ON THE FIELD DEPENDENT SURFACE RESISTANCE OBSERVED IN SUPERCONDUCTING NIQUEM CAVITIES

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Abstract

A quantitative description is presented of the non-linear current-voltage response in superconducting niobium cavities for accelerator application. It is based on a fit for a large sample of data from cavity tests of different kind. Trial functions for the surface resistance describing this non-linear relation are established by a least square data fit. Those trial functions yielding the best fit are quantitatively explained by basic physics.

INTRODUCTION AND SCOPE

If the tangential RF electric field $E$ at the cavity surface (which is very small) follows the RF magnetic field $H = B/\mu_0$ in a linear relation, their ratio, i.e. the surface resistance $R_s$, and hence the Q-value $Q \sim 1/R_s$, are constant with $B$. If, however, $R_s$ decreases or increases with $B$, a non-linear relation exists between $E$ and $H$. Therefore in this paper the according surface resistance is called “non-linear”, $R_s^{nl}$.

In fact three different regimes are observed, where the Q-value depends on $B$. In the low field region ($B < 20$ mT) the Q-value may increase with $B$ (low field Q-increase). In the intermediate field region (20-120 mT) the Q-value decreases, and beyond, incidentally, the Q-value may drop even faster. These latter two observations are named “Q-slope” and “Q-drop”.

COLLECTION AND FITTING OF EXPERIMENTAL DATA

The data consist of about 1400 quadruples ($R_s, B, f, T$) collected from cavity tests of a very broad provenience in temperature $T$, frequency $f$, shape, cell number, surface treatment, niobium quality, etc. This approach aims to cancel out stochastic factors and let prevail the fundamental parameters of the niobium metal, which may then be identified.

Individual Contributions to the Surface Resistance

The total surface resistance of the data quadruples was fitted by trial functions with the Mathematica® software. The total surface resistance is composed of a sum of the following contributions:

- the BCS surface resistance $R_s^{BCS}(f, T)$,
- the non-linear surface resistance $R_s^{nl}(B, f, T)$,
- the residual surface resistance $R_s^{res}$,
- the surface resistance describing dielectric losses $R_s^{diele}$ ($f$),
- the surface resistance describing the low-field Q-increase $R_s^{Q-inc}(B, f, T)$.

Data Fitting Procedure

Several precautions were taken for the fit. For instance, the relevant temperature $T$ is not the helium bath temperature but that of the cavity interior surface. $T$ is determined from the power flux, depending on the measured values of $B$ and $R_s$ by taking into account the heat transport properties up to the helium bath. The heat transfer depends on the thermal boundary resistance due to Kapitza and the free convection nucleate boiling, up till its film boiling limit. It also depends on the thermal conductivity and therefore on the purity of the material, which is taken into account by the residual resistivity ratio $RRR$.

A standard deviation of $\sigma = 0.35$ of the individual data for $R_s$ led to a minimum chi-square for the best guess of the fit-parameters $a_1, a_2, ...$ of
This number is reasonably compatible with the total number of 1400 data quadruples \(i\).

**Data Fitting Results**

The fitting resulted in several findings with regard to the different contributions to \(R_s\). The BCS surface resistance \(R_s^{\text{BCS}}\) was well reproduced by the well known formula

\[
R_s^{\text{BCS}}(f, T) \sim f^{1.7} e^{-\Delta/k_B T} \quad (T \to 0) \quad \text{as expected.} \quad \Delta \text{ is the superconducting energy gap and } k_B \text{ is the Boltzmann constant.}
\]

The non linear surface resistance \(R_s^{\text{nl}}\) factorizes into a temperature dependent and a field dependent part \([1]\). The temperature dependent part of \(R_s^{\text{nl}}\) could not be satisfactorily fitted other than by a constant \(c_t\) plus a linearly increasing function starting from zero at a sharp threshold at \(T' = 2.0\) K. This threshold never coincided with, though it was close to, the \(\lambda\)-temperature of liquid helium. The constant \(c_t\) was found to be independent of the frequency, while the linear increase followed a frequency dependence close to that of \(R_s^{\text{BCS}}\). The field dependent part of \(R_s^{\text{nl}}\) followed essentially the same relation as presented in ref. 1, yielding altogether

