PROGRESS ON ANALYTICAL MODELING OF COHERENT ELECTRON COOLING*

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Abstract

We report recent progresses on analytical studies of Coherent Electron Cooling. The phase space electron beam distribution obtained from the 1D FEL amplifier is applied to an infinite electron plasma model and the electron density evolution inside the kicker is derived. We also investigate the velocity modulation in the modulator and obtain a closed form solution for the current density evolution for infinite homogeneous electron plasma.

INTRODUCTION

Since the scheme of FEL-based Coherent Electron Cooling was suggested[1, 2], efforts have been made to develop an analytical model with space charge and electron velocity distribution taken into account [3-6]. Recently, progresses were made in both completing and refining the analytical model.

In order to understand the electron dynamics in the kicker, we reduce the Vlasov-Poisson equation with arbitrary initial conditions into a second ODE for infinite electron plasma with kappa-2 velocity distribution. Then we apply the closed form solution obtained from 1D FEL theory as the initial phase space density modulation at the entrance of the kicker[3]. Assuming the initial transverse spatial modulation at the entrance being uniform and the initial transverse velocity modulation being kappa-3/2, a closed form solution is obtained to describe the electron density evolution inside the kicker.

Apart from the density modulation, the velocity distribution of electrons is also modified by the moving ion in the modulator and it contributes to the initial seeding at the entrance of the FEL amplifier. Under the framework of [4], we obtained a close form solution of the current density modulation for an infinite homogenous electron plasma.

ELECTRON DENSITY EVOLUTION IN A KICKER

If the ion velocity change is negligible within one pass of the kicker, the equation of motion in the kicker is identical to that in the modulator. Assuming that the unperturbed velocity distribution of electrons has the form

$$f_0(\vec{v}) = \frac{1}{\pi^2 \beta_x \beta_y \beta_z}\left(1+\frac{v_x^2}{\beta_x^2}+\frac{v_y^2}{\beta_y^2}+\frac{v_z^2}{\beta_z^2}\right)^{-2} \quad (1)$$

and following the same procedures as shown in [4], the Vlasov-Poisson equation can be reduced to the following second ODE in the wave vector space,

$$\frac{d^2}{dt^2}\tilde{R}_1\left(\vec{k},t\right)+\omega_p^2\tilde{R}_1\left(\vec{k},t\right) = -\omega_p^2\int_{-\infty}^{\infty}\tilde{f}_1\left(\vec{k},\vec{v},0\right)e^{-\lambda\left(\vec{k},\vec{v}\right)t}d^3v \quad (2)$$

where

$$\tilde{R}_1\left(\vec{k},t\right) \equiv \tilde{n}_1\left(\vec{k},t\right)e^{-\lambda\left(\vec{k},\vec{v}_0\right)t} - \int_{-\infty}^{\infty}\tilde{f}_1\left(\vec{k},\vec{v},0\right)e^{-\lambda\left(\vec{k},\vec{v}\right)t}d^3v, \quad (3)$$

$\tilde{n}_1\left(\vec{k},t\right)$ is the Fourier component of the density modulation, $\tilde{f}_1\left(\vec{k},\vec{v},0\right)$ is the initial phase space density modulation, $\vec{v}_0$ is the velocity of the moving ion, Z_i is the ion charge, ω_p is the plasma frequency of the electrons and

$$\lambda\left(\vec{k},\vec{v}\right) \equiv i\vec{k}\cdot\vec{v} - \sqrt{\left(k_x\beta_x\right)^2+\left(k_y\beta_y\right)^2+\left(k_z\beta_z\right)^2}. \quad (4)$$

The driving term due to ion shielding is dropped from the r.h.s. of equation (2) as the solution is identical to what is found in the modulator. We left with the driving term responsible for the dispersive effects due to the initial velocity modulation. In order to proceed, we assume the phase space density modulation has the following form

$$\tilde{f}_1\left(\vec{k},\vec{v},0\right) = \frac{4\pi\delta\left(k_x\right)\delta\left(k_x\right)}{\sigma_{v_x}\sigma_{v_y}}\tilde{f}_1\left(k_z,v_z,0\right)\left(1+\frac{v_x^2}{\sigma_{v_x}^2}+\frac{v_y^2}{\sigma_{v_y}^2}\right)^{-\frac{3}{2}}, \quad (5)$$

where σ_{v_x} and σ_{v_y} are constants describing the spread of the transverse velocity modulation. From 1D FEL theory[3], the initial longitudinal phase space density modulation reads

