PERTURBATION ANALYSIS ON A FOUR-VANE RFQ

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Abstract

An important issue for high intensity RFQs (tenth of mA beam current and more) is keeping beam losses as low as possible, in order to allow reliable and safe maintenance of the machine. Typically, beam dynamics outcomes driven by these constraints result both in a RFQ length that is considerably higher than the wavelength and in an intra-vane voltage admitted variation with respect to the design value that must not exceed a few percent. Therefore an analytical tool is needed in order to foresee the effect of geometric perturbations on the voltage profile, in order to give an indication on the permitted ranges of geometrical errors in the RFQ construction. In this article a five conductors transmission line equivalent circuit for the four-vane RFQ is presented and the effects of geometrical perturbations on the voltage profile are analyzed in some particular cases. The case study is the IFMIF RFQ (125 mA deuteron current, 9.8 m length, 175 MHz frequency), whose features are particularly suitable for this kind of analysis.

RFQ MODELING VIA A FIVE-CONDUCTORS TRANSMISSION LINE

A four vane RFQ can be modelled with a five conductor (the four electrodes E1, E2, E3 and E4 plus a "virtual" ground) transmission line, whose infinitesimal element of length dz is shown in Figure 1. This topology is similar to the four conductor line introduced in [1].

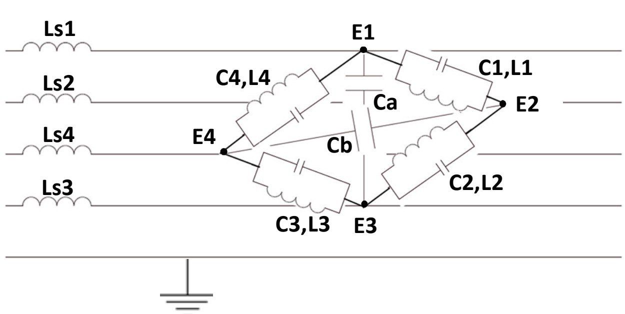

Figure1: The RFQ equivalent transmission line

In such circuits the L_{si} (i=1,..,4) are the inductances associated with the flow of longitudinal currents along the vane tip, L_i and C_i are respectively the inductances (integrated in length) associated with the flux of magnetic field through the cross-section of the RFQ and the intra-vane capacitances (per unit length) and C_a and C_b are the capacitances between opposite vanes. The circuit equations are the following

$$\frac{d\underline{\mathbf{U}}}{dz} = -j\omega \underline{\underline{\mathbf{L}}}_s \underline{\mathbf{I}}$$

$$\frac{d\underline{\mathbf{I}}}{dz} = -\left(j\omega \underline{\underline{\mathbf{C}}} + \frac{1}{j\omega} \underline{\underline{\mathbf{L}}} \right) \underline{\mathbf{U}}$$

where $\underline{\mathbf{U}} = U_1\hat{\mathbf{e}}_1 + U_2\hat{\mathbf{e}}_2 + U_3\hat{\mathbf{e}}_3 + U_4\hat{\mathbf{e}}_4$ is the vector of the intravane voltages and $\underline{\mathbf{I}} = I_1\hat{\mathbf{e}}_1 + I_2\hat{\mathbf{e}}_2 + I_3\hat{\mathbf{e}}_3 + I_4\hat{\mathbf{e}}_4$ is the vector of the longitudinal currents and

$$\underline{\underline{\mathbf{L}}}_s = \begin{bmatrix} L_{s1} & -L_{s2} & 0 & 0 \\ 0 & L_{s2} & -L_{s3} & 0 \\ 0 & 0 & L_{s3} & -L_{s4} \\ -L_{s1} & 0 & 0 & L_{s4} \end{bmatrix}$$

$$\underline{\underline{\mathbf{C}}} = \begin{bmatrix} C_1 & 0 & -C_a & -C_a - C_4 \\ -C_b - C_1 & C_2 & 0 & -C_b \\ -C_a & -C_a - C_2 & C_3 & 0 \\ 0 & -C_b & -C_b - C_3 & C_4 \end{bmatrix}$$

$$\underline{\underline{\mathbf{L}}} = \begin{pmatrix} 1/L_1 & 0 & 0 & -1/L_4 \\ -1/L_1 & 1/L_2 & 0 & 0 \\ 0 & -1/L_2 & 1/L_3 & 0 \\ 0 & 0 & -1/L_3 & 1/L_4 \end{pmatrix}$$

