Abstract

The dynamic aperture of the SPS has been measured in the presence of non linearities artificially introduced in order to simulate the operating conditions of future large superconducting colliders. These non linearities are produced by strong lumped sextupoles. By comparing the measurements with extensive computer simulations the empirical criteria which are used in the design of superconducting colliders are refined and their validity more firmly established.

Introduction

One of the main problems in the design of large hadron colliders is to ensure stability of particle trajectories over a sufficient fraction of the vacuum chamber cross section. This stability is endangered by multipolar errors in the field of the superconducting bending magnets and quadrupoles. Since reducing these errors leads to more complicated and costly designs, it is imperative to determine accurately what are the tolerable values.

Large computer programs have been written to evaluate the dynamic aperture, that is the maximum stable amplitude of the trajectories, by tracking particles along the machine lattice for a few $10^7$ or $10^8$ turns. However in real machines the beams must survive a few $10^5$ turns, and it is impossible with present day computers to simulate the trajectories for such a long time. Even if it were possible, one would never be sure to have included in the model all the perturbing effects which can conspire to reduce long term stability. Therefore experience in past or existing machines is essential. Based on this experience a set of criteria, have been proposed to judge the quality of a machine design.

The first criterion is the tune shift, that is the difference $\Delta Q$ between the tunes of a particle with a large amplitude and the particle on the central trajectory. The second is the smearing that is the rms deviation from the average value of the Courant and Snyder “invariant” evaluated after each turn. In a linear machine, both $\Delta Q$ and smear are zero. By imposing an upper limit on these two quantities, one hopes to obtain a “sufficiently linear” machine so that long term stability can be assured. At CERN studies for the design of a Large Hadron Collider (LHC) are based on $\Delta Q < 0.005$ and smear $< 0.0%$.

In order to refine these criteria experiments are being conducted on the CERN SPS, the first results of which are reported here. The principle is to introduce controlled non-linearities in an otherwise linear machine, and to compare tracking results with experimental findings for the same, well known situation.

Experimental conditions

The experiments were done while the SPS was operating in the fixed target mode. A beam of reduced intensity (about $2.10^{12}$ p) was injected at 14 GeV/c, accelerated up to 120 GeV/c and allowed to circulate at this energy for 8 sec on a magnetic flat top. The beam was kept bunched by the RF, and was distributed over the entire machine circumference. Strong non-linearities were introduced at the beginning of the flat top by exciting 8 sextupoles normally used for slow ejection at 450 GeV/c.

These sextupoles are individually powered and therefore different patterns of excitation can be chosen. In all the cases the sextupoles were distributed in two families of opposite polarity so that there was no resulting change in the machine chromaticity.

The strength of a sextupole is $B'' = 5B' = 3B_m$ for an excitation current of 140A.

An energy of 120 GeV/c was chosen because it is in this range that the non-linearities of the SPS magnetic field are minimum, the space charge phenomena are already negligible, and the effect of the sextupoles is still sufficiently important to strongly perturb the particle dynamics. In addition to the non-linearities of the bending magnet and quadrupoles which at this energy are known to be rather small, the only non linearity in the base machine comes from the four families of sextupoles used to correct the chromaticity. During these experiments it was necessary to introduce in addition a small amount of octupolar non linearity ($B'''/B_m = 3 B_m$) in both planes to stabilize the beam; in spite of the low intensity used in these experiments the resistive-wall instability would otherwise cause uncontrolled beam oscillations and losses. Both simulations and experiments show that all these unavoidable non linearities do not limit the acceptance of the SPS below the geometrical value given by the height of the vacuum chamber.

