

# SETTING THE BEAM ONTO THE REFERENCE ORBIT IN NON SCALING FFAG ACCELERATORS

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## Abstract

Described in the paper are systematic procedures to inject and keep the beam on the reference trajectory for a fixed energy, as applied to the EMMA non scaling FFAG accelerator. The notion of accelerated orbits in FFAG accelerators has been introduced and some of their properties have been studied in detail.

## INTRODUCTION

EMMA (Electron Machine with Many Applications) is a prototype non scaling FFAG built at Daresbury Laboratory. Non scaling FFAGs accelerators of EMMA type have an unprecedented potential for medical applications for carbon and proton hadron therapy. They could also represent a possible active element for an ADSR (Accelerator Driven Subcritical Reactor). This paper will summarize the procedure we call 'setting the beam onto the closed orbit'. More precisely, given an energy dependent orbit in EMMA, we describe all the surrounding apparatus required, such that to make sure the beam eventually settles onto it. In order to operate EMMA, the ALICE (Accelerators and Lasers In Combined Experiments) machine will be used as an injector and the energy of the injected beam will range from 10 to 20 MeV. ALICE will deliver a single bunch train with a bunch charge of 16 to 32 pC at a rate of 1 to 20 Hz.

ALICE is presently designed to deliver bunches, which are around 4 ps and 8.35 MeV from the exit of the booster of its injector line. These are then accelerated to energies in the range between 10 and 20 MeV in the main ALICE linac after which they are sent to the EMMA injection line. The EMMA injection line ends with a 70° septum for injection into the EMMA ring itself followed by two kickers so as to direct the beam onto the correct, energy dependent, trajectory.

After circulation in the EMMA ring, the electron bunches are extracted using what is almost a mirror image of the injection setup with two kickers followed by a 65° extraction septum. The beam is then transported to a diagnostic line, whose purpose it is to analyze in as much detail as possible the effect the non scaling FFAG has had on the bunch.

## DEFINITION AND DESCRIPTION OF THE CLOSED ORBIT

It has been shown [1] that the equations describing the particle trajectory in the median plane of a non scaling

FFAG accelerator can be derived from the Hamiltonian

$$H_e = -\sqrt{\beta_e^2 \gamma_e^2 - P_e^2} + \frac{e}{m_0 c} \int dX_e B_z(X_e; s). \quad (1)$$

Here  $X_e$  and  $P_e$  denote the reference orbit and the reference momentum, respectively,  $\beta_e$  is the relative longitudinal velocity of the particle and  $\gamma_e$  is the Lorentz factor. In addition,  $e$  is the electron charge,  $m_0$  is the electron rest mass and  $c$  is the speed of light in vacuo. The vertical component of the magnetic field  $B_z$  is given by the expression

$$B_z(X_e; s) = a(s)[X_e - X_c - d(s)], \quad (2)$$

where

$$X_c = \frac{L_p}{2 \tan(\pi/N_L)}, \quad (3)$$

and  $a(s)$  is the field gradient in the quadrupole magnets (positive for focusing and negative for defocusing ones),  $d(s)$  is the relative displacement of their magnetic center with respect to the rectangular polygon line,  $L_p$  is the length of one cell, while  $N_L$  is the total number of cells.

The static closed orbit can be calculated for a fixed value of the energy, or alternatively for constant  $\beta_e$ . In the case, where the energy varies, the above Hamiltonian (1) is the relativistic analogue of the so called Caldirola-Kanay type Hamiltonian [2], which describes the dynamics of a particle with time dependent mass. Although not strictly dissipative, this system behaves like a dissipative one and is known in conventional types of accelerators as the adiabatic damping of betatron oscillations.

The closed orbit is a periodic solution of the equations of motion following from the Hamiltonian (1) taking into account the symmetry of the lattice, which in the case of EMMA is 42-fold. However, if we consider a single super period (cell), the reference orbit can be defined as a unique solution in the Cauchy sense. It is determined by the condition that both the initial coordinate and angle are equal to the ones at the end of the cell. Numerically, the reference orbit is found by an iterative algorithm.

Let us now consider the equation for the reference orbit with a small dissipative term added

$$\frac{d^2 X_e}{ds^2} + \Gamma_e \left[ 1 + \left( \frac{dX_e}{ds} \right)^2 \right] \frac{dX_e}{ds} = -\frac{e}{m_0 c \beta_e \gamma_e} \left[ 1 + \left( \frac{dX_e}{ds} \right)^2 \right]^{3/2} B_z(X_e; s), \quad (4)$$

where the damping coefficient  $\Gamma_e$  is defined as

$$\Gamma_e = \frac{1}{\beta_e \gamma_e} \frac{d(\beta_e \gamma_e)}{ds}. \quad (5)$$