Since $\underline{\underline{\mathbf{L}}}_s \underline{\underline{\mathbf{C}}} = \frac{1}{c^2} \underline{\underline{\mathbf{I}}}_4$, [2] c being the speed of light and $\mathbf{I_4}$ the 4x4 identity matrix, in the ideal RFQ (C_i=C, L_i=L and C_a=C_b) by putting $\omega_0^2 = 1/LC$ and $h = C_a/C$ the previous equations can be written as follows

$$\frac{d^2\underline{\mathbf{U}}}{dz^2} = \left(-\frac{\omega^2}{c^2} + \frac{1}{c^2} \underline{\underline{\mathbf{C}}}^{-1} \underline{\underline{\mathbf{L}}} \right) \underline{\mathbf{U}} \triangleq \left(-\frac{\omega^2}{c^2} + \frac{1}{c^2} \underline{\underline{\mathbf{A}}} \right) \underline{\mathbf{U}} \quad (1)$$

with $$\underline{\underline{\mathbf{A}}} = \frac{\omega_0^2}{4c^2} \begin{bmatrix} 1+\frac{2}{1+h} & -1 & -\frac{1-h}{1+h} & -1 \\ -1 & 1+\frac{2}{1+h} & -1 & -\frac{1-h}{1+h} \\ -\frac{1-h}{1+h} & -1 & 1+\frac{2}{1+h} & -1 \\ -1 & -\frac{1-h}{1+h} & -1 & 1+\frac{2}{1+h} \end{bmatrix}$$

The symmetric $\underline{\underline{\mathbf{A}}}$ matrix can be made diagonal , by the matrix $\underline{\underline{\mathbf{S}}}$ in such a way that $\underline{\underline{\mathbf{S}}}^{-1} \underline{\underline{\mathbf{A}}} \underline{\underline{\mathbf{S}}} = \underline{\underline{\mathbf{k}}}^2$ with $\underline{\underline{\mathbf{k}}}^2 = c^{-2}\omega_0^2 diag(1, 0, 1/(1+h), 1/(1+h))$ and

$$\underline{\underline{\mathbf{S}}} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 0 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 0 & -1/\sqrt{2} \end{bmatrix} \triangleq \left[\underline{\mathbf{t}}_q, \underline{\mathbf{t}}_m, \underline{\mathbf{s}}_{d1}, \underline{\mathbf{s}}_{d2} \right]$$

Therefore a transformation $\underline{\mathbf{U}} = \underline{\underline{\mathbf{S}}}\hat{\underline{\mathbf{U}}}$ between modal voltages' vector $\hat{\underline{\mathbf{U}}} = U_q\hat{\mathbf{e}}_1 + U_m\hat{\mathbf{e}}_2 + U_{d1}\hat{\mathbf{e}}_3 + U_{d2}\hat{\mathbf{e}}_4 =$ and vane voltages' vector $\underline{\mathbf{U}}$ is established.

In these expressions we easily recognise the quadrupole TE_{21} and dipole TE_{11} frequencies ($f_q = f_0$ and $f_d = f_0/\sqrt{1+h}$) , and also the presence of a mode (the monopole mode), corresponding to a zero eigenvalue, which can but vanish inside a four-vane RFQ, since it would correspond to a TEM configuration. The solution of Equation (1) in modal basis can be written in matrix form, for a homogeneous segment of RFQ as:

$$\begin{bmatrix}\hat{\underline{\mathbf{U}}}(\ell)\\ \hat{\underline{\mathbf{I}}}(\ell)\end{bmatrix} = \begin{bmatrix}\cos\left(\underline{\underline{\gamma}}\ell\right) & -\sin\left(\underline{\underline{\gamma}}\ell\right)\hat{\underline{\underline{\mathbf{Y}}}}_w^{-1}\\ -\hat{\underline{\underline{\mathbf{Y}}}}_w\sin\left(\underline{\underline{\gamma}}L\right) & \cos\left(\underline{\underline{\gamma}}\ell\right)\end{bmatrix}\begin{bmatrix}\hat{\underline{\mathbf{U}}}(0)\\ \hat{\underline{\mathbf{I}}}(0)\end{bmatrix} = \underline{\underline{\mathbf{M}}}(0,\ell)\begin{bmatrix}\hat{\underline{\mathbf{U}}}(0)\\ \hat{\underline{\mathbf{I}}}(0)\end{bmatrix}$$