Experimental procedure

At the beginning of the flat top at 120 GeV/c the sextupoles are energized and the beam emittance is increased until losses occur. This is done by repetitively deflecting the beam with the fast kicker normally used to measure the tune (the Q kicker). In the absence of non linearities it is sufficiently to pulze the Q kicker twice at its maximum deflection ($5.9 \text{ mm at } B=100 \text{ m}$) to reach the machine aperture in the vertical plane. Fig. 1 shows the vertical profile of the beam obtained with the wire scanner: the deflection due to the first kick is larger than the average beam dimension and therefore after filamentation, this procedure results in a hollow beam. The subsequent application of a second kick restores a more triangular shape.
When the sextupoles are energized the acceptance shrinks. In this situation the amplitude of the kicks is reduced so as to limit the losses to 10 to 20%.

With zero current in the sextupoles the losses occur abruptly indicating that the geometrical aperture is the limitation. On the contrary when the sextupoles are energized the losses take place usually over a few seconds after the last kick, but in all cases there remain a few seconds before the end of the flat top with no visible losses. In this region the amplitude of the extreme particle in the tail of the beam distribution is measured with the wire scanner set at its maximum sensitivity, and this measurement is used to define the machine acceptance.

Experimental results

Experiment 1

In this first experiment the excitation pattern of the 8 sextupoles was such as to strongly excite the \(\frac{1}{3}\) order resonance \(3 Q_H = 80\). This results in a triangular phase space topology as can be seen from the simulation (fig.2a) as well as from the wire scanner measurement (fig.2b). The experiment was repeated for a few working points and the measured dynamic aperture appears in Table I.

![Fig.2: Effect of the proximity of the \(\frac{1}{3}\) resonance](image)

The program PATRAC \(^3\) has been used to simulate the SPS in part of the cases which have been investigated experimentally. It gives an image of the horizontal and vertical phase planes, calculates the tunes and the smear for a series of increasing initial amplitudes up to the point where the particle is eventually lost on the simulated geometrical aperture of the machine. This defines the dynamic aperture as seen by the program. In a first step synchrotron motion was ignored, all particles had a zero momentum deviation, and tracking was performed over \(2\times10^3\) turns.

The \(AQ\) and smear for a selected experimental case are plotted in Fig.3 as a function of the initial amplitude. The vertical straight line indicates the experimentally observed aperture. Values of \(AQ\) and smear corresponding to the experimental aperture are given in Tables I and II.

In experiment 1 the smear is large due to the strong influence of the \(\frac{1}{3}\) resonance which distorts the phase space. The ratio between the experimental dynamic aperture and the tracking dynamic aperture is 0.5 to 0.6. The smear at the experimental aperture varies from 0.12 to 0.18, and the tune shift from 0.003 to 0.0075. In experiment 2, the ratio between the experimental aperture and the tracking aperture is again close to 0.5, the minimum smear at the experimental aperture is around 0.04 and the minimum tune shift around 0.008.

![Fig.4: Different working points which have been explored in experiment 2](image)

Experiment 2

Here the excitation of the sextupoles was deliberately chosen to minimize the strength of the nearest \(\frac{1}{3}\) order resonance (the resulting excitation was down to that of half a sextupole) with the aim of revealing the influence of higher order resonances. Many working points were explored, and the results are given in Table II.

The geometrical vertical aperture, measured in the absence of non linearities is 14.7 mm (all apertures are normalized to 8 = 100%). Since the height of the vacuum chamber is 29 mm this indicates that the maximum closed orbit deviation augmented by any vacuum chamber misalignment is 8.3 mm, a not surprising value in the SPS at 120 GeV. The horizontal geometrical aperture is too large to be measured with the same technique. An undisturbed beam is almost round, with extreme amplitudes (just measurable on the wire scanner set at maximum sensitivity) of 5.8 mm horizontally and 4.7 mm vertically.

For points 1 to 11 the beam was kicked horizontally. Points 5 to 9 were close to the diagonal of the tune diagram, and the effect of coupling is clearly revealed: the vertical amplitude increases much more than for the other points, and approaches the geometrical vertical limit. Working points further away from the diagonal, like 2,10,11, are probably more interesting and have been analysed in more detail.

