ERROR STUDY OF A NOVEL NON-LINEAR, NONSCALING FFAG

D. J. Kelliher*, S. Machida, ASTeC, STFC Rutherford Appleton Laboratory, UK
S. L. Sheehy, John Adams Institute of Accelerator Science, Oxford University, UK †

Abstract

A novel nonlinear, nonscaling FFAG ring has been designed for proton and ion acceleration [1]. It can be used for proton and carbon therapy as well as a proton driver for various facilities such as a high intensity neutrino factory. The machine has novel features including variable energy extraction and a high repetition rate of about 1 kHz. Taking as an example the PAMELA proton ring, under study at the John Adams Institute in Oxford, we present results of an error study. A calculation of alignment tolerance is made, in which the effects of translational misalignments of the triplet magnets are included. The effect of misalignments on the dynamic aperture of the machine is investigated.

INTRODUCTION

In the magnets of a conventional scaling FFAG the magnetic field profile, with field index k, satisfy the scaling law that ensures a constant tune over the momentum range. In a typical scaling FFAG the orbit shift, determined by the field index, is of order 1m. By contrast, in a linear non-scaling FFAG, which uses just dipole and quadrupole magnets, the orbit shift is reduced to the order of cm and so much smaller magnet apertures are required. However, in this case the tune varies signficantly over the momentum range and the beam passes through many integer and half-integer resonances. In order to avoid the accumulation of orbit distortion due to resonanes, acceleration must be completed in 10-20 turns [2]. This condition of rapid acceleration means that a linear non-scaling FFAG is unsuitable for slow cycling machines such as a proton accelerator for high beam power production or particle therapy accelerators [3].

A novel non-linear non-scaling FFAG accelerator has recently been proposed that combines the stable tune of a scaling FFAG with the small beam excursion of a linear non-scaling FFAG [1]. This type of machine can be considered suitable for proton therapy, as a driver for an accelerator driven subcritical reactor, as a proton driver for neutron production or in a neutrino factory. Here we study the application of this type of FFAG to the PAMELA (Particle Accelerator for MEdicaL Applications) proton lattice. The design takes as its starting point a radial-sector FDF triplet scaling FFAG. The orbit excursion is reduced by choosing a k value in the second stability region of Hill's equation. A number of simplifications are then made to the magnet design each of which break the scaling law. The magnetic field is expanded into it multipole components and just the first few terms are included, rectangular rather than radial sector magnets are used and in each cell the magnets are aligned along a straight line rather than along the arc. Here we present the results of an error study of the PAMELA FFAG - extending the orbit distortion results already presented [4] and examining the sensitivity of the dynamic aperture to magnet misalignments.

PAMELA

The PAMELA FFAG has the following characteristics that are an advantage in hadron therapy - high repetition rate (1kHz), variable energy extraction and compact size to allow construction in a hospital. One FFAG accelerates both protons and carbon ions, to 250 MeV and 68 MeV/u respectively, while a second FFAG accelerates the carbon ions on to 400 MeV/u. In the proton FFAG the beam can be extracted at any energy in the range 70 -250 MeV in order to vary the depth of the Bragg peak in tissue by the required amount. The main lattice parameters for the proton FFAG are listed in table 1

Table 1: PAMELA proton lattice parameters

Injection Energy	30.95 MeV
Maximum extraction Energy	250 MeV
Cells	12
r0	6.251 m
Magnet length (F and D)	0.3144 m
Field index k	36.721
Long drift	1.702 m
Short drift	0.3144 m
Orbit Excursion	0.176 m

The magnet field profile is given by a polynomial fit to the scaling field including multipole terms up to decapole. A polynomial fit was found to give better agreement over the whole momentum range than a Taylor expansion [1]. The variation of the tune over the momentum range of the proton FFAG is shown in figure 1. Crucially, the variation of total tune is much less than half an integer, ensuring that no harmful resonances are crossed during acceleration. A particular operating point in tune space can be chosen by varying the field index and the ratio of the magnetic field in the F and D magnets.

ORBIT DISTORTION

Orbit distortion due to a Gaussian distribution of magnet misalignments with a standard deviation of 50 microns and a cut-off of 3 sigma is calculated. A single particle is

---

* David.Kelliher@stfc.ac.uk
† s.sheehy1@physics.ox.ac.uk

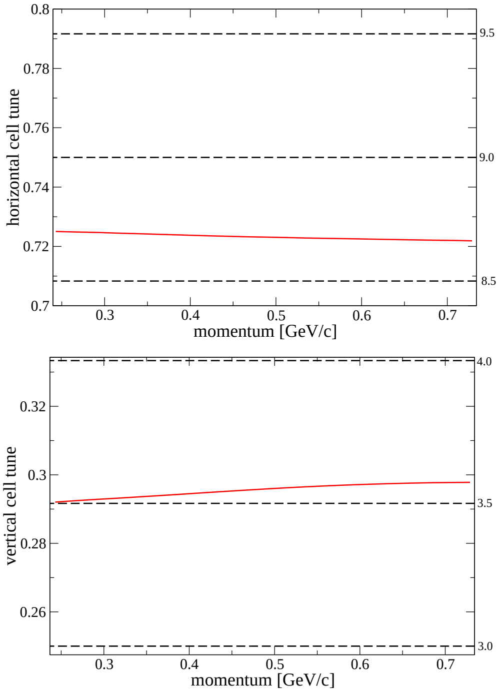

Figure 1: Variation of horizontal (top) and vertical (bottom) cell tune over momentum (solid line). Dashed lines and associated numbers show total tune in 12 cell ring.