where $\underline{\underline{\gamma}}$ is the diagonal matrix of the wave number for the above –mentioned modes and $\hat{\underline{\underline{\mathbf{Y}}}}_w \triangleq (1/j\omega)\underline{\underline{\mathbf{S}}}^t\underline{\underline{\mathbf{L}}}_s^{-1}\underline{\underline{\mathbf{S}}}\, j\underline{\underline{\gamma}} = (c^2/\omega)\underline{\underline{\mathbf{S}}}^t\underline{\underline{\mathbf{C}}}\underline{\underline{\mathbf{S}}}\,\underline{\underline{\gamma}}$ is the wave admittance matrix. This representation is particularly useful in the case of an RFQ with longitudinally varying paramaters that can be modeled as a chain of homogeneous sections.

Once the usual boundary conditions for the RFQ are assigned at the extremities $\hat{\underline{\mathbf{U}}}'(0) = \underline{\underline{\mathbf{s}}}_{e1}\hat{\underline{\mathbf{U}}}(0)$, $\hat{\underline{\mathbf{U}}}'(\ell) = \underline{\underline{\mathbf{s}}}_{e2}\hat{\underline{\mathbf{U}}}(\ell)$ and, in the case of segmented RFQ's, also at the coupling cell location $\ell_1, \ell_2 \ldots$ $\begin{bmatrix}\hat{\underline{\mathbf{U}}}(\ell_1^+)\\ \hat{\underline{\mathbf{U}}}'(\ell_1^+)\end{bmatrix} = \underline{\underline{\mathbf{s}}}_{c1}\begin{bmatrix}\hat{\underline{\mathbf{U}}}(\ell_1^-)\\ \hat{\underline{\mathbf{U}}}'(\ell_1^-)\end{bmatrix}, \begin{bmatrix}\hat{\underline{\mathbf{U}}}(\ell_2^+)\\ \hat{\underline{\mathbf{U}}}'(\ell_2^+)\end{bmatrix} = \underline{\underline{\mathbf{s}}}_{c2}\begin{bmatrix}\hat{\underline{\mathbf{U}}}(\ell_2^-)\\ \hat{\underline{\mathbf{U}}}'(\ell_2^-)\end{bmatrix}$ the solutions of the Equation are the longitudinal quadruple and dipole eigenfrequencies $\omega_{q0}, \omega_{q1}, \ldots, \omega_{qn}, \ldots, \omega_{d0}, \omega_{q1}, \ldots, \omega_{qn}, \ldots$ and orthonormal eigenvectors $\hat{\underline{\mathbf{U}}}_{qn} = \phi_{qn}\hat{\mathbf{e}}_1, n \in \mathbb{N}_0$, $\hat{\underline{\mathbf{U}}}_{d1n} = \phi_{dn}\hat{\mathbf{e}}_3, n \in \mathbb{N}_0$, $\hat{\underline{\mathbf{U}}}_{d2n} = \phi_{dn}\hat{\mathbf{e}}_4, n \in \mathbb{N}_0$.

1ST ORDER PERTURBATION ANALYSIS

In case of geometric errors in the RFQ vane and/or vessel profiles, a perturbative term $\underline{\underline{\mathbf{\delta A}}}$ in the operator $\underline{\underline{\mathbf{A}}}$ appears. On a purely geometrical point of view, this term is due to capacitance perturbations $\underline{\underline{\mathbf{\delta C}}}$ (i.e. due to mean aperture R_0 deviations from the nominal value) and/or inductance perturbations $\underline{\underline{\mathbf{\delta L}}}$ (i.e electrode height H deviations). In this case, the perturbed operator reads

$$\underline{\underline{\mathbf{\delta A}}} = \frac{1}{c^2}\delta\left(\underline{\underline{\mathbf{C}}}^{-1}\underline{\underline{\mathbf{L}}}\right) = \frac{1}{c^2}\left(\underline{\underline{\mathbf{C}}}^{-1}\underline{\underline{\mathbf{\delta L}}} - \underline{\underline{\mathbf{C}}}^{-1}\underline{\underline{\mathbf{\delta C}}}\,\underline{\underline{\mathbf{A}}}\right)$$