The dissipation term present in Eq. (4) is responsible for the adiabatic damping to the closed orbit, thus modeling a process similar to a limit cycle. Since the energy variation is usually small, we can try to solve Eq. (4) perturbatively. The approximate solution without dissipation ( $\Gamma_e = 0$ ) is

$$X_e = X_c + \frac{\langle ad \rangle}{\langle a \rangle} + A \cos \omega s + B \sin \omega s, \quad (6)$$

$$\frac{dX_e}{ds} = -\omega A \sin \omega s + \omega B \cos \omega s, \quad (7)$$

where  $\langle \dots \rangle$  imply averaging over one period,  $A$  and  $B$  are suitably determined constants, and

$$\omega = \frac{e \langle a \rangle}{m_0 c \beta_e \gamma_e}. \quad (8)$$

A small deviation  $\xi$  from the reference orbit is governed by the equation

$$\begin{aligned} \frac{d^2 \xi}{ds^2} + 3\omega^2 \tilde{X}_e \frac{dX_e}{ds} \sqrt{1 + \left( \frac{dX_e}{ds} \right)^2} \frac{d\xi}{ds} \\ + \Gamma_e \left[ 1 + 3 \left( \frac{dX_e}{ds} \right)^2 \right] \frac{d\xi}{ds} + \omega^2 \left[ 1 + \left( \frac{dX_e}{ds} \right)^2 \right]^{3/2} \xi = 0. \end{aligned} \quad (9)$$

Averaging the above equation over the relatively fast oscillations specified by the varying part  $\tilde{X}_e$  of the undamped solution (6), we obtain

$$\begin{aligned} \frac{d^2 \xi}{ds^2} + \Gamma_e \left[ 1 + \frac{3\omega^2}{2} (A^2 + B^2) \right] \frac{d\xi}{ds} \\ + \omega^2 \left[ 1 + \frac{\omega^2}{2} (A^2 + B^2) \right]^{3/2} \xi = 0. \end{aligned} \quad (10)$$

The last equation describes damped oscillations around the reference orbit with a characteristic relaxation number of turns

$$N_R = \frac{2}{\mathcal{C} \Gamma_e} \left[ 1 + \frac{3\omega^2}{2} (A^2 + B^2) \right]^{-1}, \quad (11)$$

where  $\mathcal{C}$  is the machine circumference.

For typical EMMA parameters the relaxation number of turns is roughly 100, which is nearly an order of magnitude higher as compared with the acceleration time in the energy range from 10 MeV to 20 MeV.

## GENERAL PROCEDURE

The following is a general procedure to accurately place the beam on the closed orbit of an FFAG accelerator. This procedure presents a systematic and extensive programme to determine the calibrations of the beam position monitors (BPMs) and the quadrupole magnets and their associated movers in the machine. Some of the procedures may not be required if accurate offline calibration has been performed

and is trusted to a sufficient degree of accuracy. This procedure thus represents a sort of a fall-back programme for the worst-case scenario.

We divide the process into three main topics: Calibration of the BPMs and Magnets; Symmetry Optimization; Controls Interface.

### *Calibration of the Beam Position Monitors and Quadrupole Magnets*

Calibration of the beam position monitors is an important first step in an FFAG accelerator, due to the design offsets at different energies. In an FFAG, where the quadrupole offsets and the energy play an important role in determining the closed orbit, we must also calibrate the magnets and their motion relative to the BPMs as well. To calibrate the BPM we therefore need to have an independent method of determining the BPM output versus beam position. In this case we can use the injection septum magnet to provide our independent reference. We rely on the knowledge of the beam trajectory in the transfer line preceding the septum and an adequate calibration of the septum magnet itself. The procedure involves switching off all of the quadrupole magnets and other elements between the injection septum and the first BPM in the FFAG. We can then use the septum calibration to geometrically calibrate the first BPM in the machine. Switching on one of the quadrupole magnets allows us to then calibrate both the magnetic field strength and the linear motion of the quadrupole mover, as well as the zero-position of the mover. The procedure involves varying the quadrupole current at different offset positions and recording the BPM signals. Dependent on the machine design it may not be possible to determine the center of the quadrupole magnet relative to the BPM center, in which case it must be inferred from the data available.

This can also be repeated for the second quadrupole in the cell, ideally independently. The calibration of the cell magnets allows transport into the second cell of the machine, whereby one can then bootstrap around the rest of the machine to calibrate all magnets and all BPMs. This scenario invariably relies on the calibration of the first BPM, and by extension the septum magnet calibration. This can be somewhat alleviated by using the mechanical and electrical calibration of the BPMs performed without beam, as well as the magnetic calibration of the quadrupole magnets and of the magnet movers. This data is then used to constrain the final fit of all the BPMs, and so the magnets, to within reasonable limits. The calibration can be further improved by using the extraction septum, and associated transfer line, as an additional calibration device. This should improve the calibration of the elements in the section of the machine between injection and extraction, which will obviously have knock on effects in the rest of the machine. After calibration of the machine has been performed, it is important to produce a machine response matrix - relating the BPM output at all BPMs to the effect

of one quadrupole, and repeating for all quadrupoles. This provides a wealth of information that can be used to determine errors in the lattice and to help improve the symmetry of the machine [see next section]. One can also use LOCO style concepts to improve the offline modeling of the FFAG accelerator.

