REVERSED CHERENKOV-TRANSITION RADIATION AND PROSPECT OF ITS APPLICATION TO BEAM DIAGNOSTICS*

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Abstract

We describe both analytically and numerically radiation of a small bunch intersecting the interface between vacuum and a medium which can be realized as modern metamaterial. In particular, effects of reversed Cherenkov radiation (RCR) and, especially, reversed Cherenkov-transition radiation (RCTR) are considered. As we noted earlier these phenomena can be used for detection of charged particles and diagnostics of beams. Here we continue to describe useful properties of RCTR (for example, two-thresholdness in the bunch velocity domain) in the case of the boundary between vacuum and an isotropic left-handed medium (LHM). Moreover, we investigate the bunch radiation in the case of certain anisotropic plasma-like material. We obtain conditions for RCR and RCTR effects, describe properties of RCTR, and discuss possible applications.

INTRODUCTION

In 1968 Veselago [1] introduced the concept of LHM, i.e. a medium having simultaneously negative ε and negative μ. Since that a lot of activity was given to the practical realization of LHM. The development of Pendry’s ideas [2,3] has resulted in vigorous progress in design and fabrication of metamaterials (MTMs). For example, the first left-handed MTM operated at GHz frequencies [4], today the THz and visible ranges are mastered [5]. It is reasonable to hope that the rather exotic electromagnetic properties can be realized via MTMs. It should be noted that the applications of MTMs in the area of accelerator physics are actively investigated in the last few years [6-9].

In this paper we investigate the radiation, especially RCTR, produced by a small bunch intersecting the interface between vacuum and certain artificial medium. The first part of the present paper is a further exploration of the RCTR effect in the case of isotropic LHM initiated in our recent publications [8-11]. The second part is the pioneering theoretical investigation of the RCTR effect in the case of “anisotropic plasma”.

RCTR IN THE CASE OF ISOTROPIC LHM

Here we analyze the radiation of a small bunch intersecting the interface between vacuum and isotropic LHM. We suppose that LHM is described by frequency dependent permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ (as is the case in MTMs). The theoretical aspects and first results have been presented in our recent papers [8-11]. Note that the RCTR effect in the vacuum area exhibits the most promise for the beam diagnostics for the following reasons.

The RCTR in the vacuum area is the RCR refracted at the boundary. It exists under the following conditions [8-11]

$$\beta_{\mathrm{CR}}(\omega) < \beta < \beta_{\mathrm{TIR}}(\omega),$$

$$\beta_{\mathrm{CR}}(\omega) = \frac{1}{|\operatorname{Re} n(\omega)|}, \quad \beta_{\mathrm{TIR}}(\omega) = \frac{1}{\sqrt{\operatorname{Re} n^2(\omega) - 1}}, \quad (1)$$

where $n^2 = \varepsilon\mu$. We use the following simplified model for the description of LHM:

$$\varepsilon(\omega) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2 + 2i\omega_{\mathrm{de}}\omega}, \quad \mu(\omega) = 1 - \frac{\omega_{\mathrm{pm}}^2}{\omega^2 + 2i\omega_{\mathrm{dm}}\omega}, \quad (2)$$

where ω_{pe} and ω_{pm} are, respectively, electric and magnetic “plasma” frequencies, parameters ω_{de} and ω_{dm} are responsible for losses. In this case, the area of the vacuum RCTR effect determined by (1) is presented in Fig. 1.

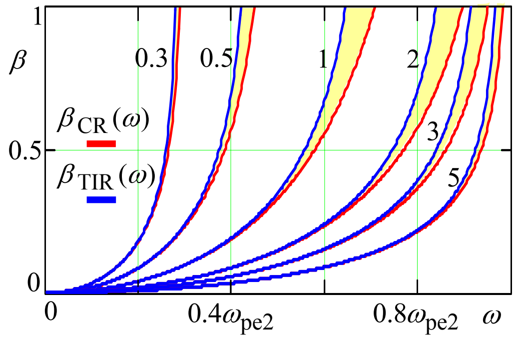

Figure 1: The velocity range of the vacuum RCTR effect (the region enclosed between $\beta_{\mathrm{CR}}(\omega)$ and $\beta_{\mathrm{TIR}}(\omega)$) depending on frequency for different ratios $\omega_{\mathrm{pm}}/\omega_{\mathrm{pe}}$ (indicated near the curves).