and, in the modal basis,

$\underline{\underline{\mathbf{\delta k}}}^2 = \frac{1}{c^2}\underline{\underline{\mathbf{S}}}^{-1}\underline{\underline{\mathbf{C}}}^{-1}\left(\underline{\underline{\mathbf{\delta L}}} - \underline{\underline{\mathbf{\delta C}}}\,\underline{\underline{\mathbf{A}}}\right)\underline{\underline{\mathbf{S}}}$, and, explicitly:

$$\underline{\underline{\mathbf{\delta k}}}^2 = -\frac{\omega_0^2}{4c^2}\begin{bmatrix}\delta L_1+\delta L_2+\delta L_3+\delta L_4 & \delta L_1-\delta L_2+\delta L_3-\delta L_4 & \sqrt{2}(\delta L_1-\delta L_3) & \sqrt{2}(\delta L_4-\delta L_2)\\ 0 & 0 & 0 & 0\\ \frac{\sqrt{2}(\delta L_1-\delta L_3)}{1+h} & \frac{\sqrt{2}(\delta L_1-\delta L_3)}{1+h} & \frac{2(\delta L_1+\delta L_3)}{1+h} & 0\\ \frac{\sqrt{2}(\delta L_4-\delta L_2)}{1+h} & \frac{\sqrt{2}(\delta L_2-\delta L_4)}{1+h} & 0 & \frac{2(\delta L_2+\delta L_4)}{1+h}\end{bmatrix}\frac{1}{L} + \\ -\frac{\omega_0^2}{4c^2}\begin{bmatrix}\delta C_1+\delta C_2+\delta C_3+\delta C_4 & 0 & \frac{\sqrt{2}(\delta C_1-\delta C_3)}{1+h} & \frac{\sqrt{2}(\delta C_4-\delta C_2)}{1+h}\\ 0 & 0 & 0 & 0\\ \frac{\sqrt{2}(\delta C_1-\delta C_3)}{1+h} & 0 & \frac{2(\delta C_1+\delta C_3+\delta C_a+\delta C_b)}{1+h} & \frac{2(\delta C_a-\delta C_b)}{1+h}\\ \frac{\sqrt{2}(\delta C_4-\delta C_2)}{1+h} & 0 & \frac{2(\delta C_a-\delta C_b)}{1+h} & \frac{2(\delta C_2+\delta C_4+\delta C_a+\delta C_b)}{1+h}\end{bmatrix}\frac{1}{C}$$

Therefore it is possible to write the explicit expression of the perturbed voltage of the RFQ

$$\hat{\underline{\mathbf{U}}} = \hat{\underline{\mathbf{U}}}_{q0} + \widehat{\delta\underline{\mathbf{U}}} = \phi_{q0}\hat{\mathbf{e}}_1 + \sum_{n=1}^{\infty} a_{qn}\phi_{qn}\hat{\mathbf{e}}_1 + \sum_{n=0}^{\infty} a_{d1n}\phi_{dn}\hat{\mathbf{e}}_3 + \sum_{n=0}^{\infty} a_{d2n}\phi_{dn}\hat{\mathbf{e}}_4$$

where

$$a_{qn} = c^2\frac{\left\langle\hat{\underline{\mathbf{U}}}_{qn}\middle|\underline{\underline{\mathbf{\delta k}}}^2\middle|\hat{\underline{\mathbf{U}}}_{q0}\right\rangle}{\omega_0^2-\omega_{qn}^2} = -\frac{\omega_0^2}{4(\omega_0^2-\omega_{qn}^2)}\int_0^\ell \varphi_{q0}\varphi_{qn}\left(\frac{\delta C_{QQ}}{C}+\frac{\delta L_{QQ}}{L}\right)dz \quad n\in\mathbb{N}$$

$$a_{d1n} = c^2\frac{\left\langle\hat{\underline{\mathbf{U}}}_{d1n}\middle|\underline{\underline{\mathbf{\delta k}}}^2\middle|\hat{\underline{\mathbf{U}}}_{q0}\right\rangle}{\omega_0^2-\omega_{dn}^2} = -\frac{\sqrt{2}\omega_0^2}{4(\omega_0^2-\omega_{dn}^2)}\int_0^\ell \varphi_{q0}\varphi_{dn}\left(\frac{\delta C_{Qd1}}{C}+\frac{\delta L_{Qd1}}{L}\right)dz \quad n\in\mathbb{N}_0$$