\[
R_s^{\text{nl}}(B, f, T) \sim \left[ 1 + cf^{1.5} \left( \frac{T - T'}{T'} \right)^{1/6} \right] \left( 1 + e^{-\xi/Bc} \right),
\]

where \(\xi\) is the coherence length, \(Bc\) is the thermodynamic critical field of niobium, and \(c\) is a fitting constant. The residual surface resistance \(R_s^{\text{res}}\) was found constant and not depending on \(f\), \(T\) or \(B\). The surface resistance describing dielectric losses \(R_s^{\text{die}}\) was best fitted by \(R_s^{\text{die}}(f) \sim f\). The surface resistance describing the low-field Q-increase \(R_s^{\text{Q-inc}}\) was best fitted by \(R_s^{\text{Q-inc}}(B, f) \sim f B^2\).

**QUANTITATIVE ANALYSIS OF THE SURFACE RESISTANCE**

**Introductory Remark**

The surface resistance \(R_s\) depends not only on the experimentally controllable variables, such as \(B\), \(f\), \(T\), but also on others, which we called “parameters”, such as the normal state conductivity \(\sigma_n\), the residual resistivity ratio \(RRR\), the penetration depth \(\lambda\), the critical temperature \(T_c\), and the thermodynamic critical field \(Bc\), etc. If we had to our disposition a complete theory for \(R_s\), we could in principle determine these parameters by a fit, and compare them with accepted values. In what follows, such a theory will be established and the parameters determined and cross-checked.

**The BCS Surface Resistance**

Instead of solving the Mattis-Bardeen integrals for the surface resistance, we base the analysis, for reasons of clarity, on the two fluid model \([2]\). \(R_s\) is described, after modification following the BCS theory by \([3]\),

\[
R_s = \mu_0^2 \omega R_s^{\text{inc}} = \lambda \sigma_0 n_0^2 \lambda^3 \left( \frac{\Delta}{k_B T} \right) e^{-\Delta/k_B T} \quad (T \to 0).
\]

\(\sigma_0\) is the conductivity just above \(T_c\), \(\lambda\) is the penetration depth, \(\lambda = \lambda_0/\sqrt{1 - (T/T_c)^4}\), \(\lambda_0\) being the penetration depth at \(T \to 0\) K.

The variables under control of the experimenter are the temperature \(T\) and the frequency \(\omega = 2\pi f\).

The conductivity \(\sigma_0\) of the normal electrons,

\[
\sigma_0 = \frac{1}{m^* \nu},
\]

depends on the mean free path \(l\), the electron mass \(m\), the Fermi velocity \(\nu\), and the density just above \(T_c\) of the normal electrons \(n_{00}\). \(\sigma_0\) depends on \(\sigma\), the electrical conductivity at room temperature, via \(RRR\),

\[
\sigma_0 = RRR \cdot \sigma.
\]

**The Non-linear Surface Resistance \(R_s^{\text{nl}}\)**

**The field dependence of \(R_s^{\text{nl}}\)**

The interface vacuum - superconductor with defect

The superconducting surface of niobium is supposed not to be perfect in a sense as not to impede the entry of magnetic flux (no Bean-Livingston barrier). It may, for instance, contain a normal conducting “defect”, acting as a “condensation nucleus”, with radius \(a\) small compared to the characteristic length scales in a superconductor (coherence length \(\xi\) and penetration depth \(\lambda\)).

![Figure 1: The superconductor loses energy inside the condensation volume \(V_c\) and gains energy inside the magnetic volume \(V_m\).](image)

