A SEMI-ANALYTICAL ALGORITHM FOR MODELLING COMPTON GAMMA-RAY BEAMS*

C. Sun† Y. K. Wu, DFELL/TUNL and Dept. of Physics, Duke University, NC 27708-0319, USA

Abstract

Compton scattering of a laser beam with a relativistic electron beam has been used to generate an intense, highly polarized, and nearly monoenergetic gamma-ray beam at several facilities. The ability of predicting the spatial and spectral distributions of a Compton gamma-ray beam is crucial for the optimization of the operation of a Compton light source as well as for the applications utilizing the Compton beam. In this paper, we present an analytical algorithm for modelling Compton scattering process. Based upon this algorithm, we developed a numerical integration code to produce smooth results for the spatial and spectral distributions of the Compton beam. This code has been used to characterize the High Intensity Gamma-ray Source (HIγS) at Duke University for varying electron and laser beam parameters as well as different gamma-ray beam collimation conditions.

INTRODUCTION

Compton scattering of a laser beam with a relativistic electron beam has been successfully used to generate an intense, highly polarized and nearly monoenergetic x-ray or gamma-ray beam with a tunable energy at many facilities [1, 2]. These unique Compton beams have been used in a wide range of basic and application research fields from nuclear physics to astrophysics, from medical research to homeland security and industrial applications [1].

The ability of predicting the spectral, spatial and temporal characteristics of a Compton gamma-ray beam is crucial for the optimization of the gamma-ray beam production as well as applications utilizing the beam. While the theory of electron-photon Compton scattering (the scattering between a monoenergetic electron and a monoenergetic laser beams with zero transverse beam sizes) are well documented in literature [3, 4], there remains a need to fully understand the characteristics of the gamma-ray beam produced by Compton scattering of a laser beam and an electron beam with specific spatial and energy distributions, i.e., the beam-beam scattering.

In this paper, we present a semi-analytical algorithm to study the Compton scattering process of a laser beam and an unpolarized electron beam in the linear Compton scattering regime. Using this algorithm, we are able to characterize a Compton x-ray or gamma-ray beam with varying laser and electron beam parameters, arbitrary collision angles, and different gamma-beam collimation conditions.

ELECTRON-PHOTON SCATTERING

In the following, we limit our analysis to head-on collisions of laser photons and electrons. Because of conservation of the momentum and energy, the photon energy after the scattering is given by

$$E_g = \frac{E_p(1+\beta)}{1 + E_p/E_e - (\beta - E_p/E_e)\cos\theta_f}, \qquad (1)$$

where E_p and E_e are the incident photon and electron energies; E_g is the scattered photon energy; $\beta = v/c$ is the speed of the incident electron relative to the speed of light; and θ_f is the scattering angle between directions of the incident electron and scattered photon.

In a laboratory frame, the angular differential cross section is given by [5, 6]

$$\frac{d\sigma}{d\Omega} = \frac{8r_e^2}{X^2}\left\{[1+P_t\cos(2\tau-2\phi_f)]\left[\left(\frac{1}{X}-\frac{1}{Y}\right)^2+\frac{1}{X}-\frac{1}{Y}\right] + \frac{1}{4}\left(\frac{X}{Y}+\frac{Y}{X}\right)\right\}\left(\frac{E_g}{mc^2}\right)^2, \qquad (2)$$

where r_e is the classical electron radius; mc^2 is the rest mass energy of electron; P_t is the degree of linear polarization of the incident photon; τ is the azimuthal angle of the polarization vector of the incident photon; ϕ_f is the azimuthal angle of the scattered photon; X and Y are Lorentz invariant quantities given by

$$X = \frac{2\gamma E_p(1+\beta)}{mc^2},\; Y = \frac{2\gamma E_g(1-\beta\cos\theta_f)}{mc^2}. \qquad (3)$$

With Eqs. (1) and (2), one is able to study the spatial and spectral distributions of a gamma-ray beam produced by Compton scattering of a monoenergetic electron and laser beams with zero transverse beam sizes, i.e., electron-photon scattering. However, in the reality, the incident electron and laser beams have finite spatial and energy distributions, which will change the distributions of the Compton gamma-ray beam. In the following section, we will present an analytical algorithm for modelling a Compton scattering process of a laser and an electron beams with Gaussian phasespace distributions, i.e., the beam-beam scattering.

