ANALYTIC EVALUATION OF THE LAMPF II BOOSTER CAVITY DESIGN

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Summary

Through the past few decades, a great deal of sophistication has evolved in the numeric codes used to evaluate electromagnetically resonant structures. The numeric methods are extremely precise, even for complicated geometries, whereas analytic methods require a simple uniform geometry and a simple, known mode configuration if the same precision is to be obtained. The code SUPERFISH, which is near the present state of the art of numeric methods, does have the following limitations: No circumferential geometry variations are permissible; there are no provisions for magnetic or dielectric losses; and finally, it is impractical (because of the complexity of the code) to modify it to extract particular bits of data one might want that are not provided by the code as written. This paper describes how SUPERFISH was used as an aid in deriving an analytic model of the LAMPF II Booster Cavity. Once a satisfactory model was derived, simple FORTRAN codes were generated to provide whatever data was required. The analytic model is described by the following five distinct sections.

Radial Sections (Terminology as defined in Ref. 1.):

<table>
<thead>
<tr>
<th>Section</th>
<th>Load Radius</th>
<th>Input Radius</th>
<th>Spacing</th>
<th>( \mu_r )</th>
<th>( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.831</td>
<td>6.30</td>
<td>7.25</td>
<td>1</td>
<td>3.456</td>
</tr>
<tr>
<td>2</td>
<td>6.30</td>
<td>5.76</td>
<td>7.25</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>5.76</td>
<td>5.51</td>
<td>7.25</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ \mu_r = (6\mu_r + 1.25)/7.25, \text{ and } \varepsilon_r \text{ is the relative permeability of the ferrite. This expression accounts for the spacers between the ferrites.} \]

TEM Sections (Coaxial):

<table>
<thead>
<tr>
<th>Section</th>
<th>Outer Radius</th>
<th>Inner Radius</th>
<th>Length</th>
<th>( Z_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.548</td>
<td>3.543</td>
<td>5.87</td>
<td>26.04</td>
</tr>
<tr>
<td>2</td>
<td>6.890</td>
<td>3.543</td>
<td>28.58</td>
<td>39.90</td>
</tr>
</tbody>
</table>

Finally, the end of the fifth section is terminated in a 7.83-pF capacitor.

The booster cavity geometry shown in Fig. 1. was designed using the rf cavity code, SUPERFISH. The analytic model for the gap end of this cavity is simply a uniform TEM transmission line terminated in a capacitor. The characteristic impedance of this TEM section was calculated from the ratio of the coaxial radii. The gap capacitance was derived from SUPERFISH data, and the length was obtained by physical measurement from the gap to the neck region of the cavity (see Fig. 1).

The analytic model for the ferrite end of the cavity is considerably more complicated. The SUPERFISH electric-field calculation of the ferrite end of the cavity is shown in Fig. 2. This figure suggests that the ferrite and insulator portions of the cavity should be modeled using radial transmission lines and the remaining portion, through the neck, by a uniform TEM.

Lossy, Radial Transmission Lines

The treatment of lossy, TEM transmission lines is found in numerous texts and will not be discussed here. The extension of the lossless radial transmission-line equations to include magnetic and/or dielectric losses follows, without proof because of space limitations.

These equations make use of the Bessel functions \( J_0, J_1, N_0, \) and \( N_1 \); the argument of each is \( kr \), where
\[ r = \text{radius at which the function is being evaluated}, \]
\[ k = 2\pi f(\mu \varepsilon)^{1/2}, \]
\[ \mu = \text{total permeability of the medium}, \]
\[ \varepsilon = \text{total dielectric constant of the medium}. \]

In the lossless case, the agreement is real and the functions are real. In the lossy case, \( k \) is modified as follows:
\[ k = 2\pi f(\mu \varepsilon)^{1/2} \left[1 - \frac{1}{\sqrt{m_0}} - j\left(\frac{1}{\sqrt{m_0}} + \frac{1}{\sqrt{q_0}}\right)\right]^{1/2}, \]
where \( q_0 \) and \( q_0 \) are the magnetic and dielectric Qs of the medium, respectively. For high-Q values (>100),\[ k = 2\pi f(\mu \varepsilon)^{1/2} \left[1 - j(0.5)\left(\frac{1}{\sqrt{m_0}} + \frac{1}{\sqrt{q_0}}\right)\right]. \]

Because the argument is now complex, the functions will be complex. The functions, however, are evaluated using the same series expansions as if the argument were real. Definitions of those expansions are collected below.