$$\tilde{f}_1\left(k_z,v_z,0\right) = -\frac{e\theta_s\gamma_z n_0 c^2}{2\pi^2\Gamma\varepsilon_0}\frac{v_z\sigma_{v_z}}{\left(\sigma_{v_z}^2+v_z^2\right)^2}e^{i\varphi_0(k_z)} \sum_{j=1}^{3}A_j\frac{1+i\hat{\Lambda}_p^2\lambda_j\left(\hat{C}\right)}{\lambda_j+i\left(\hat{C}+\frac{\beta}{\rho c}v_z\right)}e^{\lambda_j\left(\hat{C}\right)\cdot\hat{z}_{FEL}}, \quad (6)$$

where

$$\varphi_0\left(k_z\right) \equiv k_w z_{FEL} - \frac{k_z}{\gamma_z}\frac{z_{FEL}}{2} \quad (7)$$

is a phase factor appeared when transforming from the lab frame to the beam frame. Other variables have identical definition as defined in [5] and [3], i.e. k_w is the wiggler wave number, ρ is pierce parameter, θ_s is the electron rotation angle, $\Gamma \equiv 1/l_{gain}$ is the reciprocal of the FEL gain length, $\hat{z}_{FEL}$ is the FEL length in units of gain length, A_j are coefficients determined by initial modulation, λ_j are three eigenvalues solved from the polynomial equation in

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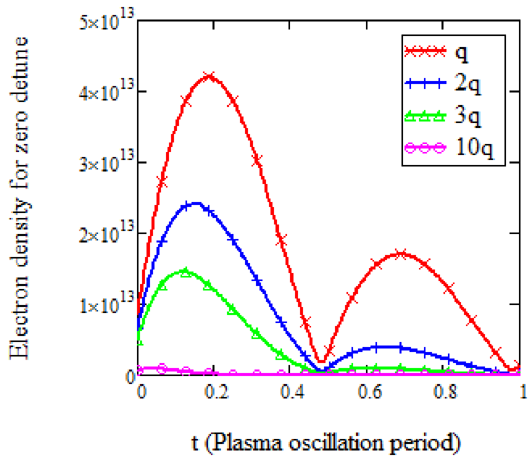

Figure 1. The electron density evolution inside kicker in wave vector domain. Curve labeled as ' Nq ' is for $\sigma_{v_z} = N\cdot 10^{-4}c$. The abscissa is time in units of plasma oscillation period and the ordinate is amplitude of the electron density modulation $n(k_z,t)$ at zero detuning , i.e. $k_z = 2\gamma_z k_w/\beta$, in units of m^{-2}.

FEL. $\hat{q}$ is parameter to describe the electron energy spread, $\hat{\Lambda}_p$ is the space charge parameter, $\hat{C}$ is the reduced detune defined as

$$\hat{C} \equiv (k_w - \frac{\omega}{2c\gamma_z^2})l_{gain} = \frac{1}{\Gamma}\left(k_w - \frac{k_z\beta}{2\gamma_z}\right),$$

and $\sigma_{v_z} \equiv \rho c\hat{q}/\beta = \beta_z$. Inserting (5) into (2) and further assuming $\sigma_{v_{x,y}} = \beta_{x,y}$, we obtain the following solution

$$\tilde{n}_1\left(\vec{k},t\right) = -Z_i 4\pi^2\delta^2\left(k_\perp\right)\tilde{\Lambda}\left(k_z\right)e^{ik_z v_{0z}t}e^{-\left|k_z\right|\sigma_{v_z}t}\left\{\cos\left(\omega_p t\right) + \frac{\sin\left(\omega_p t\right)}{\omega_p}\left[\left(\lambda_1 + i\hat{C}\right)\beta c\gamma_z\Gamma - ik_z v_{0z} + \left|k_z\right|\sigma_{v_z}\right]\right\}, \quad (8)$$

where

$$\tilde{\Lambda}\left(k_z\right) \equiv -i\frac{e^2\theta_s^2\beta^3\gamma_z n_0\left(1+i\hat{\Lambda}_p^2\lambda_1\right)e^{\lambda_1\cdot\hat{z}_{FEL}+i\varphi_0\left(k_z\right)}}{16\pi^2\rho\Gamma^2\varepsilon_0\mathcal{E}_0 a_x a_y\left(\hat{q}+\lambda_1+i\hat{C}\right)^2}F_2\left(\hat{C}\right)F_1\left(\hat{C}\right),$$