BEAM-BEAM SCATTERING

In a laboratory frame, an electron beam and a laser beam with Gaussian phasespace distributions can be described by

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† suncc@fel.duke.edu, CCSun@lbl.gov. Currently at Lawrence Berkeley National Laboratory.

$$
\begin{aligned}
f_e(x,y,z,x',y',p,t) &= \frac{1}{(2\pi)^3\varepsilon_x\varepsilon_y\sigma_p\sigma_l}\exp\left[-\frac{\gamma_x x^2+2\alpha_x xx'+\beta_x x'^2}{2\varepsilon_x}-\frac{\gamma_y y^2+2\alpha_y yy'+\beta_y y'^2}{2\varepsilon_y}-\frac{(p-p_0)^2}{2\sigma_p^2}-\frac{(z-ct)^2}{2\sigma_z^2}\right],\\
f_p(x_l,y_l,z_l,k,t) &= \frac{1}{4\pi^2\sigma_z\sigma_k\sigma_w^2}\exp\left[-\frac{x_l^2+y_l^2}{2\sigma_w^2}-\frac{(z_l+ct)^2}{2\sigma_l^2}-\frac{(k-k_0)^2}{2\sigma_k^2}\right],\ \sigma_w=\sqrt{\frac{\lambda\beta_0}{4\pi}\left(1+\frac{z_l^2}{\beta_0^2}\right)}. \qquad (4)
\end{aligned}
$$

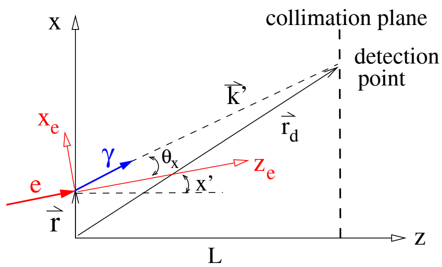

Figure 1: Geometric constraint for a scattered gamma-ray photon. The diagram only shows the projection of the constraint in the x-z plane.

Eq. (4), where p is the momentum of an electron; p_0 is the centroid momentum of the electron beam; x' and y' are the angular divergences of the electron in the x and y directions, respectively; $\alpha_{x,y}, \beta_{x,y}$ and $\gamma_{x,y}$ are Twiss parameters of the electron beam; σ_p, σ_z and $\varepsilon_{x,y}$ are the electron beam momentum spread, RMS bunch length, and emittance, respectively; k and λ are the wavenumber and wavelength of a laser photon, and k_0 is the centroid wavenumber of the laser beam; β_0, σ_k and σ_l are the Rayleigh range, and the RMS energy spread and bunch length of the laser beam.

The number of collisions occurring during a time $\mathrm{d}t$ and inside a phasespace volume $\mathrm{d}^3p\ \mathrm{d}^3k\ \mathrm{d}V$ of the incident laser and electron beams is given by [4]

$$
\begin{aligned}
\mathrm{d}N(\vec{r},\vec{p},\vec{k},t) &= \sigma_{tot}(\vec{p},\vec{k})c(1-\vec{\beta}\cdot\vec{k}/|\vec{k}|)n_e(\vec{r},\vec{p},t)\\
&\quad\times n_p(\vec{r},\vec{k},t)\mathrm{d}^3p\mathrm{d}^3k\mathrm{d}V\mathrm{d}t, \qquad (5)
\end{aligned}
$$

where $\sigma_{tot}(\vec{p},\vec{k})$ is the total Compton scattering cross section which is determined by the momenta of the incident electron and laser photon, $\vec{p}$ and $\hbar\vec{k}$; $\vec{v}_e$ and $\vec{v}_p$ are the velocities of the electron and photon, and $\vec{\beta}=\vec{v}_e/c$; $n_e(\vec{r},\vec{p},t)=N_e f_e(\vec{r},\vec{p},t)$ and $n_p(\vec{r},\vec{k},t)=N_p f_p(\vec{r},\vec{k},t)$, where N_e and N_p are the total numbers of electrons and laser photons in their respective pulses.

To calculate the spatial and energy distributions of a Compton gamma-ray beam, one need to integrate Eq. (5) over the entire collision time t and phasespace volume $\mathrm{d}^3p\ \mathrm{d}^3k\ \mathrm{d}V$. However, during the integration, the differential cross section should be used instead of the total cross section, and two constraints need to be imposed [5, 7].