$$a_{d2n} = c^2\frac{\left\langle\hat{\underline{\mathbf{U}}}_{d2n}\middle|\underline{\underline{\mathbf{\delta k}}}^2\middle|\hat{\underline{\mathbf{U}}}_{q0}\right\rangle}{\omega_0^2-\omega_{dn}^2} = -\frac{\sqrt{2}\omega_0^2}{4(\omega_0^2-\omega_{dn}^2)}\int_0^\ell \varphi_{q0}\varphi_{dn}\left(\frac{\delta C_{Qd2}}{C}+\frac{\delta L_{Qd2}}{L}\right)dz \quad n\in\mathbb{N}_0$$

with

$$\delta C_{QQ}(z) = \delta C_1(z)+\delta C_2(z)+\delta C_3(z)+\delta C_4(z)$$

$$\delta L_{QQ}(z) = \delta L_1(z)+\delta L_2(z)+\delta L_3(z)+\delta L_4(z)$$

$$\delta C_{Qd1}(z) = \frac{\sqrt{2}(\delta C_1(z)-\delta C_3(z))}{(1+h)}, \delta C_{Qd2}(z) = \frac{\sqrt{2}(\delta C_4(z)-\delta C_2(z))}{(1+h)}$$

$$\delta L_{Qd1}(z) = \sqrt{2}(\delta L_1(z)-\delta L_3(z)), \delta L_{Qd2}(z) = \sqrt{2}(\delta L_4(z)-\delta L_2(z))$$

as well as of the perturbed quadrupole frequency

$$\Delta\omega_0 \cong -\frac{c^2}{2\omega_0}\left\langle\hat{\underline{\mathbf{U}}}_{q0}\middle|\underline{\underline{\mathbf{\delta k}}}^2\middle|\hat{\underline{\mathbf{U}}}_{q0}\right\rangle.$$

APPLICATION TO THE IFMIF RFQ

In this paragraph the analytical method will be applied to the case of the IFMIF RFQ.

Table 1: IFMIF RFQ main parameters

Particles	D+	
Frequency	175	MHz
Input Current	130	mA
Energy (in-out)	0.1-5	MeV
Length *l*	9.78	m
Voltage min/max	79/132	kV
Mean aperture R_0	4.1 / 7.1	mm
Pole tip radius ρ	3.08/5.33	mm
Q_0 (SF) (z=0-z=L)	15100-	
2D Power (SF)	450	kW
Stored Energy	6.6	J
Total power P_d	1.345	MW

For our purposes, the RFQ will be treated as an homogeneous cavity (with constant voltage, mean aperture and pole tip radius) corresponding to the last cell (cell449) of the RFQ. In the following part, two case studies will be considered, under the hypothesis of ideal boundary conditions $\underline{\underline{\mathbf{s}}}_{e1} = \underline{\underline{\mathbf{s}}}_{e2} = \underline{\underline{\mathbf{I}}}_4$ and with no coupling elements. For such RFQ the parameter h is equal to 0.054 and the cavity length is such that there is a symmetric dipole free region of ±1.6 MHz around the TE_{210} mode and that the 1^{st} upper Q mode is about 700 kHz above the TE_{210} one.

In the first case, the misalignment of one electrode will be considered. In particular, let us suppose that the electrode E1 is displaced of δy=0.05 mm for all z in the interval [a, $l/2+a$] with respect to its nominal position and with a varying in the interval [0, $l/2$] . This case corresponds to δR_{0E1}=0.05 mm, and consequently to a perturbation of the capacitances C_1 and C_4. The capacitance perturbations can be written as $\delta C_1=\delta C_4=2\chi_{R0}\delta R_{0E1}(z)$ and $\delta C_2=\delta C_3=0$ with χ_{R0}=7.6 MHz/mm in this case [1].The explicit expression of the quadrupole voltage perturbation can be written as follows (here α_n is equal to 2 if n=0 and 1 otherwise)

$$\frac{\widehat{\delta \mathbf{U}}}{\phi_0} = \left(\frac{\ell}{\lambda}\right)^2 \frac{8\chi_{R0}\Delta R_0}{f_0}\sum_{n=1}^{\infty}\frac{[\sin(\frac{n\pi}{2}+\frac{n\pi a}{\ell})-\sin(\frac{n\pi a}{\ell})]\cos(\frac{n\pi z}{\ell})}{n^3\pi}\hat{\mathbf{e}}_1 +$$

$$-\frac{\sqrt{2}}{2}\frac{\chi_{R0}\Delta R_0}{f_0}\sum_{n=1}^{\infty}\frac{\alpha_n[\sin(\frac{n\pi}{2}+\frac{n\pi a}{\ell})-\sin(\frac{n\pi a}{\ell})]\cos(\frac{n\pi z}{\ell})}{n\pi\left[h-(1+h)(n\lambda/2\ell)^2\right]}(\hat{\mathbf{e}}_3+\hat{\mathbf{e}}_4)$$

and the corresponding diagram is shown in Figure2 for the case of Q component and in Figure3 for D component. It can be noticed that this results is useful in the dimensioning of the tuners system and/or in the determination of the required geometrical tolerances [1].