a_x, a_y and a_z are Debye radius in the modulator. $F_1\left(\hat{C}\right)$ and $F_2\left(\hat{C}\right)$ are factors associated with the electron modulation from the modulator and are defined as

$$F_1\left(\hat{C}\right) \equiv \int_0^\infty \xi\sin\xi e^{i\left(\frac{\omega}{\beta c}-k_w-k\right)z+\left(i\tilde{v}_{0z}\xi-f_\perp\right)a_\psi} f_\perp^{-3}\left[1+a_\psi f_\perp\right]d\xi,$$

$$F_2\left(\hat{C}\right) \equiv \begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\ \lambda_1 & \lambda_2 & \lambda_3\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2\end{pmatrix}^{-1}\begin{pmatrix}-i\left(\hat{\Lambda}_p^2+\hat{q}^2\right)\\ 1\\ -i\hat{C}\end{pmatrix},$$

where $a_\psi \equiv a_z\omega/\beta c\gamma_0$ and

$$f_\perp\left(\xi\right) = \sqrt{\xi^2 + \left(x/a_x + v_{0x}\xi/\beta_x\right)^2 + \left(y/a_y + v_{0y}\xi/\beta_y\right)^2}.$$

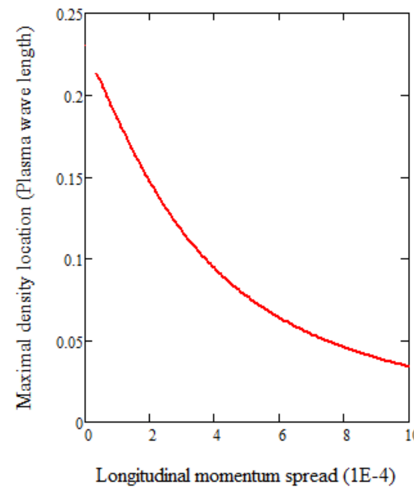

Figure 2. The location for maximal electron density modulation. The abscissa is the momentum spread and the ordinate is the location for maximal electron density variation.

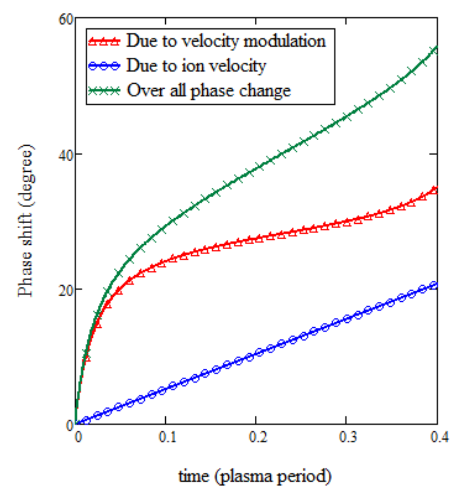

Figure 3. The phase slippage of the electron wave-packet with respect to ion. The abscissa is time in units of plasma period and the ordinate is the phase slippage from its initial value at the entrance of the kicker in units of degree. The ion has velocity of $0.5\sigma_{vz}$.

Figure 1 shows an example calculation of $\tilde{n}_1\left(k_z,t\right)$ at $k_z = k_0 \equiv 2\gamma_z k_w/\beta$ for various energy spreads in the FEL. It is worth noticing that the location of the maximal density modulation depends on the energy spread in the kicker. Actually, the maximal location can be derived from equation (8) by requiring $\frac{d}{dt}\left|\tilde{n}_1\left(k_0,t\right)\right| = 0$. Figure 2 shows one example calculation of the maximal modulation location as a function of the energy spread in the FEL. Apart from the amplitude, the phase slippage of the wave-packet with respect to the ion can also be calculated from equation(8). Figure 3 shows one example calculation of the phase slippage.

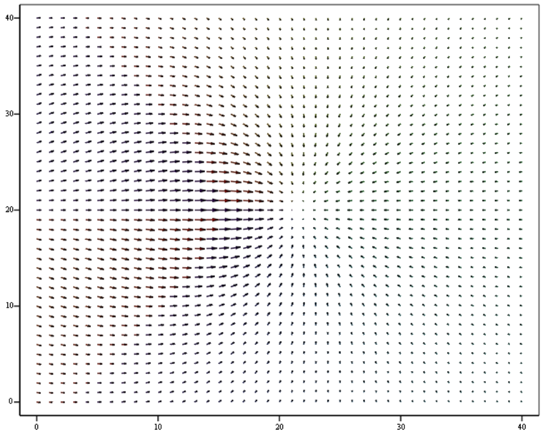

Figure 4 Dynamic current density modulation induced by an moving ion at $\omega_p t = \pi$ as calculated from (13). The abscissa is the position along the ion's moving direction from $-0.3a$ to $0.3a$ and the ordinate is the transverse coordinates in the same range. The ion is sitting in the center (20,20) moving with $v_0 = \bar{\beta}$.