### Symmetry Optimization

In order to inject the beam, we choose the most relaxed lattice first and we do not turn acceleration on until we have developed an advanced understanding of what happens at several fixed energies.

The way we propose to inject the beam and to setup the reference trajectory is to look for piecewise symmetry on the BPMs. So, initially, we inject the beam and look at the available BPMs or YAG screens for a  $2\pi$  symmetry. We ensure this is satisfied by moving the quadrupole positions and, thereby, changing the bending of the quadrupoles whilst keeping the focusing constant. When this is achieved, we move onto  $\pi$  symmetry and arrange the same thing there.

Subsequently, we look at identifying  $\pi/7$  and hence  $\pi/21$  symmetry. Or, in the case of acceleration, we may relax this to  $2\pi/21$  symmetry given that cavities are only present in every other cell. A sketch of what is meant by symmetry arrangement is given in Fig. 1 below.

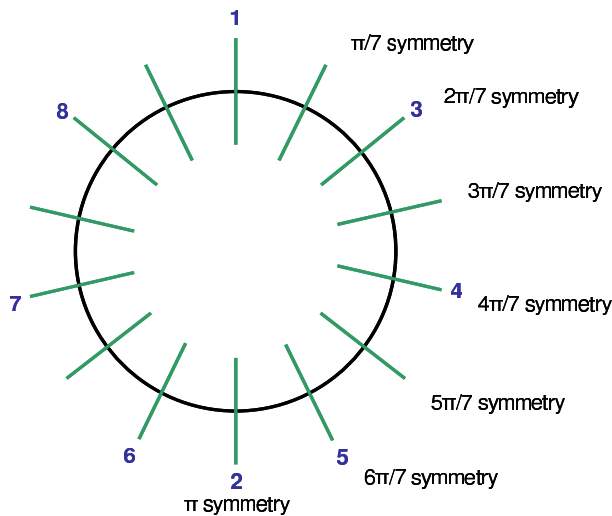


Figure 1: Illustration of EMMA symmetries.

Immediate commissioning plans involve beam injection and transport through four sectors of the EMMA ring. In this case, symmetry can only be ensured for  $\pi/7$ , or  $2\pi/7$  in the case of acceleration.

### Controls Interface

A control interface panel used to put the beam on the closed orbit should contain automatic procedures to avoid repeating manual operations and should be flexible enough

to allow the operator to monitor and optimize the process. Two distinctive part are to be considered, namely calibration of the BPMs and symmetry optimization. For both of them, it is important to note that the EMMA magnet transverse positions can be individually adjusted, whereas only three power supplies are available. Therefore as mentioned above, the focusing and defocusing magnets can be varied independently in each cell but all the cells will be identical.

Since the calibration is done several times, most of the tasks will be overtaken manually. The control panel must contain each BPM reading and the position and current of the magnets located nearby. For each BPM a table of data or a graph relating the BPM signal magnitude to the magnet position should be created. This will create a set of data accessible later on by the operator (or by any routine) when performing other measurements using the BPMs (e.g. tune measurement, orbit correction, etc.).

The symmetry optimization panel should contain a procedure for finding the optimal magnets' position in order to match the symmetries described above ( $2\pi, \pi, \dots, \pi/21$ ). The option of moving either all the magnets from the same family by the same distance or allowing individual positioning will be implemented. The procedure building the response matrix must be implemented as well. It will simply consist in slightly moving each magnet one at a time and saving all BPM readings. Misalignments can then be targeted and in a first stage manually addressed. Later on an automatic correction scheme can be added to ease iterating optimization of the symmetry.

This procedure can be tested before the actual commissioning of the machine using the online model created on site [3].

## CONCLUDING REMARKS

The notion of accelerated orbits and adiabatic damping in FFAG accelerators has been introduced. Further, an estimate of the relaxation time of small oscillations around the reference trajectory has been presented.

Systematic procedures to determine the calibrations of the beam position monitors and the quadrupole magnets and their associated movers in the machine have been described. These procedures are being implemented in a control panel interface, which will essentially facilitate the forthcoming commissioning of the EMMA non scaling FFAG.

## REFERENCES

- [1] J.K. Jones, B.D. Muratori, S.L. Smith and S.I. Tzenov, Progress in Physics 1 (2010) 72-82.
- [2] S.I. Tzenov, "Contemporary Accelerator Physics", World Scientific, Singapore (2004).
- [3] Y. Giboudot, D. Kelliher, F. Meot and S. Tygier, "The EMMA Experimental FFAG Online Commissioning Code", These Proceedings.