THE ANALYSIS OF TUNABLE DIELECTRIC LOADED WAKEFIELD ACCELERATING STRUCTURE OF RECTANGULAR GEOMETRY*

I. Sheynman#, A. Altmark, S. Baturin, Saint-Petersburg ElectroTechnical University «LETI», Saint-Petersburg, Russia
A. Kanareykin, Euclid TechLabs, LLC, Solon, Ohio, U.S.A.

Abstract

The analysis of Cherenkov radiation generated by high current relativistic electronic beam passing through a rectangular waveguide with the multilayered dielectric loading is carried out. One ceramic layer of the structure possesses ferroelectric properties, which allow the waveguide frequency spectrum to be controlled by varying the permittivity of this ferroelectric using external electric field applied. On the basis of decomposition on orthogonal eigenmodes of a rectangular multilayered waveguide the analytical expressions are obtained and numerical modelling of wakefield electromagnetic fields including the radial deflecting forces are studied.

INTRODUCTION

The field of advanced accelerators is in search of novel technologies to allow progress in particle accelerators for high energy physics experiments. Techniques based on the Dielectric Wakefield Accelerator (DWFA) concept are some of the most promising to date in terms of their potential to provide high gradient accelerating structures for future generation linear colliders. These structures may be excited by a high current electron beam or an external high frequency high power RF source and have been under intensive study in recent years [1-7]. The basic RF structure is very simple — a dielectric loaded waveguide with an axial vacuum channel is inserted into a conductive sleeve. A high charge, (typically 20-40 nC), short, (1-4 mm) electron drive beam generates TM_{01} mode electromagnetic Cherenkov radiation (wakefields) which propagating through the waveguide vacuum channel is used to accelerate a less intense beam pulse following at an appropriate distance.

The purpose of the given work is research a perspective rectangular waveguide (Fig. 1), providing a number of technological and constructive advantages in comparison with a traditional for DWFA cylindrical waveguide. Use of a wide flat bunch in rectangular waveguides allows to achieve also increase in a charge of relativistic electronic bunches and a current passed through a waveguide. Besides, the increase in width of a bunch leads to decrease in a direction of a plane of a bunch of rejecting transverse fields in a waveguide, connected with displacement of a bunch from a waveguide axis that weakens transverse instability of a bunch and, finally, also conducts to increase in a critical current [5-8].

Our work brought to a focus to detailed elaborations of a conclusion of expressions for fields created by an electron bunches in a rectangular multilayer waveguide and research of their frequency spectrum and spatial structure.

We describe the tunable waveguide design. Creation of a frequency-tunable accelerating structure on the basis of a rectangular dielectric waveguide (Fig. 1) is carried out by introduction of ferroelectric layers 2 between dielectric ceramic plates 1 and a metal environment 3. Introduction of additional ferroelectric layer to the rectangular waveguide results in to control frequency spectrum structure.

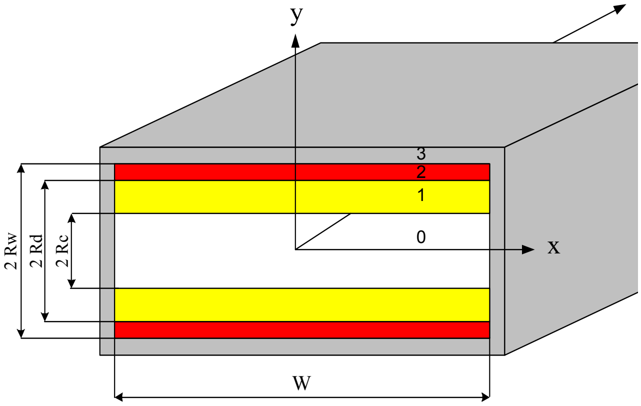

Figure 1: Tunable rectangular wakefield structure.

For cylindrical accelerating structure with dielectric filling the tunable scheme a frequency spectrum of a waveguide with the help of ferroelectric film put on an outer side of a dielectric waveguide, has been offered by authors of article in [6], [7]. The solution of tunable rectangular waveguide has been offered by authors of article in [6]. Here we present some improvements to the field calculations on the basis of field decompositions for LE and LM modes.

In articles [1-3] calculation of amplitudes of fields in a rectangular waveguide has been spent on the basis of the approached expression through normalized shunt impedance *R*/*Q*. Here we represent the strict decision of a problem on the basis of decomposition on eigenfunctions of transverse operators of Helmholtz equations for electric and magnetic fields.