We suppose that the surface is exposed to an RF magnetic field \(B\). Inspecting Fig. 1, at the interface between the normal conducting defect and the circumjacent superconductor, being of type II, the Cooper
pair density increases relatively rapidly away from the defect on a characteristic length scale $\xi$, defining the condensation volume $V_c$. The shielding action against $B$ by the Meissner effect will become effective even further away on a length scale of typically $\lambda > \xi$, defining the magnetic volume $V_m$. Hence, compared to a situation where the transition is abrupt, the superconductor must provide less condensation energy, independent of $B$, and gains diamagnetic energy, proportional to $B$, up to a finite $B_{c1}^* = B_{c1}$, where the energy balance is equalized. For $B > B_{c1}$, entry of magnetic flux is favoured, because this lowers the total energy. Hence, the energy balance becomes negative once the applied magnetic field $B$ exceeds $[1]$

$$B_{c1}^* = \sqrt{\frac{V_c}{V_m}} B_c$$

or, equivalently

$$V_c B_c^2 = V_m B^2$$

As outlined in ref. 1 an increase of $B$ by $dB$ leads to an increase of $V_c$ by

$$dV_c = \frac{\nu V_m}{B_c^2 - (\kappa B)^2} dB.$$  

Derivation of the field-dependent surface resistance

An increase of the condensation volume $dV_c$ is accompanied by an increase of the electron density $d n_{\text{el}}$

$$dn_{\text{el}} = n_0 n_d(T) dV_c$$

$n_d(T)$ being the defect volume density, which may depend on the temperature $T$. Differentiating eq. 5 and using eqs. 9 and 10, we obtain

$$d \sigma_{\text{el}} = \frac{e^2}{m v_F} d n_{\text{el}} = \frac{e^2}{m v_F} n_0 n_d(T) dV_c =$$

$$= \frac{e^2}{m v_F} n_0 n_d(T) \frac{2V_m}{B_c^2 - (\kappa B)^2} dB.$$

By the definition of the surface resistance $R_s$, in analogy to eq. 4, the dissipated power per unit area $p = R_s (B/\mu_0)^2/2$ increases with the conductivity as

$$dp = \frac{1}{4} \omega^2 \lambda(T)^3 B^2 d \sigma_{\text{el}}$$

Using eq. 11,

$$dp = \frac{1}{2} \omega^2 \lambda(T)^3 \frac{e^2}{m v_F} n_0 n_d(T) \frac{V_m}{B_c^2 - (\kappa B)^2} dB =$$

$$= \text{const} \cdot n_d(T) \frac{B^3}{B_c^2 - (\kappa B)^2} dB$$

$$\text{const} = \frac{1}{2} \omega^2 \lambda(T)^3 \frac{e^2 n_0}{m v_F} \frac{V_m}{\sigma_{\text{el}}} = \frac{1}{3} \pi \omega^2 \lambda(T)^6 \sigma_{\text{el}}$$

Integrating $dp$ from the threshold field $B_0$ (taken for convenience as zero) to the magnetic field amplitude $B$, one obtains

$$p = -\text{const} \cdot n_d(T) \frac{B^2}{2\kappa^2} + \frac{B^2 \ln \left(1 - \kappa \left(\frac{B}{B_C}\right)^2\right)}{2\kappa^4},$$

and, by definition of the surface resistance,

$$R_s^{\text{nl}} = \frac{p}{\frac{1}{2} \left(\frac{B}{\mu_0}\right)^2} = -\text{const} \cdot n_d(T) \mu_0^2 \left[ \frac{1}{\kappa^2} + \frac{\ln \left(1 - \kappa \left(\frac{B}{B_C}\right)^2\right)}{\kappa \left(\frac{B}{B_C}\right)^2} \right].$$

After expansion of the logarithm and the addition of a temperature independent term for the residual resistance $R_{\text{res}}$ (c.f. below), eq. 15 reads

$$R_s^{\text{nl}} = \left( R_{\text{res}} + \text{const} \cdot n_d(T) \mu_0^2 \left[ \frac{1}{\kappa^2} + \frac{\ln \left(1 - \kappa \left(\frac{B}{B_C}\right)^2\right)}{\kappa \left(\frac{B}{B_C}\right)^2} \right] + \frac{\left(\frac{\mu_0 B}{B_{c1}}\right)^4}{4} + \cdots \right); B_C = B_{c1} \bigg| T = 0 \left[ 1 - \left(\frac{B}{B_C}\right)^2 \right].$$

As suggested by the singularity of eq. 15, the surface resistance will grow rapidly above $B = B_{c1}/\kappa$, describing thus the $Q$-drop.

