A FIXED FIELD ALTERNATING GRADIENT ACCELERATOR WITH LONG STRAIGHT SECTIONS*

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Abstract

The lattice of a Fixed Field Alternating Gradient (FFAG) accelerator normally has high symmetry. The whole ring consists of many identical cells which can be a simple FODO, doublet or triplet focusing unit. There is, however, no real reason for an FFAG lattice to have high symmetry, except for a linear nonscaling design which relies on high symmetry to avoid betatron resonances. We propose an FFAG lattice design with a superperiod that makes it possible to have long straight sections for injection, extraction and rf cavities. We discuss how to introduce a superperiod structure. The impact on dynamic aperture as well as the advantage of having long straight sections is presented.

INTRODUCTION

The lattice of a fixed field alternating gradient (FFAG) accelerator [1] consists of many identical small cells with a minimum focusing structure such as FODO, doublet, triplet, or pumplet [2], where F is a focusing magnet, D is a defocusing magnet and O is a drift space in between. One of the reasons not to choose a more complicated structure as is commonly found in the modern synchrotron lattice may be attributed to the historical background of the development. The FFAG accelerator was invented a few years after the invention of an alternating gradient focusing synchrotron. A synchrotron lattice at that time was similar to what we see as an FFAG lattice now. The CERN Proton Synchrotron has 20 identical cells and the BNL Alternating Gradient Synchrotron has 24 identical cells, which were both designed and constructed in the 1950s. Since then the synchrotron lattice evolved with the introduction of many new ideas. Among them, a long straight insertion is a very practical idea which enables us to install a variety of devices in an accelerator. Beam injection and extraction from the ring becomes much easier with enough space, which is an obvious advantage. The dispersion suppressor in the arc and absence of finite dispersion functions in a long straight section are features commonly seen in the modern synchrotron. These optics can be calculated using software with various functions for optimization by introducing many families of bending and focusing magnets. On the other hand, all activities on FFAG accelerators were frozen in the middle of the 1960s. There was no chance of the lattice design of an FFAG incorporating the new feature in the optics design of a synchrotron.

In fact, there is no need to keep the lattice structure with very high symmetry in an FFAG other than in a linear nonscaling FFAG. The operating tune is fixed independently of beam momentum in a scaling FFAG and in a nonlinear nonscaling FFAG and can be chosen anywhere in tune space. Although the high symmetry of the lattice helps to reduce the number of systematic resonances, it does not matter if a machine is operated at some distance from a resonance. The synchrotron lattice with only two or three superperiods proves that is the case. Only a linear nonscaling FFAG lattice is an exception. Because there is no chromaticity correction, the transverse tunes change in a wide range, from around 0.4 to 0.1 in terms of tune per cell. Keeping very high symmetry with many small unit cells is the only way to avoid crossing systematic resonances.

Nonetheless, there were not many activities to break the high degree of symmetry in scaling and nonlinear nonscaling FFAG lattices since the development restarted. It is mostly due to the lack of a design tool. Let us take the design of a scaling FFAG as an example. The magnet of a scaling FFAG has a field profile of r^k, where r is the radial coordinate with respect to the machine centre and k is the field index defined as $k = (r/B)(dB/dr)$. Unlike a synchrotron, the design orbit is not determined by a piecewise constant magnetic field and the curvature of an orbit is a continuous function of longitudinal position. Although, at the beginning of the optics design, we must make an assumption of a piecewise constant field and obtain a rough idea of orbit shape and optics properties of an FFAG accelerator, the only way to find the exact orbit and optics is firstly to calculate the orbit iteratively and secondly to calculate the optics based on the linear fields along the orbit. It is not difficult to start with a guess and obtain the exact orbit and optics using the exact FFAG field profile. However, a proper software code is needed when the lattice has a complicated structure with many families of magnets.