MATHEMATICAL MODEL

For reception of analytical expressions for calculation

---

*Work supported by Ministry of Education and Science of the Russian Federation, the program "Scientific and scientific-pedagogical personnel of innovative Russia" and the Russian Foundation for Basic Research (09-02-00921)

#isheinman@yandex.ru

of accelerating fields in rectangular accelerating structure we w Q ake advantage of representation of a $\Delta f/f$, % ıe form of superposition of two independent types of waves: LE with $E_y = 0$ and LM with $H_y = 0$. The electron bunch goes in the vacuum channel along an axis of a waveguide. We will search for decisions in a system, moving together with a charge with a speed v: $\zeta = z - vt$.

From system of the Maxwell equations for LE waves we will receive the Helmholtz equation for longitudinal electric field component E_z.

$$\frac{\partial^2 E_z}{\partial \zeta^2} + \hat{L}E_z = \frac{-e}{\varepsilon_0 \varepsilon(y)} \frac{\partial n}{\partial \zeta},$$

where $\hat{L} = \frac{1}{\left(1-\varepsilon(y)\beta^2\right)}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ is an operator describing transverse electric field structure, e is an electron charge, n is a concentration, $\beta = v/c$. The decision for a dot charge can be presented in the form of decomposition on eigenfunctions of this operator.

For LE waves the accelerating field can be found as

$$E_z = \sum_{n,m} \frac{q\left(1-\beta^2\right)\Psi_{n,m}(x_0,y_0)\Psi_{n,m}(x,y)}{\varepsilon_0 \int \Psi_{n,m}{}^2(x,y)\left(1-\varepsilon(y)\beta^2\right)dxdy} \cos\left(k_{z\,n,m}(\zeta - z_0)\right)$$

where q is a dot bunch charge, $\Psi_{n,m}(x,y)$ are eigenfunctions of operator $\hat{L}$, (x_0, y_0) are charge coordinates in a vacuum channel, $k_{z\,n,m}$ are eigennumbers of $\hat{L}$. The other field components can be determined from Maxwell equations.

For LM waves the Helmholtz equation can be received for component H_x:

$$\frac{\partial^2 H_x}{\partial \zeta^2} + \hat{M}H_x = \frac{ev}{1-\varepsilon(y)\beta^2}\left(\frac{\partial n}{\partial y} - \frac{n}{\varepsilon(y)}\frac{\partial \varepsilon(y)}{\partial y}\right),$$

where $\hat{M} = \frac{1}{\left(1-\varepsilon(y)\beta^2\right)}\left(\frac{\partial^2}{\partial x^2} + \varepsilon(y)\frac{\partial}{\partial y}\left[\frac{1}{\varepsilon(y)}\frac{\partial}{\partial y}\right]\right)$ is an operator describing transverse magnetic field structure H_x.

Finding the decision for wake fields of a dot charge in the form of decomposition on own functions of this operator and using received from the Maxwell equations connection between electric and magnetic fields, for LM waves we will have:

$$E_z = \frac{-q}{\varepsilon_0 \varepsilon(y)} \sum_{n,m} \left[\frac{\cos\left(k_{z\,n,m}(\zeta - z_0)\right)}{\left(k_{z\,n,m}\right)^2 \int \frac{\left(1-\varepsilon(y)\beta^2\right)}{\varepsilon(y)} \Phi^2_{n,m}(x,y)dxdy} \times \right.$$
$$\left. \times \frac{\partial \Phi_{n,m}(x,y)}{\partial y} \frac{\partial \Phi_{n,m}(x_0,y_0)}{\partial y} \right].$$

Here $\Phi_{n,m}(x,y)$ are eigenfunctions of operator $\hat{M}$, $k_{z\,n,m}$ are eigennumbers of $\hat{M}$.

NUMERICAL CALCULATOINS

On the basis of the received expressions we numerically investigated of a spectrum and wake fields created by a Gaussian relativistic electron bunch propagating along waveguide axes with a charge $q = 10$ nC and energy of particles $W = 200$ MeV, $\sigma_x = \sigma_y = 0.01$см, $\sigma_z = 0.2$ cm for a rectangular waveguide (Fig. 1). Parameters of the waveguide are corresponding to parameters, projected to use in Argonne Wakefield Accelerator of Argonne National Laboratory [4], $w = 2.3$ cm, $R_c = 0.3$ cm, $R_d = 0.4135$ cm, $R_w = 0.4235$ cm, $\varepsilon_1 = 16$, $\varepsilon_2 = 400$, $f = 13.625$ GHz.

The schedule of a longitudinal component of the electric field E_z, depending on distance behind it $\zeta = z - vt$ is submitted on Fig. 2 (the bunch is located in a point $\zeta = 0$).

Results of calculation on the basis of the received expressions show good coincidence to numerical calculation on the basis of programs CST Particle Studio and MAFIA®.

