

# RESONANT TRANSITION RADIATION INDUCED BY AN ULTRASHORT ELECTRON BUNCH FROM ALUMINUM FOIL STACK

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## Abstract

Resonant transition radiation (RTR) driven by a femtosecond electron beam is being studied. An aluminum foil stack with vacuum spacers is used as the radiator. With a 27 MeV electron bunch with pulse duration at ~ 100 fsec incident normally on the aluminum foil stack, high photon yields in hard X-ray regime can be obtained. Characteristics of the radiation such as emission spectrum, spatial distribution are calculated. The dependence of RTR photon yields on beam size and bunch length are also studied.

## INTRODUCTION

When a charge particle travels at constant velocity through the boundary of two media of different dielectric constants, radiation will be induced around the boundary. This phenomenon is called Transition Radiation (TR) which was first studied by V. L. Ginzburg in 1946 [1]. Transition radiation has broad emission spectrum ranging from far infrared to hard x-ray. Since the characteristic of such radiation depends on the kinetic energy of a charge particle. Therefore, it is often used to characterize the spatial distribution and the energy of a relativistic electron beam.

Intense and tunable X-ray sources have wide range applications in many areas. So far, synchrotron radiation is the most popular choice for this kind of X-ray sources. However, ring-based synchrotron light sources are large scale and expensive facilities that is not available in many areas in the world. Therefore, alternative methods to generate X-ray source with brilliance that are comparable to 3<sup>rd</sup> generation synchrotron light source have been proposed [2-3]. It is of considerable interest to study transition radiation for X-ray generation because the beam energy required by such source is much lower than a ring-based machine. Although the spectrum of TR is broadband, the possibility of generating narrow line X-ray had been investigated [4]. The experiments of single mode resonant transition radiation in soft X-ray region were first carried out by Koji Yamada et al. [5]. A periodic interface structure was used to enhance photon yield and obtain angular dependent single mode soft X-ray by diminishing the period of the multilayer target.

In recent years, researchers have raised the concern about the application of coherent x-ray sources. In comparison with incoherent light, coherent light has the advantages of detectable phase information and much higher intensity. A series of experiments on coherent transition radiation in far-infrared region have been

performed by using sub-picosecond electron bunch [6]. Condition for coherence of the radiation in free space is known that the electron bunch length must be shorter than half of the radiation wave length. For transition radiation, however, it is not emitted from the beam itself but from the media induced by the beam. In this paper, we derive the equations to calculate resonant transition radiation (RTR) from foils stack driven by an ultra-short electron bunch. The spectrum and spatial distribution are shown in the following section. Finally, the factors affecting the photon yield of RTR is also investigated.

## RTR EMITTED FROM FOIL STACK BY THE ELECTRON BUNCH

The electric field of TR, which is emitted by a single electron passing through the interface between two kinds of media, is given by [7]

$$E_r(z) = \frac{iq\theta^2}{2\pi^2\omega} \times (L_1 - L_2) \exp\left(i\frac{\omega}{c}\sqrt{\epsilon_2\mu_2}z\right) \quad (1)$$

where  $\omega$  is the angular frequency of the radiation,  $c$  is the speed of light in vacuum, and  $\theta$  is the angle with respect to the electron path which is perpendicular to the boundary surface.  $L_1$  and  $L_2$  are proportional to the formation lengths of media 1 and 2. In x-ray region, it can be approximated by

$$L_i \approx \frac{1}{\gamma^{-2} + \theta^2 + (\omega_{pi}/\omega)^2} \quad (2)$$

where  $\omega_{pi}$  is the plasma frequency of the medium. When we begin to consider multi-foils TR emitted by electron bunch, Eq. (1) should be modified and multiplied by some factors. By shifting the origin of the coordinate to the center of the electron bunch, the bunching factor at  $t=0$  can be expressed as

$$B_j = \exp\left(-i\frac{\omega}{c}r_j\theta - i\frac{\omega}{v_j}z_j\right) \quad (3)$$

Here, we assume that the radiation emitted by each electron at boundary is plane wave. The approximation  $\sin\theta \approx \theta$  is applied in above equations. More details are discussed in the following.

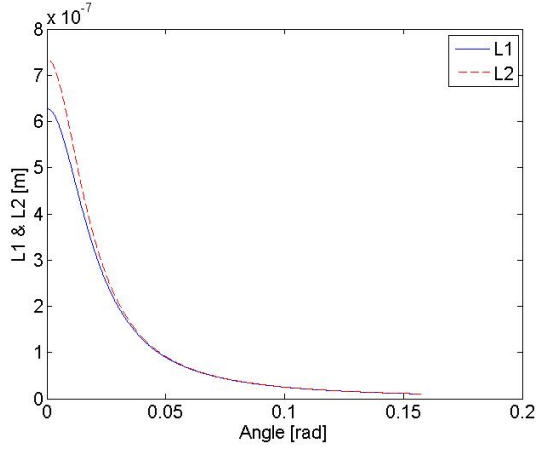


Figure 1: The angular dependence of  $L_1$  and  $L_2$ . The photon energy here is 3keV and the  $\gamma$  is 53.