As one can see from Fig. 1, vacuum RCTR is a double threshold effect in both frequency and bunch velocity domain. In other words, if ω is fixed, then RCTR occurs within the range of velocities $\beta_{\mathrm{CR}} < \beta < \beta_{\mathrm{TIR}}$; if β is fixed, then RCTR occurs within the frequency range $\omega_- < \omega < \omega_+$, where $\operatorname{Re} n^2(\omega_-) = 1 + \beta^{-2}$, $\operatorname{Re} n^2(\omega_+) = \beta^{-2}$. Moreover, varying the “plasma”

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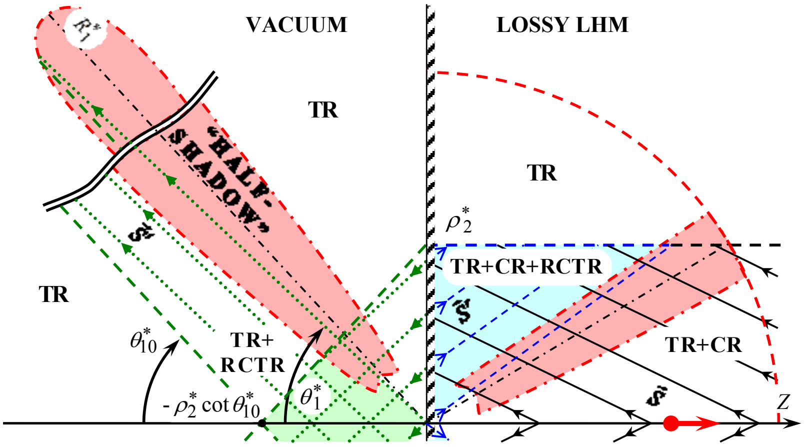

Figure 2: Areas of significance of the different parts of the electromagnetic field in the case of lossy isotropic LHM. The solid, dashed and dotted lines are parallel to the Poyting vector of the RCR ($\vec{S}$), reflected RCTR ($\vec{S}^r$) and transmitted RCTR ($\vec{S}^t$), respectively. Vacuum RCTR exist inside the region hatched by the dotted lines. The maximum size of this region is determined by R_1^*. Dash-dotted lines indicate boundaries of "half-shadow" areas: TR and RCTR can be considered separately outside these areas only.

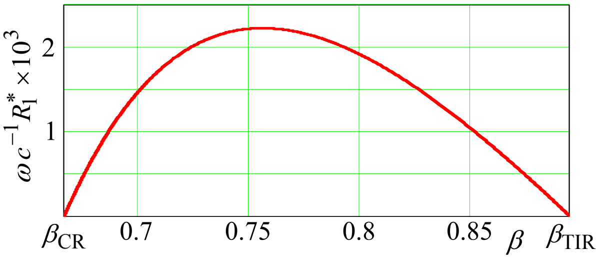

Figure 3: Dependence of R_1^* (in units of $c\omega^{-1}$) on β. Medium parameter is $n = -1.5 + i0.01$.

frequencies, the width of the RCTR area can be modified. For example, very narrow width can be attained.

These peculiarities can be applied to the beam diagnostics. Choosing the proper MTM and fixing the operating frequency, the desired lower and upper thresholds can be attained. This allows the design of a double threshold detector separating particles within the predetermined range. Alternatively, varying the operating frequency, the RCTR threshold frequencies can be measured and the beam energy can be estimated.

