New Developments on the Generation of Arbitrary Polarized Radiation from Insertion Devices

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Abstract

The complete description of the polarization of a beam of radiation is described in terms of the total energy and three polarization rates. The polarization characteristics from conventional undulators and wigglers is recalled. A presentation is made of some new Insertion Devices that were proposed and/or built to generate circular polarization and more generally to improve the control of polarization. They are the asymmetric and elliptical wigglers and the helical and crossed undulators.

I. INTRODUCTION

The polarization of the synchrotron radiation is essentially linear with electric field in the horizontal plane of the electron or positron orbital motion. The availability of free straight sections on existing storage rings and the construction of new synchrotron sources that will accommodate a large number of Insertion Devices (ID) has motivated a world wide effort to design and build new exotic IDs capable of generating an arbitrary state of polarization. Of particular interest is the circular polarization. Applications extend from natural or magnetic dichroism, spin polarized photoemission, magnetic scattering...

II. GENERALITIES ON THE POLARIZATION

Each electron of the beam generates a wave of electromagnetic field. This wave nearly becomes a plane wave by passing through a monochromator. Its polarization state is a pure state which means that it is entirely described by two complex amplitudes (one for the horizontal and vertical plane). The polarization state typically depends on the observer and electron positions and velocities. In practical situations, the radiation beam is generated by an ensemble of electrons each having its own position and angle. Furthermore, the radiation beam is integrated over some area and angle by a detector. As a result, the description of the polarization by means of the two complex amplitudes becomes unsufficient. One must deal with a statistical sum of pure states. The most suitable method to deal with such cases is the density matrix formalism. The density matrix describing the polarization state is a 2x2 hermitian matrix[3]. In the following I shall prefer the Stokes-Poincare representation. It is largely sufficient to make a zoological classification of the various polarizations of radiation, however this may be not as practical as the density matrix if one wants to treat the transformation of the polarization through any scattering or polarization sensitive absorption. Other formalisms exists such as the Jones Matrices. Of course all these representations are equivalent and one can find four independent energy-like quantities that completely define the polarization. In the Stokes representation, they are the total intensity I (integrated over the slit aperture) and three polarization rates, p1 = (I1 - I3) / I, p2 = (I45 - I135) / I, p3 = (I1 - I3) / I, Ix (Iy) is the energy linearly polarized in the horizontal (vertical) plane, I45 (I135) is the energy linearly polarized at 45 (135) degrees with the horizontal and vertical directions, I1 (135) is the energy circularly polarized with right (left) orientation. Note the following equalities: I=Ix + Iy = I45 + I135 =I1 + I35. Each of the three polarization rates defined above is a dimensionless quantity between -1 and 1. A pure state of polarization is such that the sum of the square of the three rates is exactly equal to 1. This is trivially verified if all 6 partial intensities Ix, Iy, I45, I135, I1, I35 are zero except for one. The proof can be extended to the most general pure elliptical state of polarization by decomposing the intensity over the complex amplitudes[4]. The statistical averaging of the different polarization present in a beam is made by summing separately each contribution to the 6 partial intensities described above. As a result of some convexity property one easily show that the sum of the square of the three polarization rates is less than 1 becoming equal to zero for completely depolarized radiation. In the following I shall define the amount of depolarization by introducing the rate of unpolarized radiation ρ0 such that in any condition:

\[ 1 = \rho_0^2 + \rho_1^2 + \rho_2^2 + \rho_3^2 \]
\( \rho_0 \) is a dimensionless quantity between 0 and 1. If \( \rho_0 = 0 \) (1), the radiation is fully polarized (depolarized). In the most general case all four polarization rates depend on the electron beam (energy, sizes, angular spreads, position and angle), the magnetic field of the ID, the photon energy and the detector (aperture, position). Fully polarized radiation can only be obtained from a filament monoenergetic electron beam and with a point detector.