VELOCITY MODULATION IN THE MODULATOR

As the electron density modulation at the modulator in wave vector domain is solved[4], it is straightforward to insert it back to the Vlasov equation and obtain the phase space density in the wave vector domain.

$$\tilde{f}_1(\vec{k},\vec{v},t) = iZ_i \frac{\omega_p^2}{k^2} \frac{e^{-i\vec{k}\cdot\vec{v}t}}{1+\omega_p^2/\lambda^2} \vec{k}\cdot\frac{\partial}{\partial \vec{v}} f_0(\vec{v}) \left\{ \frac{1-e^{i\vec{k}\cdot\vec{v}t}}{i\vec{k}\cdot\vec{v}} + \frac{e^{(\lambda+i\vec{k}\cdot\vec{v})t}\,\omega_p/\lambda}{(\lambda+i\vec{k}\cdot\vec{v})^2+\omega_p^2} \left[\left(\lambda + i\vec{k}\cdot\vec{v} - \frac{\omega_p^2}{\lambda}\right)\sin\omega_p t - 2\omega_p\left(1+\frac{i\vec{k}\cdot\vec{v}}{2\lambda}\right)\left(\cos\omega_p t - e^{-(\lambda+i\vec{k}\cdot\vec{v})t}\right) \right] \right\}, \quad (9)$$

where $\lambda \equiv \lambda(\vec{k},\vec{v}_0)$ is defined in equation (4). Fourier transformation of equation (9) is generally difficult to be carried out analytically. In an homogeneous electron plasma, the current modulation, i.e. the first moment of the phase space density, can be expressed as

$$\vec{j}_1(\vec{k},t) = \int_{-\infty}^{\infty} \vec{v}\tilde{f}_1(\vec{k},\vec{v},t)d\vec{v} = \vec{j}_s(\vec{k},t) + \vec{j}_d(\vec{k},t), \quad (10)$$

where $\vec{j}_s(\vec{k},t) = -\vec{v}_0\tilde{n}_1(\vec{k},t)$ is the static part due to the fact that the solution is given in the frame of the moving ion. The first time derivative of the dynamic part caused by the interaction between the ion and electrons reads

$$\dot{\vec{j}}_d(\vec{k},t) = \frac{i\vec{k}}{k^2} Z_i\omega_p^2 e^{\lambda t}\cos(\omega_p t) - i\vec{V}Z_i\omega_p\sin(\omega_p t)e^{\lambda t}, \quad (11)$$

where

$$\vec{V} = \vec{\nabla}_k\sqrt{(k_x\beta_x)^2+(k_y\beta_y)^2+(k_z\beta_z)^2} \quad (12)$$

Fourier transformation of (11) leads to the expression for the current density modulation in spatial domain,

$$\vec{j}_d(\vec{x},t) = \frac{Z_i\omega_p^2 a}{2\pi^2}\int_0^t d\tau(\vec{x}+\vec{v}_0\tau)\left\{ \frac{2\sin\omega_p\tau}{\left(\bar{\beta}^2\tau^2+|\vec{x}+\vec{v}_0\tau|^2\right)^2} + \frac{\cos\omega_p\tau}{|\vec{x}+\vec{v}_0\tau|^2}\left[\frac{\omega_p\tau}{\bar{\beta}^2\tau^2+|\vec{x}+\vec{v}_0\tau|^2} - \frac{\arctan\left(|\vec{x}+\vec{v}_0\tau|/\bar{\beta}\tau\right)}{a|\vec{x}+\vec{v}_0\tau|} \right] \right\} \quad (13)$$

, where a is the Debye radius and $\bar{\beta}$ is the velocity spread for the homogeneous electron beam. Figure 4 shows the vector plot of an example calculation of equation(13).

SUMMARY

The preliminary analytical model has been developed for all three sections of CeC. Despite some of the assumptions used in the derivation can be unrealistic, the model contains major physics processes, i.e. space charge effects, Landau damping and energy modulation in the FEL. It can be used to estimate the scaling law and bench mark simulation codes. Further improvements of the analytical model involve investigating the diffraction effects in the FEL, the transverse evolution of the wave-packet in the kicker and finite beam size effects.

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