It should be noted that real MTM always possesses nonzero losses resulting in the decay of RCTR in MTM and in vacuum area as well. The result of our rigorous consideration is presented in Fig. 2. RCTR can be observed in vacuum at distance $R \le R_1^*$, where

$$R_1^* = \frac{8(\beta^2 - {\rm Re}\, n^2 \beta^2 + 1)({\rm Re}\, n^2 \beta^2 - 1)}{\omega c^{-1} \beta^4 ({\rm Im}\, n^2)^2} . \qquad (3)$$

At $R > R_1^*$ RCTR is diminished at least by the factor of e^2. One can observe the interference in the RCTR at $0 < \theta \le \theta_1^*$ given that $R \le \rho_2^* \cot\theta_{10}^*$, where

$$\rho_2^* = \frac{4\sqrt{{\rm Re}\, n^2 \beta^2 - 1}}{\omega\beta\, |{\rm Im}\, n^2|}, \quad \cot\theta_{10}^* = \sqrt{\frac{\beta^2 - {\rm Re}\, n^2\beta^2 + 1}{{\rm Re}\, n^2 \beta^2 - 1}}, \qquad (4)$$

$$\theta_1^* = \theta_{10}^* - \delta\theta_1^*, \quad \delta\theta_1^* = \sqrt{c/(\omega R_1^*)} .$$

The dependence of R_1^* on β is presented in Fig. 3. As one can see, RCTR on the GHz frequencies can penetrate vacuum a distance about several meters which is more than sufficient for observation. Note that the "half-shadow" areas indicated in Fig. 2 are regions where RCTR and TR can not be considered separately.

RCTR IN THE CASE OF ANISOTROPIC PLASMA-LIKE MEDIUM

Here we present the results of investigation of the RCTR effect occurring when a bunch traverses from vacuum into nonmagnetic anisotropic medium described by diagonal tensor $\hat{\varepsilon}$ with components $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_\perp$, $\varepsilon_{zz} = \varepsilon_\parallel$. It is supposed that $\varepsilon_\perp$ and $\varepsilon_\parallel$ are characterized by a plasma-like dispersion:

$$\varepsilon_\perp(\omega) = 1 - \frac{\omega_{\rm p\perp}^2}{\omega^2 + 2i\omega_{\rm d\perp}\omega}, \quad \varepsilon_\parallel(\omega) = 1 - \frac{\omega_{\rm p\parallel}^2}{\omega^2 + 2i\omega_{\rm d\parallel}\omega} . \qquad (5)$$

We found that three conditions must be satisfied for generation of RCTR in the vacuum area.

(a) CR must be generated in the medium: $\min(\omega_{\rm p\perp}, \omega_{\rm p\parallel}) < \omega < \max(\omega_{\rm p\perp}, \omega_{\rm p\parallel})$;

(b) CR must be reversed to reach the boundary: $\omega < \omega_{\rm p\perp}$;

(c) RCR must penetrate vacuum through the boundary, i.e. it may not undergo the total internal reflection: $\omega^2 < \Omega^2 = \omega_{\rm p\parallel}^2 (1-\beta^2)/2 + \sqrt{\omega_{\rm p\parallel}^4 (1-\beta^2)^2/4 + \omega_{\rm p\parallel}^2 \omega_{\rm p\perp}^2 \beta^2}$.

To satisfy requirements (a) and (b) one should provide $\omega_{\rm p\parallel} < \omega_{\rm p\perp}$. Taking (c) into account, we obtain the following condition of the vacuum RCTR effect:

$$\omega_{\rm p\parallel} < \omega < \Omega(\beta), \quad \text{or} \quad \beta > \beta_{\rm RCTR}(\omega), \qquad (6)$$

where

$$\beta_{\rm RCTR}(\omega) = \sqrt{\omega^2 \omega_{\rm p\parallel}^{-2} (\omega^2 - \omega_{\rm p\parallel}^2)(\omega_{\rm p\perp}^2 - \omega^2)^{-1}} .$$

For the demonstration of this effect we consider the spatial distribution of the magnetic field Fourier harmonic on the angle θ at $R = \text{const}$ (Fig. 5). We use two

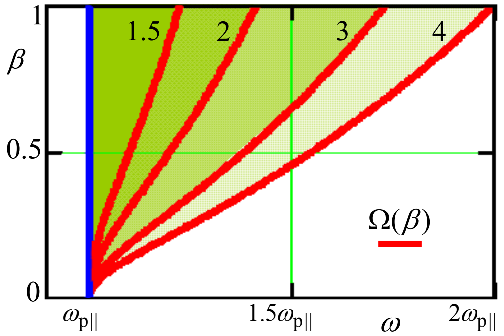

Figure 4: Areas of the vacuum RCTR effect (region enclosed between $\omega = \omega_{p||}$ and $\Omega(\beta)$) in the case of anisotropic plasma (5), (6). The value of ratio $\omega_{p\perp} / \omega_{p||}$ is indicated near each curve.

