POTENTIAL FORMS FOR ELECTROSTATIC AND MAGNETIC CYLINDRICAL LENS AND TRACKING OF CHARGED PARTICLE

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Abstract

A cylindrical lens is mainly used for focusing and transporting low energy beam. Some analytical forms of scalar potential have been formulated to evaluate electric and magnetic field and its derivatives on the central axis, which help in evaluation of potential and field in the region about the central axis. They are, subsequently, used to analytically find out the optical properties of the lens as well as in tracking of charged particles through section method in which a thick lens is divided into large number of thin weak lenses arranged transverse to the central beam line. The field computation technique with given lens parameters and section method of calculating optical parameters of thick lens described in the paper turn into a tool to design an electrostatic or a magnetic cylindrical lens with more accuracy as per the requirements.

INTRODUCTION

Either electrostatic or magnetic cylindrical lenses are used for focusing low energy beam from an ion source. To evaluate the optical parameters of these lenses, potential and field distribution is needed either by analytical or numerical methods. In absence of closed form analytical expressions for the distribution, people mostly use numerical method to calculate the potential and field distribution with good accuracy.

It is easier to construct a solenoid lens with a cylindrical symmetry and also it does not have any break-down problem except entangling of the beam emittances in x- and y-planes because of the rotation of the beam about the axis. If the initial beam to be transported is round then entangling of the sub-phase spaces in (x-z) and (y-z) plane is of less concern. Using the same simple potential form, field can be evaluated for two cylinder lens for immersion type lens.

We shall consider ways of generating the potential distribution for lens with two electrodes (two equal ½-parts of a solenoid). If the electrode potentials are V_1 and V_2, we can express the axial potential in the form of eq. (1), which satisfy the Laplace equation. The potential and the field distribution can also be known using eq. (2) around the central z-axis inside a lens. The first term in eq. (1) is vanishes if V_1=V and V_2=-V.

The field is evaluated from $\xi(r,z)=-\mathrm{grad}(\varphi(r,z))$ and expression for $\varphi(0,z)$ is obtained for electrostatic (ES) (magnetostatic (MS)) configuration of lens parameters like diameter D, spacing S, applied electro(magneto)-motive-force on the right and left conductors (poles) in Fig. 1 are V (NI/2) and -V (-NI/2) respectively.

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$$\varphi(z) = (V_1+V_2)/2 + (V_2-V_1)/2\ \varphi(z) \qquad (1)$$

$$\varphi(r,z) = \sum_n [(-1)^n/(n!)^2](r/2)^{2n}\varphi^{(2n)}(z) \qquad (2)$$

POTENTIAL AND FIELD FORMS

A ES cylindrical (MS Glaser solenoid) lens fits inside (on) a non-magnetic metal beam pipe. The lens parameters set the design of the lens of certain focal length, F for a given beam of particular average energy and Bρ. The iron shielded (hatched area) Glaser solenoids lens are shown in the middle of Fig. 1 below.

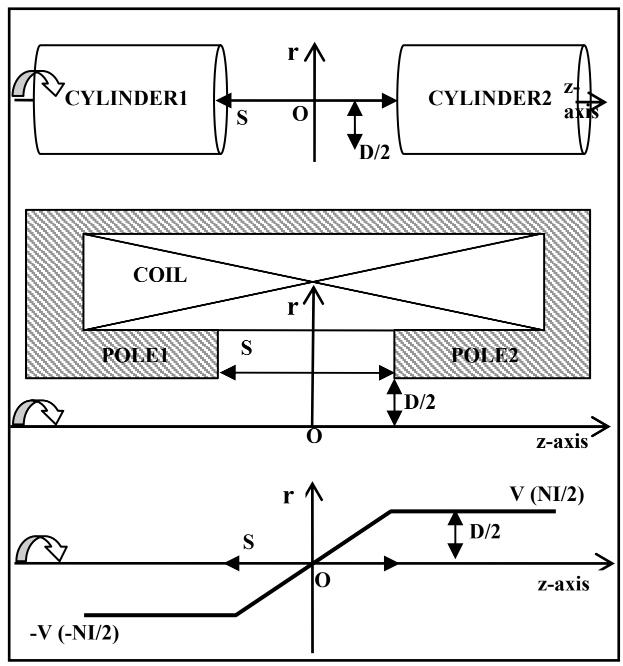

Figure 1: Sketch of ES Cylinder lens of immersion type on the upper figure, Glaser solenoids with poles in the middle and electro(magneto)-motive-force, M_F charged on the cylinders (poles) as shown in the lowest figure.

