ERROR ANALYSIS OF SURFACE RESISTANCE FITS TO EXPERIMENTAL DATA

ABSTRACT: Superconducting material properties such as energy gap, mean free path or residual resistance are commonly extracted by fitting experimental surface resistance data. Depending on the measurement setup, both, temperature range and the number of points are limited. In order to obtain significant results, systematic as well as statistical uncertainties have to be taken into account. In this contribution we discuss the impact of systematic and statistical errors on BCS fit parameters. In particular, past measurements have yielded contradictory conclusions that, we believe, result from the use of insufficient data in the necessary temperature range. Furthermore, this study is applied to the boundary conditions of the Quadrupole Resonator and its measurement accuracy.

SYSTEMATIC ERRORS

ERRORS ON dB SCALE

RF power is typically measured in units of dBm. The conversion from dBm to Watts if given by
\[ P[W] = 10^{\frac{P[\text{dBm}]}{10}} \]
An offset of x dBm leads to
\[ P[\text{dBm}] = 10 \ln(P[W]) = 10^x \]
Looking at the relative error
\[ \frac{\partial P[\text{dBm}]}{P[\text{dBm}]} = \frac{x \ln(10)}{10} \approx 23\% \]
\( \Rightarrow \) constant relative error: 2.3 % per 0.1 dB offset

IMPACT ON BCS PARAMETERS

In General: constant relative error \( \beta \)
\( \Rightarrow \) multiplication of \( R_s \) with \( \beta \)
\( \Rightarrow \) affects only \( a \) and \( R_{\text{res}} \)
\( \Rightarrow \) applicable to all multiplicative formulas
\( \Rightarrow \) good approximation for QPR and cavity tests

CONCLUSIONS

Systematic errors
\( \Rightarrow \) on dB level \( \Rightarrow \) constant relative error 2.3 % per 0.1 dB
\( \Rightarrow \) constant relative errors only affect parameters \( a \) and \( R_{\text{res}} \)

Statistical uncertainties
\( \Rightarrow \) Impact depends on temperature range of fit
\( \Rightarrow \) Error amplification factor \( \alpha \) independent of input error
\( \Rightarrow \) Linear decrease of \( \alpha \) with increasing number of data points
\( \Rightarrow \) 1.5 – 2.1 K \( \Rightarrow \) only \( R_{\text{res}} \) has sufficient accuracy
\( \Rightarrow \) 2.0 – 4.5 K \( \Rightarrow \) \( \alpha > 1 \) but acceptable for all fit parameters

REFERENCES