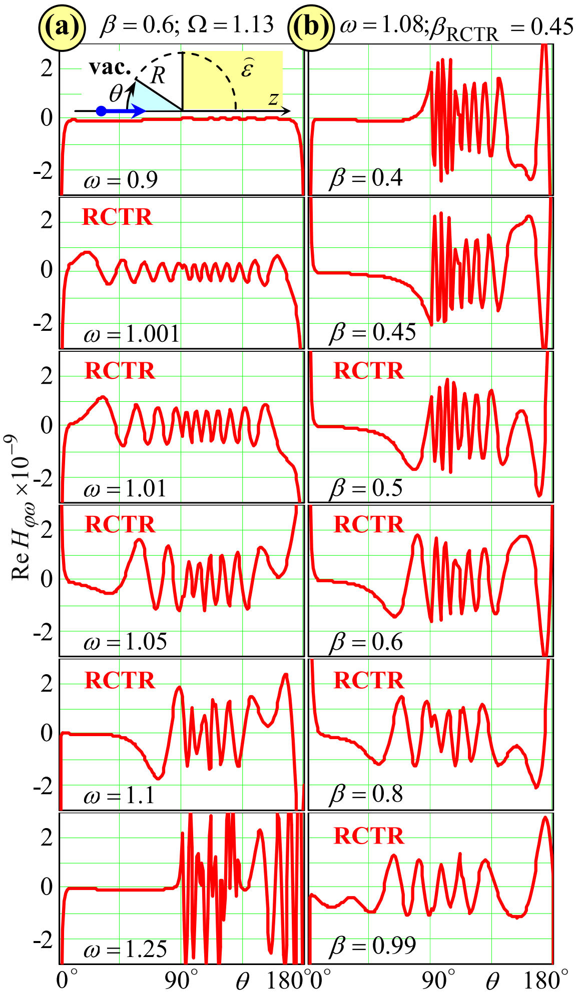

Figure 5: The modification of the dependence $\mathrm{Re}\,H_{\varphi\omega}(\theta)$ with increasing ω for $\beta = \mathrm{const}$ (a) and with increasing β for $\omega = \mathrm{const}$ (b). All frequencies are in units of $\omega_{p||}$; $\omega_{p||} = 2\pi \cdot 10\,\mathrm{GHz}$, $\omega_{p\perp} = 1.5$, $R = 14$ cm

possibilities for the demonstration: to vary ω at $\beta = \mathrm{const}$ (Fig. 5 (a)) or to vary β at $\omega = \mathrm{const}$ (Fig. 5 (b)). Our numerical calculations confirm the theoretical predictions of Eq. 7. Note that RCTR is concentrated basically in the vicinity of the boundary. However, RCTR generated at frequencies close to $\omega_{p||}$ can be observed near the bunch motion line. This fact and other properties of radiation can be useful for diagnostics of metamaterials, for example, for determination of the effective plasma frequency.

In conclusion, it should be underlined that reversed Cherenkov-transition radiation in the vacuum area can be useful for diagnostics of beams. In the case of the vacuum – isotropic LHM interface RCTR has two thresholds in respect to the charge velocity. This allows selection of particles having energy (velocity) within the predetermined range. In the case of the boundary between vacuum and anisotropic plasma like material the RCTR has only the lower threshold. This allows selection of particles with energy exceeding some predetermined value. Moreover such material can be convenient for measurement of energy (velocity) of particles because the barrier velocity is a nearly linear function of frequency.

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