Electric and Magnetic Scalar Potential

We assume the origin of the z-axis at the centre of the gap between the cylinders (mid-length of the solenoid). We will describe here two combined ES and MS forms of scalar potential along the axis represented by φ_1 [1(p.285), 2(p.36)] and φ_2 [1(p.49), 2(p.38)], which depend on the geometry of cylinderical lens (solenoid) and electro(magneto)-motive-force M_F.

$$\varphi_1(z) = \frac{M_F D}{\pi S}\int_0^\infty \frac{\sin(S\cdot r/D)\cdot\sin(2\cdot r\cdot z/D)}{r^2\cdot I_0(r)}dr \qquad (3)$$

Where $I_0(x)$ is the modified Bessel function of first kind and order zero. The integration is in radial direction using the dummy variable r.

$$\varphi_2(z) = \frac{M_F D}{2\pi S}\left\{z_+ \cdot \tan^{-1}(z_+) - z_- \cdot \tan^{-1}(z_-)\right\} \quad (4)$$
$$z_+ = (2z+S)/D, \quad z_- = (2z-S)/D$$

Where $\varphi_2(z)$ is a monotonic function of z and goes to ±1 as z approaches $\pm\infty$. It is deduced from electric scalar potential form given by Szilagyi [3] for two apertures at the two ends. This uses a superposition of the potentials for two thin apertures with equal diameter D separated by a distance S.

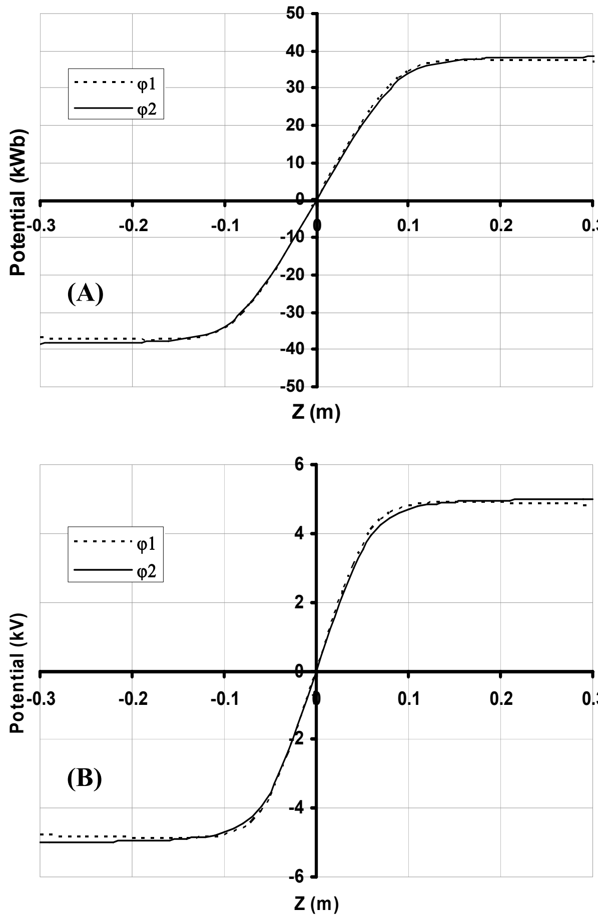

Figure 2: Plot of electric(A) and magnetic (B) scalar potentials on the central z-axis.