Analysis and interpretation

The program PATRAC \(^3\) has been used to simulate the SPS in part of the cases which have been investigated experimentally. It gives an image of the horizontal and vertical phase planes, calculates the tunes and the smear for a series of increasing initial amplitudes up to the point where the particle is eventually lost on the simulated geometrical aperture of the machine. This defines the dynamic aperture as seen by the program. In a first step synchrotron motion was ignored, all particles had a zero momentum deviation, and tracking was performed over \(2\times10^3\) turns.

The \(AQ\) and smear for a selected experimental case are plotted in Fig.3 as a function of the initial amplitude. The vertical straight line indicates the experimentally observed aperture. Values of \(AQ\) and smear corresponding to the experimental aperture are given in Tables I and II.

In experiment 1 the smear is large due to the strong influence of the \(\frac{1}{3}\) resonance which distorts the phase space. The ratio between the experimental dynamic aperture and the tracking dynamic aperture is 0.5 to 0.6. The smear at the experimental aperture varies from 0.12 to 0.18, and the tune shift from 0.003 to 0.0075. In experiment 2, the ratio between the experimental aperture and the tracking aperture is again close to 0.5, the minimum smear at the experimental aperture is around 0.04 and the minimum tune shift around 0.008.

Fig.4 shows the different working points which have been explored in experiment 2. The thick dot represents the small amplitude tune measured before the emittance had been increased by repetitive kicking. The line indicates the tune of large amplitude particles, as calculated by the tracking program. The cross is the position of the particle corresponding to the experimentally found aperture, so that the continuous line represents the surviving particles, while the dashed part represents the particles which survive in the tracking but not in the real machine.

In figure 3 (corresponding to point 10) the effect of the strong 5th order resonance \(5Q_H = 133\) is clearly shown. The tune is locked to the resonance while the amplitude is varied across the stable islands. These islands are well visible in the phase space plots, and a similar picture is obtained for point 2. In both cases 2 and 10 the particles are lost in the real machine at an amplitude much smaller than that corresponding to this resonance. In fact for point 2 the real dynamic aperture seems to be determined by the
<table>
<thead>
<tr>
<th>Working point (Fig. 4)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track Ap. mm</td>
<td>27.5</td>
<td>29.5</td>
<td>29</td>
<td>15.5</td>
<td>17.5</td>
<td>18.2</td>
<td>23</td>
<td>31</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Exp. Ap. mm</td>
<td>18.4</td>
<td>13.9</td>
<td>13</td>
<td>15</td>
<td>17.5</td>
<td>18.2</td>
<td>21.4</td>
<td>13.2</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{q}$ at Exp. Ap.</td>
<td>.012</td>
<td>.008</td>
<td>.011</td>
<td>.031</td>
<td>.008</td>
<td>.016</td>
<td>.08</td>
<td>.038</td>
<td>.06</td>
<td></td>
</tr>
<tr>
<td>Smear at Exp. Ap.</td>
<td>.055</td>
<td>.04</td>
<td>.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vert. beam size mm</td>
<td>11.9</td>
<td>8.8</td>
<td>8.9</td>
<td>11.2</td>
<td>12.6</td>
<td>12.5</td>
<td>11.7</td>
<td>10</td>
<td>6.9</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table II

Sextupole excitation +++--

Conclusions

The criteria which have been proposed to assure long term beam stability in large hadron colliders have been submitted to experimental tests in the CERN SPS. In the conditions of the experiments the application of these criteria ensures stability for a few seconds of machine time, that is for a few $10^6$ turns. Long term tracking over 10^5 turns reproduces well the observations provided that synchrotron oscillations and closed orbits are taken into account. Further experiments are in progress to extend the time scales involved to a few hours.

Acknowledgements

It is a pleasure to thank the SPS operation crews, and in particular K.Cornells, A.Faugier, R.Lauckner and D.Thomas whose expertise and collaboration was essential to set up the machine in the conditions required by these experiments.

References

[2] The large Hadron Collider in the LEP Tunnel, CERN 87-05