The first constraint is the geometric one, which assures the gamma-ray photon generated at the location $\vec{r}$ can reach the location $\vec{r}_d$ shown in Fig. 1. The projection of this constraint in the x-z and y-z planes is given by

$$
\theta_x+x'=\frac{x_d-x}{L},\ \theta_y+y'=\frac{y_d-y}{L}. \qquad (6)
$$

Here, θ_x and θ_y are the projections of the scattering angle θ_f in the x-z and y-z planes, i.e., $\theta_x=\theta_f\cos\phi_f$, $\theta_y=\theta_f\sin\phi_f$ and $\theta_f^2=\theta_x^2+\theta_y^2$, where θ_f and ϕ_f are the angles defined in the electron coordinate system (x_e, y_e, z_e) in which the electron is incident along the z_e-axis direction (Fig. 1). x' and y' are the angular divergences of the incident electron, i.e., the angles between the electron momentum and z-axis. L is the distance between the collision plane and the detection plane (or the collimation plane). Note that a far field detection (or collimation) has been assumed, i.e., $L\gg|\vec{r}|$ and $L\approx|\vec{r}_d|$.

The second constraint is the energy conservation. Due to the finite energy spread of the electron beam, the gamma-ray photon with an energy of E_g can be scattered from the electron with an energy of $\bar{\gamma}mc^2$ and scattering angle of θ_f. Mathematically, this constraint can be expressed as

$$
\delta(\bar{E}_g-E_g),\text{where } \bar{E}_g=\frac{4\bar{\gamma}^2E_p}{1+\bar{\gamma}^2\theta_f^2+4\bar{\gamma}E_p/mc^2}. \qquad (7)
$$

Thus, the spatial and energy distributions of a Compton gamma-ray beam is given by

$$
\begin{aligned}
\frac{\mathrm{d}N(E_g,x_d,y_d)}{\mathrm{d}\Omega_d\mathrm{d}E_g} &\approx N_eN_p\int\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\delta(\bar{E}_g-E_g)c(1+\beta)\\
&\quad\times f_e(x,y,z,x',y',p,t)\\
&\quad\times f_p(x,y,z,k,t)\mathrm{d}x'\mathrm{d}y'\mathrm{d}p\mathrm{d}k\mathrm{d}V\mathrm{d}t, \qquad (8)
\end{aligned}
$$

where $\mathrm{d}\Omega_d=\mathrm{d}x_d\mathrm{d}y_d/L^2$, and $\mathrm{d}\sigma/\mathrm{d}\Omega$ is the differential Compton scattering cross section. Note that a head-on collision between electron and laser beams has been assumed, and the density function $f_e(\vec{r},\vec{p},t)$ in Eq. (5) has been replaced with $f_e(x,y,z,x',y',p,t)$ of Eq. (4) under the approximation $p_z\approx p$ for a relativistic electron beam. In addition, the integration $\int\cdots f_p(\vec{r},\vec{k},t)\mathrm{d}^3k$ is replaced with $\int\cdots f_p(x,y,z,k,t)\mathrm{d}k$, where $f_p(x,y,z,k,t)$ is defined in Eq. (4). Integrations over $\mathrm{d}k_x$ and $\mathrm{d}k_y$ have been carried out since the differential cross section has a very weak dependency on k_x and k_y for a relativistic electron beam.

Assuming head-on collisions for each individual scattering event, neglecting the angular divergences of the laser beam and replacing x' and y' with θ_x and θ_y, we can integrate Eq. (8) over $\mathrm{d}V$, $\mathrm{d}t$ and $\mathrm{d}p$ to yield Eq. (9), where

$$
\begin{aligned}
&\xi_x=1+(\alpha_x-\frac{\beta_x}{L})^2+\frac{2k\beta_x\varepsilon_x}{\beta_0},\ \zeta_x=1+\frac{2k\beta_x\varepsilon_x}{\beta_0},\\
&\sigma_{\theta x}=\sqrt{\frac{\varepsilon_x\xi_x}{\beta_x\zeta_x}},\xi_y=1+(\alpha_y-\frac{\beta_y}{L})^2+\frac{2k\beta_y\varepsilon_y}{\beta_0},\\
&\zeta_y=1+\frac{2k\beta_y\varepsilon_y}{\beta_0},\ \sigma_{\theta y}=\sqrt{\frac{\varepsilon_y\xi_y}{\beta_y\zeta_y}},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}N(E_g,x_d,y_d)}{\mathrm{d}E_g\mathrm{d}x_d\mathrm{d}y_d} &= \frac{r_e^2L^2N_eN_p}{4\pi^3\hbar c\beta_0\sigma_\gamma\sigma_k}\int_0^\infty\int_{-\sqrt{4E_p/E_g}}^{\sqrt{4E_p/E_g}}\int_{-\theta_{xmax}}^{\theta_{xmax}}\frac{1}{\sqrt{\zeta_x\zeta_y}\sigma_{\theta x}\sigma_{\theta y}}\frac{\gamma}{1+2\gamma E_p/mc^2} \\
&\times\left\{\frac{1}{4}\left[\frac{4\gamma^2E_p}{E_g(1+\gamma^2\theta_f^2)}+\frac{E_g(1+\gamma^2\theta_f^2)}{4\gamma^2E_p}\right]-2\cos^2(\tau-\phi_f)\frac{\gamma^2\theta_f^2}{(1+\gamma^2\theta_f^2)^2}\right\} \\
&\times\exp\left[-\frac{(\theta_x-x_d/L)^2}{2\sigma_{\theta_x}^2}-\frac{(\theta_y-y_d/L)^2}{2\sigma_{\theta_y}^2}-\frac{(\gamma-\gamma_0)^2}{2\sigma_\gamma^2}-\frac{(k-k_0)^2}{2\sigma_k^2}\right]\mathrm{d}\theta_x\mathrm{d}\theta_y\mathrm{d}k. \qquad (9)
\end{aligned}
$$