Electric Field and Magnetic Induction

The radial component of the induction is given by $\xi_r(r,z) \approx (r/2)(\partial^2\varphi(z)/\partial z^2)$, which is used to derive $\xi_z(r,z)$ using the azimuthal **curl**, $(\partial\xi_z/\partial r - \partial\xi_r/\partial z) = 0$, which is expressed in series form by eq. (2). The field, $\xi(z) = -\partial\varphi(z)/\partial z$ corresponds to electric field if $M_F = V_2 - V_1$ and magnetic inductions, $B(z) = -\mu_0\xi(z)$ and M_F=NI Amp-turn of the solenoid on the z-axis deduced from the corresponding φ's are given by eqs. (5) and (6), which are utilized to obtain the off-axis field distribution and the particle optics in the lens. The potentials and the fields along the axis of the cylinder (solenoid) for S=D=0.1m and M_F=10kV (80kA-turn) are shown in Fig. 2 and 3 respectively.

$$\xi_1(z) = \frac{-M_F}{\pi S}\int_0^\infty \frac{2\sin(S\cdot r/D)\cdot\cos(2\cdot r\cdot z/D)}{r\cdot I_0(r)}dr \quad (5)$$

$$\xi_2(z) = \frac{-M_F}{\pi S}(P-Q) \quad (6)$$

$$P = \frac{z_+}{1+z_+^2} + \tan^{-1}(z_+) \quad (6a)$$

$$Q = \frac{z_-}{1+z_-^2} + \tan^{-1}(z_-) \quad (6b)$$

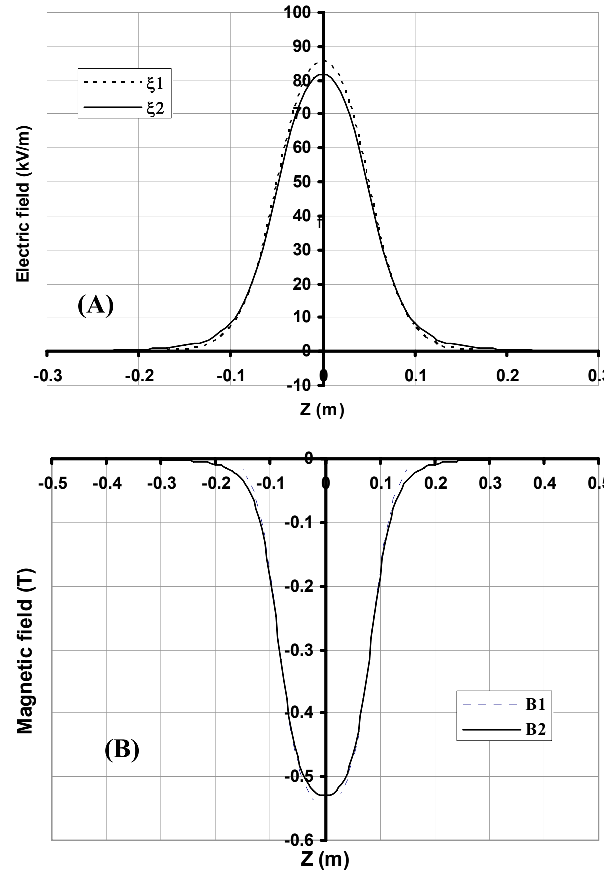

Figure 3: Plot of electric field (A) and magnetic induction (B) on the central z-axis.

PARTICLE TRACKING IN LENSES

We know that a charged particle moving along the central axis para-axially in a cylindrical ES lens, the particle gets radial kick $qE_r(z)$ inward or outward as $\varphi''(z)>$ or <0 respectively. The particle get inward or outward kick at the rising or falling B(z) region respectively when it moves in the direction of B(z). The motion of the particle is properly described by eq. (7) in both electric and magnetic cases. In the electric case $T(z) = (\sqrt{3}\varphi'(z))/(4\varphi(z))$ and $R = r\varphi^{1/4}(z)$, which is called the reduced ray. The reduced ray maches to the actual ray when optical evaluation is done for sectionized thin weak lenses. In the magnetic case $T(z) = B(z)/(2B\rho)$, where $B\rho$

and R=r are the beam rigidity and the actual radial position respectively.

$$R'' + T^2(z)R = 0 \qquad (7)$$

The lenses are generally thick but we have assumed a thin weak lens for proper formulation and description of the optical properties of the lens for para-axial rays for which the radial change inside the lens is assumed to be negligible. So, the thick lens is sectionized into a large number of thin lenses of width 'd' in which the above assumptions are valid. The solution of the eq. (7) is given in matrix form in eq. (8).