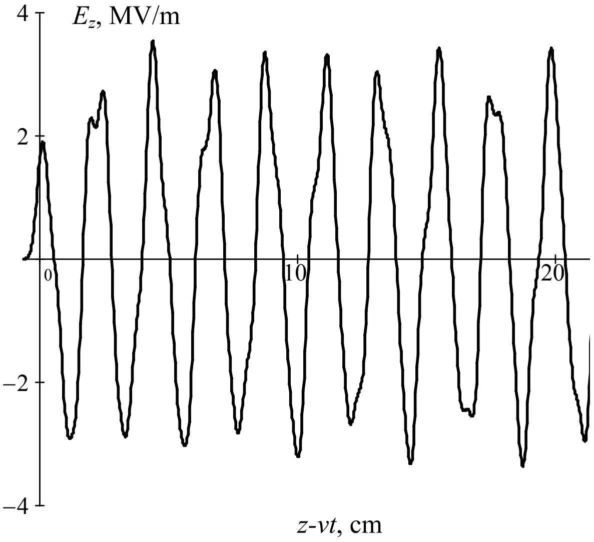

Figure 2: Dependence of accelerating field E_z from distance $\zeta = z - vt$ in the waveguide.

On Fig. 3 dependences of quality factor Q and tunability from a thickness ferroelectric films (permeability of a ferroelectric material changes in a range ±10 % from average value $\varepsilon_2 = 400$) are presented. Base frequency of a waveguide (frequency LM_{11} mode), the sizes of vacuum channel R_c and w and dielectric permeability $\varepsilon_1 = 16$ do not vary. Calculation R_d and R_w is made for every thickness film of a ferroelectric material.

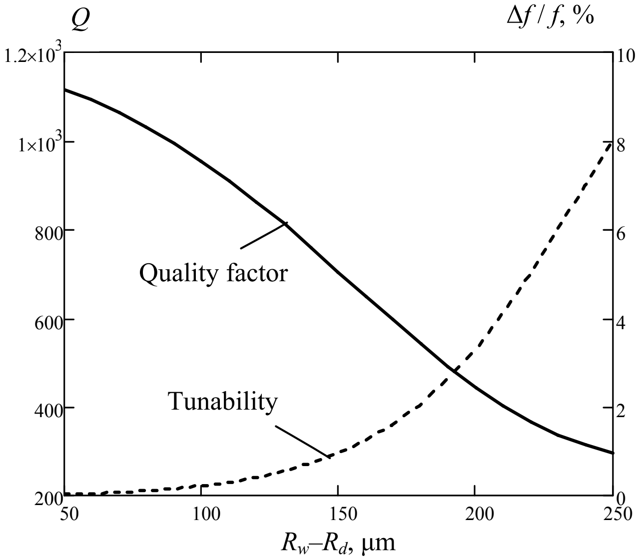

Figure 3: Quality factor and tunability vs. ferroelectric layer thickness.

Presence of a ferroelectric layer with high value of dielectric permeability causes increase in energy losses in a waveguide and sharply accrues with growth of a thickness of a layer of a ferroelectric material. This effect conducts to restriction of an admissible thickness ferroelectric films and limits possibilities of operative frequency adjustment of wakefield waveguide.

SUMMARY

Frequency control of any accelerating structure is a fundamental issue. Synchronization between the electron beam velocity and the phase velocity of the accelerating field must be maintained in order for the bunch to gain energy. Even for a simple wakefield accelerator using a single drive bunch the tuning technology can be useful to adjust the phase of the accelerated beam relative to the drive beam. At the same time, for the more complex schemes required for practical high energy dielectric based accelerators frequency adjustment of the DLA structure is a critical issue.

The rectangular geometry for the tunable accelerating structure based on the double-layer ferroelectric technology has been studied with this paper. The rectangular approach provides a number of technological and constructive advantages described above.

REFERENCES

[1] L. Xiao, W. Gai, X. Sun. Phys. Rev. E, 65 (2001) 016505
[2] T-B. Zhang, J. L. Hirshfield, T. C. Marshall, B. Hafizi Phys. Rev. E. 56 (1997) 4.
[3] C. Jing, W. Liu, L. Xiao, W. Gai, P. Schoessow, T. Wong. Phys. Rev. E 68 (2003) 016502.
[4] P. V. Schoessow, J. B. Rosenzweig. "Slab Symmetric Dielectric Micron Scale Structures for High Gradient Electron Acceleration" PAC-1999, New York, p. 3624–3626 (1999).
[5] A. Tremaine, J. Rosenzweig, P. Schoessow, W. Gai. Phys. Rev. E 56 (1997) 7204.
[6] A. Altmark, A. Kanareykin, I. Sheinman. "A Double-Layered, Planar Dielectric Loaded Accelerating Structure" PAC-2003, Portland, USA, pp.1897-1899 (2003).
[7] A. M. Altmark, A. D. Kanareykin, and I. L. Sheinman. Techn. Phys., 50 (2005) 1.