tracked from the injection momentum to the maximum extraction momentum in 1000 turns assuming a constant energy gain in each of eight zero-length cavities. The closed orbit at the injection momentum is used to initialise tracking. The particle coordinates are recorded once per turn in the middle of a long drift. The orbit distortion is defined as the maximum difference in the recorded coordinates with and without magnet misalignments. Thirty error patterns are studied for statistical purposes. For this study the tracking code Zgoubi [5] is chosen since it is able to model FFAG magnets. In order to facilitate calculations a python interface to Zgoubi is used [6].

The horizontal orbit distortion due to horizontal misalignments is calculated at 15 horizontal cell tunes in the range $0.7-0.75$. Similarly, the vertical orbit distortion due to vertical misalignments is calculated in the range and $0.25-0.3$. As expected, the results shown in Fig. 2 show that the orbit distortion in the horizontal and vertical plane are each maximum in vicinity of integer total tunes, i.e. at a horizontal cell tune of 0.75 and a vertical cell tune of 0.25. Assuming the symmetry of the lattice is broken, the spike in orbit distortion near a cell tune of 0.71 may be driven by the systematic resonance $2*Q_x = 17$ where Q_x is the total horizontal tune of the lattice. This should be confirmed in further studies by calculating the beta function modulation at this resonance.

By constrast, the spike in the vertical orbit distortion near a cell tune of 0.27 is likely to be due to the coupling resonance at this point. Vertical distortion in the presence of the sextupole field creates a skew quadrupole term that drives a resonance at $\nu_x + \nu_y = 1$. The onset of coupling during acceleration is shown in Fig. 3.

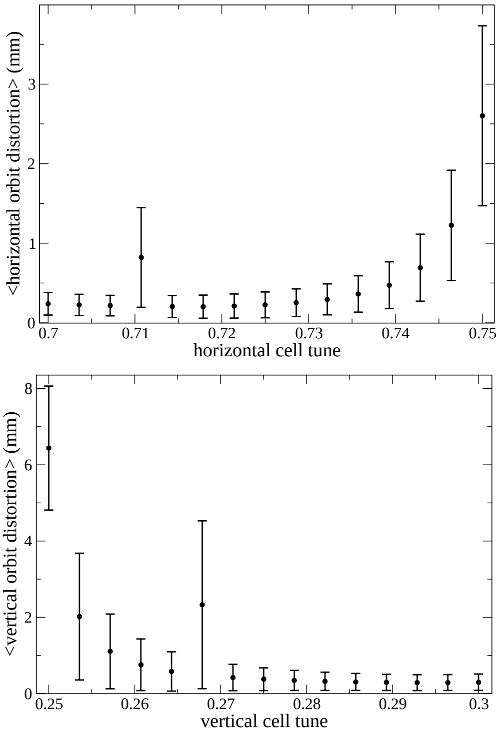

Figure 2: Mean of the maximum horizontal (top) and vertical (bottom) orbit distortion calculated for 30 misalignment patterns with standard deviation of 50 microns over a range of cell tunes. The error bars represent the standard deviation of the distortion results. In the top figure the vertical cell tune is fixed at 0.292 while in the bottom figure the horizontal cell tune is fixed at 0.73.

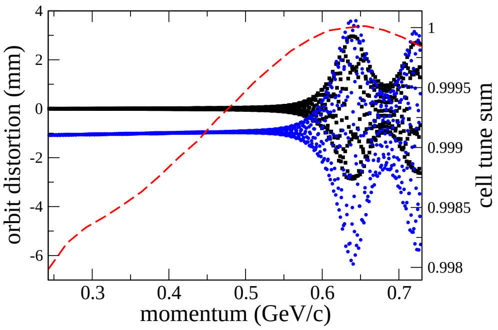

Figure 3: Horizonal (black crosses) and vertical (blue dots) orbit distortion over the momentum range showing the effect of coupling. The sum of the tunes, $\nu_x+\nu_y$, is calculated in the ideal lattice at 20 fixed energies over the momentum range (red dashed line, axis on right).

The minimum orbit distortion shown in the figure is 0.2 mm in the horizontal plane and 0.3 mm in the vertical plane. Defining the amplification factor as the ratio of the maximum orbit distortion to the magnitude of magnet misalignments, these figures suggest the ratio is 4 in the horizontal plane and 6 in the vertical plane. These low amplification factors imply that magnet misalignments at the level of even 100 microns result in a tolerable level of orbit distortion (<1 mm).