Smooth profiles of the field and potential along the axis are divided into large number of small stepped profile. Each step represents a weak thin lens as change in radial movement is very small. The effect of the individual weak lenses is evaluated and combined by matrix multiplication method to get optical property of the thick lens. The total focal length due to all the thin lenses (for magnetic solenoid lens, [4]) including a drift space of width 'd' in between the thin lenses gives very accurate tracking of particles in the thick lens. The rotation of the particle is also discussed in the reference [4].

$$\begin{pmatrix} R_f \\ R'_f \end{pmatrix} = \begin{pmatrix} \cos(Td) & \dfrac{\sin(Td)}{T} \\ -\sin(Td) & \cos(Td) \end{pmatrix} \begin{pmatrix} R_i \\ R'_i \end{pmatrix} \qquad (8)$$

The optical property of an ES cylinder lens of immersion type is evaluated using the similar method. The focal length for the thin lens here is conventionally written as $f=1/(T\sin(Td))= 1/(T^2d)$ and the position of the principal plane is $z_p=(\cos(Td)+1)/ (T\sin(Td))$.

Let V_0=10kV be the initial potential to attain the initial kinetic energy by the particle (proton here) and it passes through a 2-cylinder lens with potential V_1=-5kV and V_2=+5kV. The focal length F=192.5cm of the lens is given by eq. (9) if the thickness of the individual n=866 thin lens be d=0.1cm and taking $(\varphi(z_n)/\varphi(z_{n-1}))^{1/4}=1$ for thin lens. Adopting the procedure of reference [4] and using the difference formula for general focal length $1/f=(\xi(z_n)-\xi(z_{n-1}))/4(V_0-V_1+\varphi(z_n))$ and integrating by matrix method the particles are tracked along the axis and the focal length F=190.4cm is obtained. The track of proton is depicted inside the lens in Fig.4. If the proton is tracked numerically solving the Lorentz equation in the analytically obtained potential and field distribution, the focal length is a little more than 210cm depending on the launching radius of the particle exactly along the central axis, which is far from satisfying the Gaussian and para-axial ray condition keeping the lens parameters same.

$$\frac{1}{F} = \sum_n \frac{1}{f_n} = \sum_n \left(\frac{\sqrt{3}\xi(z_n)}{4(V_0 - V_1 + \varphi(z_n))} \right)^2 d \qquad (9)$$

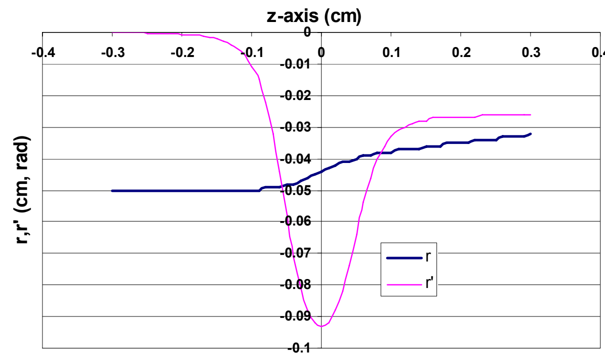

Figure 4: Plot of proton tracking of energy 10 keV in the cylindrical ES lens.

CONCLUSION

This method was used in [4] to evaluate the optical properties of Glaser solenoid using the magnetic field obtained by eq. (6) and the measured actual field for the same magneto-motive-force (Amp-turn). The position of the focus and the principal plane matched within ~3% difference. The theory proposed in this paper for the electrostatic lens consisting of two cylinders of immersion type also promises to give accurate optical properties and thus the method turns into a tool to design cylindrical electrostatic lens and magnetic solenoid lens for various applications.

The method can further be extended to investigate for designing electrostatic and magnetic Einzel lenses accurately. Study of a rotationally symmetric combined electrostatic and magnetic lens by this method will prove to be vital for its probably accurate design.

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