\[ J_0(kr) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n(0.5kr)^{2n}}{(n!)^2}, \]
\[ J_1(kr) = \sum_{n=1}^{\infty} \frac{(-1)^n(0.5kr)^{2n-1}}{(n!)^2}, \]
\[ N_0(kr) = 2 \left[ \log \gamma + \log(n(0.5kr)) + \sum_{n=1}^{\infty} \frac{(-1)^n(0.5kr)^{2n}}{(n!)^2} \right], \]
and
\[ N_1(kr) = 2 \left[ \sum_{n=1}^{\infty} \frac{(n\gamma + 0.5)(-1)^n(0.5kr)^{2n-1}}{(n!)^2} \right], \]
where \( \log \gamma = 0.5772157 \) (Euler's constant), and
\[ FN = \log \gamma + \log(n(0.5kr)) - \sum_{m=1}^{n} \left(\frac{1}{m}\right). \]

The radial transmission-line equations depend on \( Z_0, \Theta \) and \( \Phi \). The equations for evaluating these parameters follow:

Define
\[ R_{01} = \text{Re}(J_0) - \text{Im}(N_0), \quad I_{01} = \text{Im}(J_0) + \text{Re}(N_0), \]
\[ R_{02} = \text{Re}(J_0) + \text{Im}(N_0), \quad I_{02} = \text{Im}(J_0) - \text{Re}(N_0), \]
\[ R_{11} = \text{Re}(J_1) - \text{Im}(N_1), \quad I_{11} = \text{Im}(J_1) + \text{Re}(N_1), \]
\[ R_{12} = \text{Re}(J_1) + \text{Im}(N_1), \quad I_{12} = \text{Im}(J_1) - \text{Re}(N_1), \]
as a general expression.

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\[ R_{01} = \text{Re}(J_0) - \text{Im}(N_0), \quad I_{01} = \text{Im}(J_0) + \text{Re}(N_0), \]
\[ R_{02} = \text{Re}(J_0) + \text{Im}(N_0), \quad I_{02} = \text{Im}(J_0) - \text{Re}(N_0), \]
\[ R_{11} = \text{Re}(J_1) - \text{Im}(N_1), \quad I_{11} = \text{Im}(J_1) + \text{Re}(N_1), \]
\[ R_{12} = \text{Re}(J_1) + \text{Im}(N_1), \quad I_{12} = \text{Im}(J_1) - \text{Re}(N_1), \]
as a general expression.

The equivalent beam-load impedance is added across \( C_3 \). The amplifier arm is 21 in. long and its characteristic impedance is 37.7 \( \Omega \). The characteristic impedance of the horizontal section remains 39.9 \( \Omega \), with an outer radius of 6.89 in. and an inner radius of 3.543 in. The lengths that replace the 28.58-in. length of Section 5 of the cavity analytic model are as follows:

<table>
<thead>
<tr>
<th>Section 4 to ( C_2 ) - 6.506 in.</th>
<th>( C_2 ) to ( C_4 ) - 6.0 in.</th>
<th>( C_4 ) to ( C_0 ) - 9.0 in.</th>
</tr>
</thead>
</table>

Note that the cavity is shortened about 1 in. by the addition of the amplifier. Sections 1 through 4 of the cavity analytic model remain intact as the tuning variable.

The most significant parameter that can be calculated from the complete model is the rf amplifier load impedance. That is the impedance seen looking into the model at the terminals across \( C_3 \). A scaled drawing of the resulting booster cavity and amplifier is shown in Fig. 4.
Calculation Results and Conclusions

Several FORTRAN codes were written based on the analytic models. The first code was used to optimize the ferrite dimensions. The second code was used to locate the amplifier and size the coupling capacitor. Another code was written that gave wall-loss heat distribution over the cavity. Peak and average power-dissipation density was mapped through the ferrite by another code. Finally, the amplifier plate load over the acceleration cycle was plotted.

The calculated results show that the cavity can be tuned from 50.3 to 59.2 MHz by varying the control current in the bias coils so that the relative permeability of the ferrite changes from 2.64 down to 1.43. The maximum rms dissipation density occurring in the ferrite during the accelerating cycle is 1.25 W/cm³, and the average over the cycle is 200 mW/cm³. The rf amplifier plate-load resistance stays within the limits 500 to 700 Ω. In the areas evaluated by these analytic techniques, the cavity-amplifier package design has a comfortable safety margin.

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References