III. WIGGLERS

The polarization of the radiation generated by a typical N periods wiggler is described by simply summing every 6 partial intensities over each 2N source points. Conventional wigglers generate a radiation essentially horizontally linearly polarized in the plane of the orbit which gradually becomes depolarized as the observer moves away from the orbit plane. Note the difference with bending magnet radiation which becomes completely circularly polarized off axis. The depolarization originates from the statistical average of purely left (originating from a source point of positive field) and purely right (originating from a source point of negative field) radiation with equal probability. In any direction of observation, \( \rho_2 \) and \( \rho_3 \) are equal to zero. On axis \( \rho_1 \) dominates unless the electron beam emittance is extremely large in which case \( \rho_0 \) dominates. Two modifications have been used to restore the circular polarization. They are the asymmetric[5] [6] and Elliptical[7] [8] wigglers.

A. Asymmetric Wiggler

The asymmetric wiggler is made of a non sinusoidal magnetic field. Figure 1. presents one possible magnetic design.

\[ \begin{array}{c}
\rightarrow \downarrow \leftarrow \\
\leftarrow \downarrow \rightarrow
\end{array} \]

Figure 1. Schematic of an asymmetric wiggler

In an asymmetric wiggler, the two source points per period seen by an observer do not present the same absolute value (as for a conventional wiggler). Ideally the field corresponding to one of the source points is large while the field on the second source point is small and of the opposite sign. The polarization is most easily analyzed by mean of a vertical field vs. horizontal angle diagram such as the one shown in Figure 2. The intersection of any vertical line (corresponding to some direction of observation) with the closed curve generally defines two points the vertical coordinate of which is the magnetic field of the source points. In fact the curve is closed N times on itself defining a total of 2N source points. In some very special cases 4,6... source points per period can be seen.

![Field vs. Angle Diagram of an asymmetric wiggler](image)

Figure 2. Field vs. Angle Diagram of an asymmetric wiggler

As can be anticipated from Figure 2., the circular polarization rate depends not only on the vertical angle of observation but also on the photon energy and horizontal angle of observation.

B. Elliptical Wiggler

The elliptical wiggler can be understood as a conventional vertical field wiggler to which a small horizontal field of identical periodicity has been added in such a way that the electron trajectory is a flat ellipse.

\[ \begin{array}{c}
\rightarrow \downarrow \leftarrow \\
\leftarrow \downarrow \rightarrow
\end{array} \]

Figure 3. Schematic of an Elliptical Wiggler

The radiation generated on the axis is the sum of the one generated by a positive and negative field but seen from below and above (respectively) the orbit plane. As a result they both present the same circular orientation and they do not cancel out as for a conventional wiggler.
The choice between asymmetric and elliptical wigglers is largely dictated by engineering complexity. The asymmetric wiggler is much simpler to build and does not require any special vacuum chamber even though potentially less efficient than the elliptical wiggler.

IV. UNDULATORS

A. Linear Undulator

The radiation generated by a single electron over each period of motion typically interferes, resulting in an enhancement of the emission at some particular photon energy. This phenomenon is predominant in undulators while difficult to observe on a wiggler (except for a filament electron beam and a point detector). The transition between undulators and wigglers is usually determined by the deflection coefficient \( K = 0.934 \frac{B(T)}{\lambda_0 \text{[cm]}} \). B is the peak sinusoidal vertical field. \( \lambda_0 \) is the spatial period. Undulators (wigglers) typically correspond to \( K < (> 2 \text{ to } 3 \). As for a wiggler, the radiation from an undulator is essentially horizontally linearly polarized on axis. Off axis the polarization is still linear but inclined with respect to the horizontal plane. In any direction of observation \( \rho_3 = 0 \). The highest brightness from an undulator is typically obtained by selecting the part of the radiation emitted around the main central axis. Figure 1 presents the flux and polarization rates on the fundamental peak of a conventional linear undulator (length 1.6 m, period 30 mm \( K = 0.67 \)) seen through a 1*1 mm slit placed 30 m away from the source. The ESRF electron beam has been used: Energy = 6 GeV, Current = 100 mA, horizontal (vertical) Emittance = 7 E 9 m (7 E 10 m), horizontal (vertical) beta function = 27 m (11 m).

Figure 5. Flux and polarization rate on the fundamental peak from a linear undulator from [10]

Note the very small amount of depolarization which is typical of undulator radiation generated by small emittance electron beams. Various schemes have been proposed and used to generate other polarization states on the central axis, they are described below.