The subscript  $j$  in Eq. (3) represents each electron in the electron bunch, and  $v$  is the velocity of the electron in  $z$  direction. We include the effect of energy spread here, so the other two factors  $f_1$  and  $f_n$ , which are single foil and multi foil factor [8], also should be modified and written as

$$f_{1j} = -1 + \exp(-\mu_1 l_1 + i\varphi_{1j}) \quad (4)$$

$$f_{nj} = \frac{1 - \exp[-N\sigma + iN(\varphi_1 + \varphi_2)]}{1 - \exp[-\sigma + i(\varphi_1 + \varphi_2)]} \quad (5)$$

where

$$\sigma = \mu_1 l_1 + \mu_2 l_2 \quad (6)$$

$$\varphi_{ij} = \frac{l_i \omega}{2cL_{ij}}, \quad L_{ij} = \frac{1}{\gamma_j^{-2} + \theta^2 + (\omega_{pi}/\omega)^2} \quad (7)$$

$\mu_1$  and  $\mu_2$  are the absorption coefficients of each medium in x-ray region.  $N$  is the numbers of the foils. In our case, we use the aluminum foil to be the radiator with thickness  $l_1$ , and each foil is separated by a distance  $l_2$ . The spacing is the vacuum, so  $\mu_2$  and  $\omega_{p2}$  are zero. With Eq. (1) ~ Eq. (5), radiation energy emitted per unit radian per unit frequency is given by

$$\frac{d^2W}{d\omega d\theta} = \frac{q^2 \theta^2}{\pi^2 c} \left| \sum_j^{N_0} \Delta L_j \times B_j \times f_{1j} \times f_{nj} \right|^2 \quad (8)$$

where  $\Delta L_j$  is  $(L_{1j} - L_{2j})$ , and  $N_0$  is the numbers of electrons.  $L_{ij}$  is dependence on the angle and the frequency. In figure 1,  $\Delta L_j$  is approach to zero as the angle is larger. It implies that the energy of RTR is almost concentrated in a small cone. When we apply the boundary conditions to derive the Eq. (1) and Eq. (8), the

space charge effect of the electrons in the bunch is neglected. The charge field and radiation field of each electron are derived independently. The electric field of each electron is added together to obtain the spectra energy density of RTR for ultra-short electron bunch. If the radiation field emitted by each electron is completely incoherent, the spectra energy density is proportional to the number of particles in the bunch. By comparing the calculation results with the completely incoherent situation, the coherence of RTR emitted by ultra-short electron bunch can be estimated.

## CALCULATION RESULTS OF RTR FOR THE ELECTRON BUNCH

In this section, the calculation results of RTR for electron bunch is shown. The electron bunch is generated from the design of thermionic cathode rf gun driver linac, and the parameters of the electron bunch we assess are listed in Table 1. From Eq.(4) and Eq.(5), we can derive the resonance conditions of RTR. The twenty pairs of the 261nm aluminum foil and 272nm vacuum spacing are used to evaluate the 3keV fundamental mode. As shown in figure 2(a), there is a single peak at 33.8mrad for 3keV photons.