DYNAMIC APERTURE

Since the PAMELA lattice is a non-linear FFAG with multipole components up to decapole, it is important to establish that the dynamic aperture is sufficient to accommodate the required beam emmitance. The 2 sigma normalised beam emittance from the injector to the PAMELA FFAG is approximately 16π mm mrad and 11π mm mrad in the horizontal and vertical planes respectively [7]. These beam sizes constitute the required acceptance of the FFAG.

A calculation of the dynamic aperture at various points in tune space allows a suitable operating point to be chosen. In ref. [1] it was shown that in the ideal lattice a dynamic aperture in the range $30-40\pi$ mm mrad can be achieved at a number of points in tune space. In that study the dynamic aperture at the injection energy was calculated to study the case where the beam size is at its largest in physical space, no acceleration was included. For the purposes of this paper, acceleration over the momentum range is included (as in the previous section) and the effects of misalignments on dynamic aperture are investigated.

The algorithm employed to calculate the dynamic aperture involves tracking a particle with small initial amplitude of 1 π mm mrad normalised emittance and, if the particle surives to the end, increasing the amplitude in steps of 1 π mm mrad. This process is repeated until the particle is lost. The dynamic aperture is given by the highest emittance particle that survives tracking. In each transverse plane the initial coordinate of the particle is given by $-\sqrt{2J/\gamma_{x,y}}$ where J is the action variable and $\gamma_{x,y}$ is the horizontal and vertical Twiss parameter.

The dynamic aperture with acceleration is first calculated in the case of the ideal lattice. In figure 4, the results at horizontal cell tunes in the range $0.7-0.75$ are shown. It is clear that the dynamic aperture is greater than the required acceptance, apart from in the region of the $\nu_x+\nu_y=1$ coupling resonance, the location of which is indicated by the vertical line in the figure. The effect of horizontal misalignments on the dynamic aperture is also calculated. The same set of 30 error patterns used in the orbit distortion study are used. However, in this case just three points in horizontal tune are investigated. The resulting decrease in the mean dynamic aperture compared with the ideal case, and the standard deviation of the results, is at its most pronounced when the cell tune is 0.75, i.e. when the total horizontal tune is an integer. By contrast, at cell tune 0.725 the decrease in the mean dynamic aperture is modest (1.1 π mm mrad) and the standard deviation is just 2.7 π mm mrad.

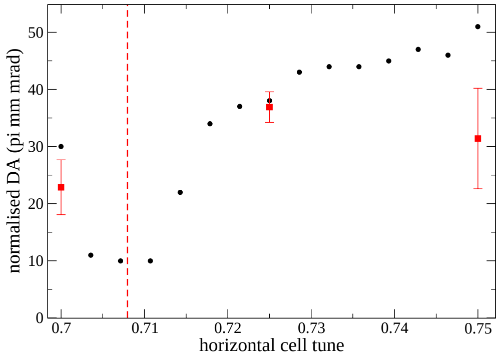

Figure 4: Dynamic aperture in the ideal lattice calculated at a range of horizontal cell tunes (black circles). The mean of the dynamic aperture calculated for 30 horizontal misalignment patterns with standard deviation 50 microns is shown at three points (red squares). The error bars represent the standard deviation of the results. The vertical cell tune is fixed at 0.292. The vertical dashed line represents the location of the coupling resonance $\nu_x+\nu_y=1$.

DISCUSSION

This error study of a non-linear, non-scaling FFAG, taking PAMELA as an example, indicates that a tolerable orbit distortion and a sufficient dynamic aperture can be achieved by careful selection of the operating point in tune space. The orbit distortion results show that, in addition to the integer total tune resonance driven by misalignments, coupling resonances and systemic resonances introduced by symmetry breaking must be considered. Future work in this error study should include a full 2D scan over the tune space and the addition of magnetic field imperfections of the triplet magnets.

ACKNOWLEDGEMENT

The authors would like to thank Dr. T. Yokoi for helpful comments.

REFERENCES

[1] S. L. Sheehy, K. J. Peach, H. Witte, D. J. Kelliher and S. Machida, Phys. Rev. ST Accel. Beams, 13 (2010) 040101

[2] S. Machida and D. J. Kelliher, Phys. Rev. ST Accel. Beams, 10 (2007) 114001

[3] S. Machida, Phys. Rev. ST Accel. Beams, 11 (2008) 094003

[4] S. L. Sheehy and D. J. Kelliher, proc. FFAG09, Fermilab, September 2009

[5] The ray-tracing code Zgoubi, F. Méot, NIM-A 427 (1999) 353-356 ; Zgoubi users' guide, F. Méot, S. Valero, CEA DSM DAPNIA/SEA-97-13 (1997) ; http://sourceforge.net/projects/zgoubi/

[6] PyZgoubi interface to Zgoubi, S. Tygier, D. Kelliher ; http://sourceforge.net/projects/pyzgoubi/

[7] M. Aslaninejad, private communication (2010)