B. Helical Undulator

The most obvious method to restore the circular polarization of the radiation issued from an undulator is to use a helical or elliptical magnetic field geometry. From a technological point of view, a large variety of technical solutions exist. For small values of the undulator deflection parameter \( K \), the ellipticity of the radiation is nearly identical to the helicity of the magnetic field. In other words, the electric field of the radiation is nearly proportional to the magnetic field components. Figure 6 presents the flux and polarization rates on the fundamental peak from a helical undulator. The electron beam, slit, undulator length and period are identical to those of Figure 5. The peak helical field is 0.17 in order to keep the same fundamental energy.

Figure 6. Flux and polarization rate on the fundamental from a helical undulator from [10]
Helical undulators have already been used a few times for FEL applications but very seldom as synchrotron radiation source\[15\]. Possible reasons are the more complicated magnet technology and the incompatibility of ultra-high vacuum with a circular vacuum chamber to which the field geometry is particularly suited.

C. Crossed Undulator

An other interesting approach to restore the circular polarization is to use a crossed undulator\[16\][17]. It is an arrangement of two undulators placed successively on the electron beam but rotated by 90 degrees around the beam axis (see Figure 7.).

![Crossed Undulator Schematic](Figure 7. Schematic of a Crossed Undulator)

Understanding of the polarization characteristics is not a trivial matter. One way to understand it is to analyze in the time domain the radiation wave generated by a single electron as it crosses the full device. It is made of two successive N periods sinewaves with orthogonal orientation. The spectrum of such a quasi periodic wave presents the usual undulator harmonic peaks, but also an oscillation of the polarization characteristics along the spectrum. Figure 8. presents the flux and polarization rates on the fundamental peak from a crossed undulator. The electron beam, slit, total undulator length and period are identical to those of Figure 5. Each undulator segment is 0.8 m long.

![Flux and Polarization Rate](Figure 8. Flux and polarization rate on the fundamental from a crossed undulator from [10])

The high sensitivity of the polarization to the photon energy and angle of observation results in a significant depolarization. This effect has been analyzed in detail for a crossed undulator emitting around 9 keV at ESRF[10]. Even with the low emittance beam of the ESRF, the majority of the radiation generated on each peak of the spectrum is depolarized. Elliptical polarization can be observed only on the high energy side of the peak. One faces a trade-off between flux and circular polarization rate (60% rate for 50% of the peak flux in Figure 8.). Higher polarization rates could be observed by closing the slit but again at the cost of the flux. The phenomenon decreases with electron energy, beam emittance and slit aperture. Experiments looking for circular polarization from a crossed undulator will require extensive alignment and care. The prime advantage of the crossed undulator is its potential rapid switching of polarization orientation (left to right) which can be accomplished by a short electromagnet three pole section (dispersive section) placed between the two undulators.

V. FLIPPING OF THE POLARIZATION

Experiments that make use of the circular polarization very often require a switching mechanism of the orientation from left to right. This operation can be done by playing with some optical element in the beamline or by directly operating on the magnetic field of the ID. The modification of the magnetic field should always be considered as a last alternative since it is very likely to result in a disturbance of the electron beam closed orbit that is detrimental to the large number of other users of such a facility. The higher the electron energy, the lower the perturbation.

Polarization switching can be done with a permanent magnet asymmetric wiggler by periodically inclining the trajectory vertically by mean of a two or four magnets bump placed on each side of the device. In an elliptical wiggler or in a helical undulator, the phasing between the horizontal and the vertical magnetic field flip the polarization. This can be accomplished by a longitudinal displacement of some permanent magnet assembly (a few seconds) or by using an electromagnet (a few hundredths of a second). As we have seen above, the crossed undulator offers a very elegant method for flipping the polarization by means of a dispersive section. If a high photon energy is envisaged, an electromagnet undulator are inefficient and permanent magnets must be used. In that case, an interesting scheme can be used to switch the helicity of the field by separating the magnet responsible for the vertical and horizontal component of the magnetic field on two different jaws (see Figure 9.). Such a device can accommodate a conventional flat vacuum chamber.
A helical undulator is presently being built at ESRF based on this concept[18].

VI. CONCLUSION

New wigglers have been successfully tested in the past years that were built in order to generate circular polarization. New undulator schemes presently under construction are expected to be tested in the coming years. The large worldwide effort to build synchrotron radiation sources will undoubtedly result in an improvement of the ID technology in all aspects and especially the control of the polarization which is a new important demand from the user community.