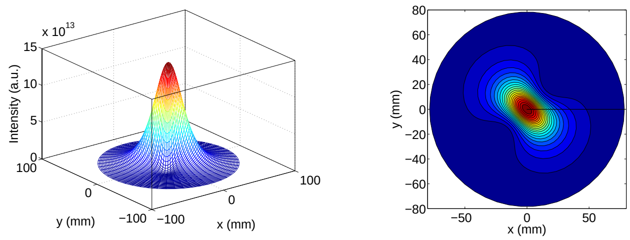

Figure 2: Calculated spatial distribution of a Compton gamma-ray beam at a transverse plane 30 meters downstream from the collision point. The left figure is a 3D plot of the distribution and the right one is its contour plot. The gamma beam with the peak energy of about 9 MeV is produced by a 515 MeV electron beam scattering with a 545 nm linearly polarized laser beam. The polarization vector of the laser beam has an azimuthal angle of 45 degree with respect to the horizontal plane.

$$
\theta_f=\sqrt{\theta_x^2+\theta_y^2},\ \theta_{xmax}=\sqrt{4E_p/E_g-\theta_y^2},\ \sigma_\gamma=\frac{\sigma_{E_e}}{mc^2},
$$

$$
\gamma=\frac{2E_gE_p/mc^2}{4E_p-E_g\theta_f^2}\left(1+\sqrt{1+\frac{4E_p-E_g\theta_f^2}{4E_p^2E_g/(mc^2)^2}}\right), \qquad (10)
$$

and σ_{E_e} is the RMS energy spread of the electron beam.

The integrations with respect to k, θ_y and θ_x in Eq. (9) must be carried out numerically. Thus, a numerical integration Compton scattering code in the C++ computing language (CCSC) has been developed to evaluate the integral of Eq. (9).

The spatial and spectral distributions of Compton gamma-ray beams calculated using this code are illustrated in Figs. 2 and 3. More applications of this code can be found in [8, 9]. This code has been benchmarked with our another Compton scattering code based upon a Monte Carlo simulation algorithm [9, 10].

CONCLUSIONS

In this paper, we present an analytical algorithm to model a gamma-ray beam produced by Compton scattering a laser beam and an electron beam with Gaussian phasespace distributions. Based upon this algorithm, we have developed a numerical integration code to produce smooth results for the spatial and spectral distributions of a Compton beam. This code has been successfully used to characterize the High Intensity Gamma-ray Source (HIγS) at Duke University for varying electron and laser beam parameters as well as different gamma-ray beam collimation conditions.

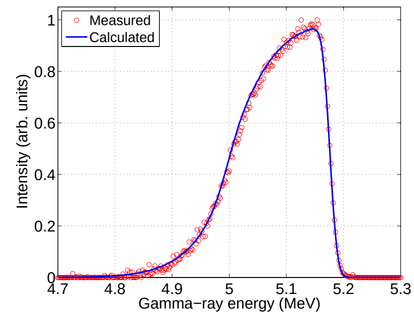

Figure 3: Comparison of measured and calculated Compton gamma beam spectra. The gamma-ray beam is produced by Compton scattering of a 466 MeV electron beam and a 790 nm laser beam at the HIγS facility. The energy spread of the electron beam is 0.1%, and horizontal and vertical emittance are 7.8 and 1.0 nm-rad, respectively. The collimator with an aperture radius of 12.7 mm is placed 60 meters downstream from the collision point.

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