The FFAG lattice with a long straight section is not just an option for a design, but seems to be an essential design ingredient for future FFAG designs, especially to ease the injection and extraction components and the rf cavity. A tight space for an rf cavity causes significant orbit distortion when permeability materials are coupled with leakage fields of lattice magnets [3]. To increase the bunch intensity, H^- charge exchange injection is preferable and a long straight section is needed to accommodate the system. The advantage of large horizontal acceptance becomes clearer when an extraction system for a large emittance beam is available. In this paper, we will show the design criteria first. We will show an example.

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SCALING LAW

We will focus on the way to make a long drift section for a scaling FFAG. The same procedure can be applied to a nonlinear nonscaling FFAG based on the scaling FFAG design [4].

A scaling FFAG has two conditions [5]. One is that the orbits for different momenta are isomorphic. The other is that the betatron function scales linearly with orbit radius. These two conditions can be satisfied with the magnetic field profile of

$$B_z = B_{z,0}\left(\frac{r}{r_0}\right)^k F(\theta) \quad (1)$$

where $B_{z,0}$ is the vertical magnetic field at the radius of r_0 and r_0 is the reference radius. Notice that $F(\theta)$ is an arbitrary function describing the dependence on the azimuthal coordinate. Focusing structure like FODO, triplet and pumplet etc. are simple examples of $F(\theta)$. However, $F(\theta)$ can be a more complicated function as long as the overall focusing gives stable motion.

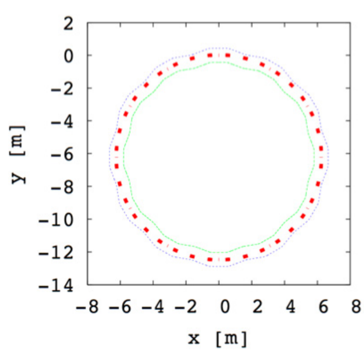

Figure 1 (a): Footprint of a 12-fold symmetry FFAG lattice. Red blocks show magnets and green and blue lines are orbits of 0.243 GeV/c and 0.729 GeV/c, respectively.

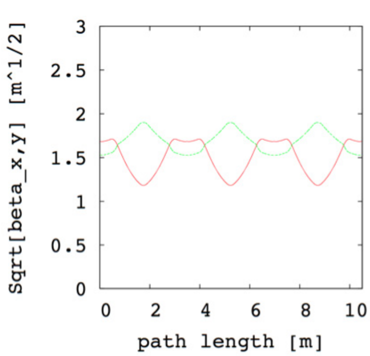

Figure 1 (b): Lattice functions of a quarter of the ring. The red and green lines show horizontal and vertical beta functions respectively.

FOUR-FOLD SYMMETRY LATTICE

As a first example, we examine the possibility of inserting a long straight section in a 12-fold symmetry lattice. In this example, each cell has a triplet focusing and the physical length of F and D magnets is equal. This can be used as a proton FFAG accelerating from 30 MeV (0.243 GeV/c) to 250 MeV (0.729 GeV/c). The footprint and the beta functions of the original lattice are depicted in Fig. 1 (a) and (b), respectively. By squeezing the magnets at four corners, relatively long straight sections appear, that gives the lattice four-fold symmetry. Figure 2 (a) shows the footprint and the beta functions after squeezing. Because the field profile follows Eq. (1), transverse tunes are constant over the momentum range.

The previous example demonstrates it is possible to have a long straight in an FFAG accelerator by moving magnets without introducing extra knobs. For some applications, it is already good enough. However, the quite noticeable change after introducing a long straight section is the modulation of the beta functions. The minimum and maximum beta functions become smaller and bigger, respectively. Fine adjustment of lattice parameters with a greater number of magnet families is required to flatten the beta functions.

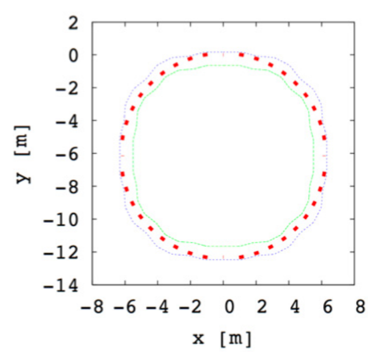

Figure 2 (a): Footprint of 4-fold symmetry FFAG. Orbits looks more square shape.