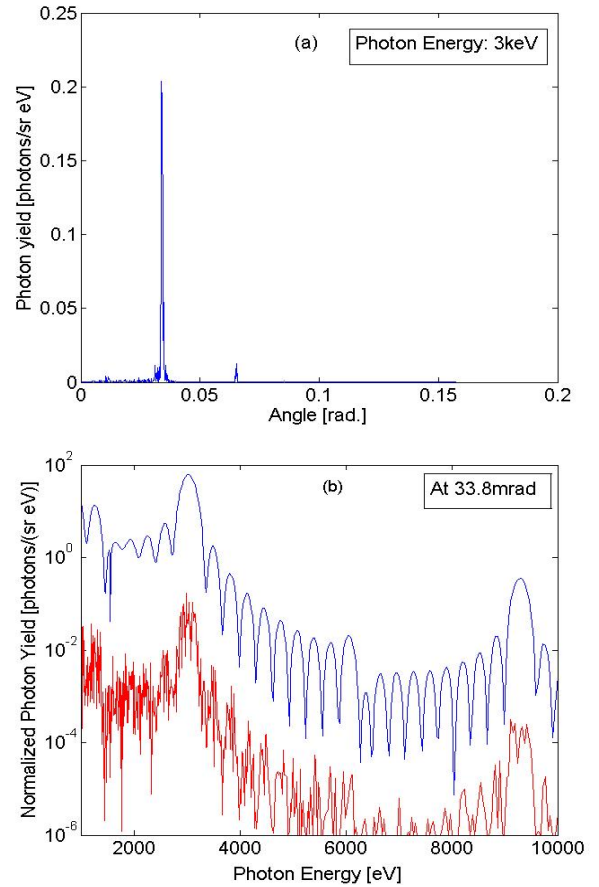


Figure 2: The spatial and spectra distribution of RTR for electron bunch. The electron bunch contains 1179 particles, and each particle represents single electron. The blue line in figure 2(b) represents the case of ideal bunch.

Figure 2(b) is the spectrum of RTR at 33.8mrad. Although the simulation result of electron bunch is used to examine the spectrum of RTR, the trend of spectra distribution shown in figure 2(b) is similar to the case of the ideal electron bunch. The ideal electron bunch we use here means that the bunch length and beam spot size are far smaller than the wavelength of radiation. The largest peak occurs at 3keV which coincides with our expectation. The space charge effect is not considered here.

The bunch charge we assess is 29pC, and the number of particles is 1179. In the other words, each particle in our simulation represents  $1.56 \times 10^5$  electrons. As a result, there are  $1.56 \times 10^5$  electrons emit intense coherent radiation at the same point. To avoid this error in the calculation, the charge of each particle is replaced with the charge of single electron, and therefore the number of electrons in the bunch we assess is only 1179. For examining the influence of the bunch factor on RTR, we use General Particle Tracer (GPT) to generate several electron bunches. Each electron bunch has uniform distribution in x-y plane and the numbers of the electron are uniformly distributed in the z direction. The calculated results for dependence of 3keV photon yield on bunch length are shown figure 3. According to Eq. 8 and the discussion in previous section, the mean influence on photon yield of RTR is the distribution of electrons in z direction. The transverse distribution of electrons affects the photon yield of RTR slightly, so we tune the bunch length with fixed spot size to observe the variation of photon yield. As the green line indicating in figure 3, there is no variation on photon yield for different bunch length. Although the influence of transverse distribution of electron on the photon yield is slight, the diameter of spot size still needs to be focused bellow 30nm to obtain the intense radiation. The photon yield of the electron bunch with 30nm and 0.3nm spot size is less than the electron bunch with  $30 \mu\text{m}$  spot size when the bunch is approximately larger than 10 nm. The reduction is due to the partial coherence and destructive interference.

Table 1: The parameters of electron bunch we assess

Electron Bunch	
Energy	27.16 MeV
Charge	29.46 pC
Particle number	1179
Bunch length	96.63 fs
Beam size	30.49 $\mu\text{m}$
Standard $\gamma$ deviation	0.17

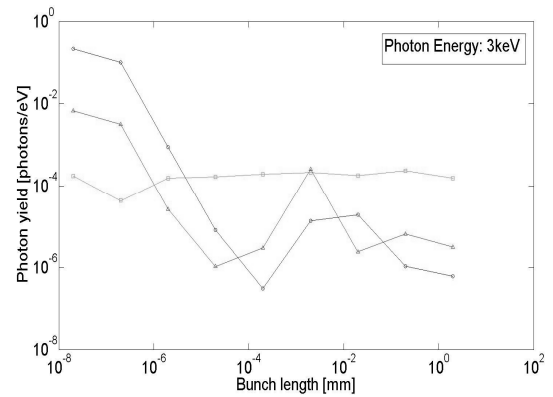


Figure 3: Dependence of photon yield on bunch length. The electron bunches with  $30 \mu\text{m}$ , 30nm, and 0.3nm spot size correspond to green line, red line and blue line.

## CONCLUSION

We have derived the equations to calculate RTR emission from a foil stack which is driven by an electron bunch. We have demonstrated how to design a periodic target to obtain the desired photon energy at specific emission angle. The spatial distribution and energy of each electron is considered simultaneously. The results are shown in figure 2 when an ultra-short electron bunch is used. Because of the energy chirp of the electron bunch, from the thermionic cathode rf gun driver linac we use is too small to affect the emission of RTR, the influence of spatial distribution of electrons on RTR is dominant. Furthermore, the transverse distribution of electrons affects the photon yield is much less than the longitudinal distribution. The diameter of spot size still need to be less than sub-micron meter to obtain the intense RTR emitted at boundary surface.

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