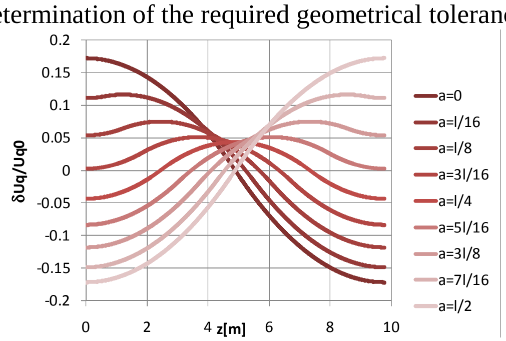

Figure 2 The voltage perturbations (Q component)

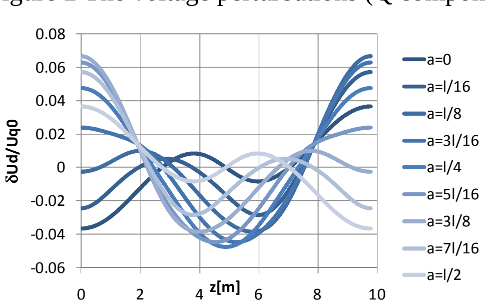

Figure 3 The voltage perturbations (D component)

The second case takes into account the capacitance and inductance variations due to the thermal-induced deformations in the RFQ, as calculated by bthermo-structural simulations [4]. The cooling channel scheme for each Super-Module (composed of 6 assembled, 0.55 m length, mechanical modules) is designed in such a way that the cooling water runs along the first three 0.55 m modules (1.65 m thermal module), while it is reversed in the remaining three modules.

The temperature increase along each thermal module creates a non-uniform deformation along z and the symmetry of the cooling system is such that only quadrupole perturbations appear (Figure 4).

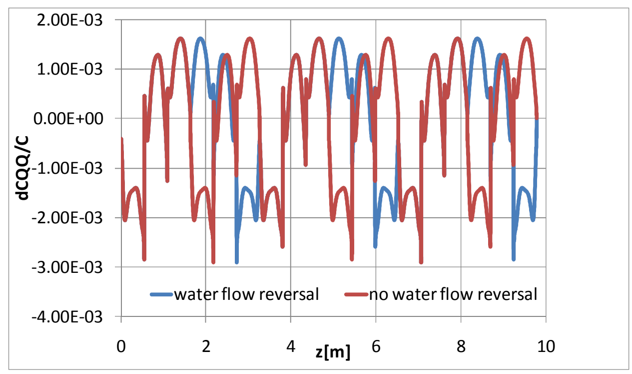

Figure 4: The δC_{QQ} thermal-induced contribution

The corresponding $\delta U_q/\phi_0$ is shown in Figure 5, where it is compared with the case of no water flow reversal between adjacent thermal modules; the comparison between the two quadrupole voltage profiles justifies the choice of the reversal of the water flow.

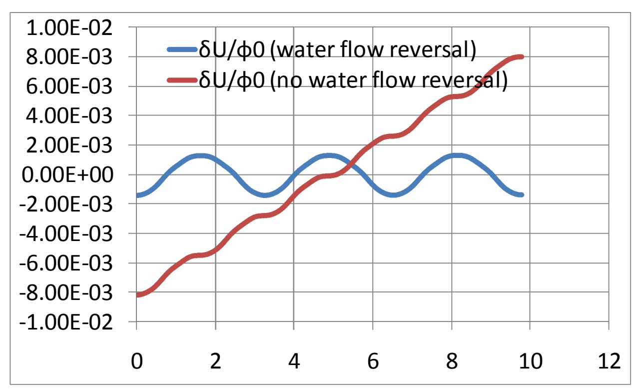

Figure 5: $\delta U_q/\phi_0$ in the case of δC_{QQ} thermal-induced perturbation.

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