In the low field region, another contribution to the surface resistance originates from the transition per RF half cycle of the defects from the superconducting into the normal conducting state and back again. As this phase transition is of first order, in the presence of a magnetic field, the latent heat $L$ per square meter is dissipated to the niobium lattice and further to the helium bath, independent of the RF magnetic field amplitude $B$. This additional loss results in the surface resistance

$$R_s^{\text{dec}} = \frac{2 m_1 B_{c1}^2}{\pi B^2}$$

which describes the low field Q-increase.

**The temperature dependence of $R_s^{\text{nl}}$**

Proximity effect in the NbO/Nb composite

It is well known that the niobium surface consists of a composite of a niobium matrix that comprises among other elements dissolved gases as interstitials, such as oxygen, and various oxides of niobium [4]. In what follows only the niobium monoxide (NbO) will be considered. NbO is, compared to Nb, a weak superconducting metal with a transition temperature $T_c = 1.38$ K. Other relevant parameters for NbO are shown in Table 1, in comparison with those for Nb. They are the superconducting coupling constant $N\lambda$, the Debye
temperature $\Theta_0$, and the electron density $N$ near the Fermi surface.

A composite of NbO/Nb on top of, and in close contact with, the niobium bulk will be subject to the superconducting proximity effect [5]. It follows that the composite exhibits a transition temperature $T_{SN}$ in between the transition temperatures of the two constituents, called “N” for the weak superconductor (NbO) and “S” for the strong superconductor (Nb). The limit, when the typical extensions of N and S are small compared with the coherence length, is considered here (Cooper limit). The superconducting coupling constants $NV$ for Nb and NbO are determined from their respective critical temperatures $T_C$ via the BCS formula

$$T_C = 1.14\Theta_D e^{-\frac{1}{3Nv}} , \quad (18)$$

$\Theta_D$ being the respective Debye temperatures. The electron density $N$ for Nb is calculated from the Fermi velocity $v_F$, 

$$v_F = \frac{\hbar}{m} (3\pi^2 N)^{1/3}, \quad (19)$$

taken from the literature [6]. The electron density $N$ of NbO is taken from the literature as well [7]. The average coupling constant of the composite in the Cooper limit is given by

$$\langle NV \rangle_{eff} = \frac{N_N v_N + N_S v_S}{N_N + N_S} \quad (20)$$

$v_N$ and $v_S$ being the volumes, $N_N$ and $N_S$ the electron densities, and $N_N v_N$ and $N_S v_S$ the superconducting coupling constants of the N and S components, resp.

Once $\langle NV \rangle_{eff}$ is known, the critical temperature $T_{SN}$ is calculated from eq. 20, taking for $\Theta_0$ the value for NbO (Fig. 2), which is the dominant constituent in the composite.

**Percolation effect in the NbO/Nb composite**

Supposing we are increasing the helium bath temperature $T$ from the critical temperature $T_c = 1.38$ K of NbO. Due to the proximity effect, by the presence of Nb (S), the NbO is still superconducting. Increasing $T$ further, the NbO in the composites with the smallest volume fraction of Nb will first become normal conducting. The Nb in the composite still remains superconducting, but it does not yet form a continuous superconducting path among itself. Increasing the temperature even more, the NbO in the composites with a larger volume fraction of Nb will become normal conducting up to the point, where the Nb in the composite forms a continuous superconducting path among itself. This situation is identical with a so-called percolation threshold.

![Figure 2: Critical temperature of the NbO/Nb composite in the Cooper limit of the proximity effect vs. the volume fraction $x = v_S/(v_N + v_S)$ of the S component (Nb).](image)

It should be noted that as soon as the Nb of the composite forms a continuous superconducting path, the NbO of the composite fragments into normal conducting defects of small size. Hence it provides the small condensation nuclei needed for the entry of magnetic flux, as described before, already at a very small RF magnetic field $B$.

However, as long as the Nb in the composite does not create a superconducting path, the entire composite itself represents a normal conducting defect of so large a size that the entry of magnetic flux is prohibited up to large $B$.

Percolation thresholds of composites were extensively studied in recent years. For example, the “void percolation threshold” for continuum percolation, as it is called in the literature, was investigated for a distribution of overlapping spheres (N) with equal radius and voids (S) in between [8]. The threshold was computed to lie at a volume fraction of $v_S/(v_N + v_S) = 0.0301 \pm 0.0003$. Above this threshold the voids are connected.

