\hbar\omega$ what means the quantization axes of the spin before and after emitting photon are not strictly parallel. Use $\hat{n}_i$ and $\hat{n}_f$ to denote the quantization axes before and after emitting photon, then one has

$$\hat{n}_f \simeq \hat{n}_i + \delta E\frac{\partial\hat{n}}{\partial E} = \hat{n}_i + \frac{\delta E}{E}\left(\gamma\frac{\partial\hat{n}}{\partial\gamma}\right) \tag{12}$$

where $\gamma\frac{\partial\hat{n}}{\partial\gamma}$ is another significant quantity in radiative polarization that involves the spin diffusion rate. It is called spin chromaticity function. Hereto, we quote the end result, Derbenev-Kondratenko formula [2], for the equilibrium degree of the radiative polarization:

$$P_{eq} = \frac{8}{5\sqrt{3}}\frac{\left\langle\oint\frac{d\theta}{|\rho|^3}\hat{b}\cdot\left[\hat{n} - \gamma\frac{\partial\hat{n}}{\partial\gamma}\right]\right\rangle}{\left\langle\oint\frac{d\theta}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left|\gamma\frac{\partial\hat{n}}{\partial\gamma}\right|^2\right]\right\rangle} \tag{13}$$

where $\hat{b}$ is the binormal vector of the orbit. The integral over θ is around the circumference of the ring and the angle brackets are over the orbital phase space. The expression is widely used to determine the equilibrium polarization of electron beam. One should always grasp the two important quantities, spin quantization axis and spin chromaticity function, in the expression. They contain all the detailed information of the accelerator structure.

THE SLIM ALGORITHM AND ITS APPLICATION TO HLS

SLIM [6] is an algorithm which uses thin lens approximation to calculate the spin polarization of a beam in an electron storage ring. It firstly obtains the equilibrium closed orbit. Then the polarization directions, namely quantization axes, along the particle orbit are calculated. After that, the 8×8 transfer matrices are constructed and the spin chromaticity functions along the ring are evaluated. At last, the equilibrium polarization is worked out according to the expression (13).

First of all, the input parameters should be converted into corresponding ones of thin lenses before the simulation is carried out. Since errors are brought in when each thick lens is replaced with thin lens, there would be accumulated some considerable change of the lattice in a superperiod. So the parameters of thin lenses should be further matched that its lattice in common with the thick lens lattice as much as possible. The following two figures show the results of matching and simulation.

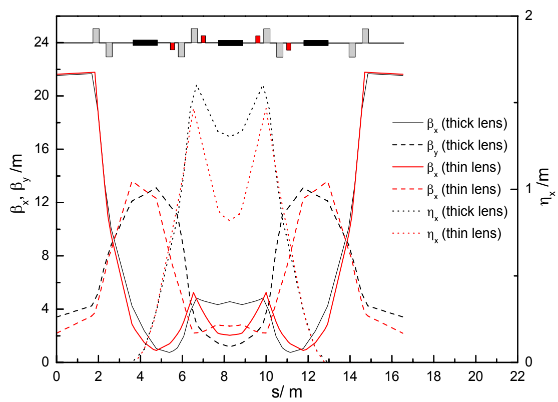

Figure 1: Comparison between thick and thin lens lattices.

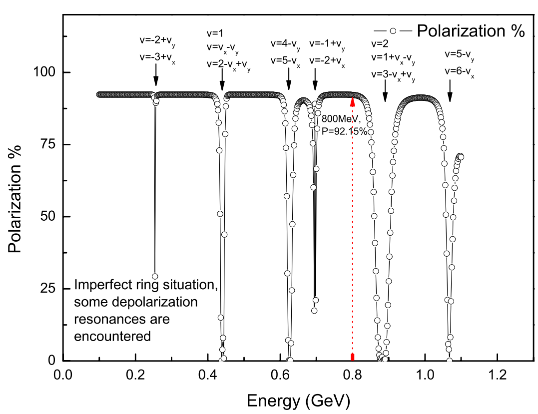

Figure 2: Polarization versus energy.

Fig. 1 is the comparison between thick lens lattice and thin lens lattice that has been matched. Fig. 2 is the calculation result of polarization versus energy when a random distribution of vertical orbit kickers is introduced to simulate field imperfections.

RESULTS AND DISCUSSION

From Fig. 1 we can see, the thin lens lattice, after matched, is on the whole in accordance with thick lens lattice though there are some distinctions in some places. Specially, dispersion function of the matched thin lens lattice is a little lower than the thick one in the middle region of a period. This would result in a bit decrease of the momentum compact factor. Never mind! Note here the transverse tunes are the most important things what we need accurately to analyze the frequency spectra of depolarization resonances. The small discrepancy of lattice functions in some places is admissible.

With this matched thin lens lattice, the polarization versus beam energy is simulated as showed in Fig. 2. A red dashed line, in 0.8 GeV position, is marked out in the figure. The corresponding polarization is 92.15%. The beam in HLS works at 800 MeV and 2.58/3.58 transverse tunes while the spin precession frequency is 1.8155. The theoretical analysis shows, the beam keeps away from a variety of spin depolarization resonances and is able to build up high polarization.

Furthermore, the specific types of resonances at some energy are designated in the corresponding locations in Fig. 2. We can see the integer resonances occur at the energies near 0.44 GeV and 0.88 GeV. We also note that, since HLS works at difference resonance tunes, each place of depolarization consists of at least two resonances. The superposition of resonances would enlarge the strength of depolarization.

CONCLUSION

In conclusion, we first reviewed the dynamics of radiative polarization by starting with the single-particle spin dynamics. Then with the generalization to multi-particles spin dynamics, as well as the spin-flip processes considered, the polarization conclusion was seen to be obviously. After that, an algorithm of spin transfer matrix was used to simulate the situation of beam polarization in HLS. In the calculations about equilibrium polarization, two quantities, spin quantization axis and spin chromaticity function, are extremely important. An appropriate understanding about these two quantities is necessary to catch on polarization theory. One should always bear in mind that, they are vector fields but not constant vectors. In the end, the simulation result was remarked. We got the conclusion that the beam in HLS, working at 800MeV and 2.58/3.58 transverse tunes, could build up high polarization.

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