FITTING BY CODE

The fine adjustment should be done numerically with the aid of advanced computer software. Since we have a clear understanding of the optics dependence through Eq. (1), we will restrict attention to adjusting the magnet parameters for a single value of momentum.

The procedure for the adjustment will be the following. Given the initial configuration of the lattice, the closed orbit and beta functions can be calculated. Any initial configuration can be tried, but obviously a configuration which is not so much different from the highly symmetric one and has stable optics has a better chance of leading to the solution. If the beta functions are larger than we expect, the phase advance is not what we want, and/or the beta functions are largely modulated, the magnet strength is varied. At this stage, we introduce families of magnets and each family can have a different strength. It is, however, still preferable to keep some local symmetry to find the solution. We could make the magnet length and the distance between magnets as other free parameters for the fitting. However, we will show in the following examples of fitting only with variations in magnet strength.

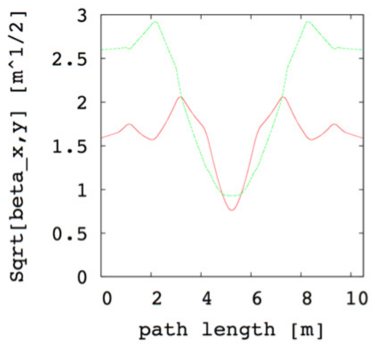

Figure 2 (b): Lattice functions of a quarter of the ring before flatting beta functions.

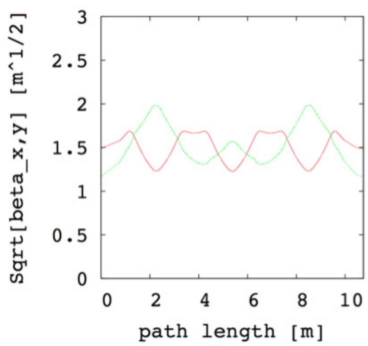

Figure 3: Lattice functions of a quarter of the ring after flatting beta functions.

Using this fitting process, the beta function in Fig. 2 (b) is flattened as shown in Fig. 3. In this example, three families of F magnets and two families of D magnets are defined as free parameters. The fitting goal is to reduce the maximum beta functions in horizontal and vertical directions separately. It leads to flatter beta functions. The parameters of both designs are listed in Table 1.

DYNAMIC APERTURE

Despite rich nonlinearities in a scaling FFAG lattice, the dynamic aperture is always very large if the tune is not located near a systematic resonance. If this is a result of a high degree of lattice symmetry, a lattice with superperiod may not preserve larger dynamic aperture. This has been checked with particle tracking for the lattice we described.

The dynamic aperture was explored in the horizontal direction with fixed vertical initial amplitude of 10 π mm mrad (normalized). A particle was tracked for 1000 turns to define the aperture. In order to compare dynamic aperture with similar tunes, the 12-fold lattice is retuned to (3.367, 3.225). The dynamics aperture becomes 4,000 π mm mrad (normalized) for the four-fold lattice while is it 13,000 π mm mrad for the 12-fold case.

Table 1: Machida parameters

parameters	12 fold symmetry	4 fold w/o fitting	4 fold w/ fitting
Magnet length [m]	0.262	0.262	0.262
Drift length [m]	0.829	1.964 (long) 0.687 (short)	1.964 (long) 0.687 (short)
Tune (H,V)	(3.213,2.383)	(3.130,2.346)	(3.205,3.124)
Radius [m]	6.251	6.251	6.251

SUMMARY

Without violating the scaling FFAG conditions, we have shown that long straight sections can be introduced into the lattice. Numerical optimization minimizes the modulation of the beta functions so that physical aperture does not deteriorate and there is only small reduction in dynamic aperture. A long straight section of 2 m or more makes injection and extraction much easier.

In this report, long straight sections are inserted in the scaling FFAG lattice with moderate k value. However, our final goal is to combine this technique and FFAG lattice with relatively high k value [6]. This should open up many applications for an FFAG accelerator.

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