Applied to the preceding, we identify the spheres with the NbO part of the composite (N), and we identify the voids with the Nb part of the composite (S). We would therefore expect a continuous path of Nb (S) to exist, if $v_S/(v_N + v_S) > 0.0301$. Inspecting Fig. 2, this situation corresponds to a transition from superconducting to normal of the NbO in the composite at $T > T' = 2.015$ K, which we call percolation temperature.

In conclusion, only for temperatures $T > T'$, magnetic flux will enter and make the condensation volume $V_c$ grow under the action of $B$, as described by eq. 8.

Hence $R_{SN}^0$ will vanish below $T'$. For a constant number of defects becoming normal conducting in every temperature interval, the effective defect volume density $n_d(T)$ increases linearly with $T$. Hence, for $T > T'$, and $n_{d0}$ being the defect volume density at $T_c$ of niobium,

$$n_d(T) = n_{d0} \frac{1}{1 + e^{(T - T')/\omega}} \frac{T - T'}{T_c - T'} , \quad (21)$$

with step width $\omega$.  

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For temperatures \( T < T' = 2.015 \text{ K} \), the normal conducting NbO will contribute to the residual surface resistance. We take into account these considerations by the term \( R_{\text{res}}' \) in eq. 16.

Table 2: Fit parameters obtained by \( \chi^2 \) minimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Error interval</th>
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<th>Error interval</th>
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<tbody>
<tr>
<td>( \lambda ) [nm]</td>
<td>(86, 89)</td>
<td>( A/k_BT )</td>
<td>(1.68, 1.73)</td>
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<td>( \text{RRR} )</td>
<td>(480, 530)</td>
<td>( B_c ) [mT]</td>
<td>(190, 220)</td>
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<td>( R_{\text{res}} ) [n( \Omega )]</td>
<td>(1.2, 2.0)</td>
</tr>
<tr>
<td>( R_{\text{NbO}} ) [n( \Omega )]</td>
<td>(19, 28)</td>
<td>( L ) [J/m(^2)]</td>
<td>&lt; 5( \times )10(^{-12})</td>
</tr>
<tr>
<td>( w ) [K]</td>
<td>&lt; 0.03</td>
<td>( T' ) [K]</td>
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<td>( T_c ) [K]</td>
<td>(8.8, 10.2)</td>
<td>( n_{\text{H}_{2}O} ) [m(^{-3})]</td>
<td>(0.6, 1.1)( \times )10(^{24})</td>
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<td>( \varepsilon''d ) [m]</td>
<td>(0.17 ... 0.35)( \times )10(^{-12})</td>
<td>( k )</td>
<td>(1.740, 1.745)</td>
</tr>
<tr>
<td>( l )</td>
<td>(0.98, 1.01)</td>
<td>( n )</td>
<td>&lt; 1.1</td>
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<td>( \eta )</td>
<td>(1.58, 1.60)</td>
<td>( \times ) error defined for ( \chi^2 &lt; 1300 )</td>
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</tbody>
</table>

Other Contributions to the Surface Resistance

Per definition the residual surface resistance is taken as constant,

\[
R_{\text{res}} = \text{constant}. \tag{22}
\]

It will predominantly originate from trapped magnetic flux.

The surface resistance describing dielectric losses is described by

\[
R_{\text{die}} = \alpha_0\varepsilon''\varepsilon_0\mu_0^2(E/B)^2d; l \approx 1, \tag{23}
\]

with \( \varepsilon'' \) the dissipation factor, \( E/B \) the ratio of electrical and magnetic field, and \( d \) the thickness of the dielectric layer.

In summary, the total surface resistance is composed of the sum of eqs. 4, 16, 17, 22 and 23.

Determination of Fit Parameters

In Table 2 the fitted values of the relevant parameters are compiled. Although the error is relatively large, as expected from the large variety of sample cavities, a consistency with accepted values can generally be stated.

Figure 3 shows the \( Q \) vs. \( B \) curves, obtained with the fitted parameters, for different helium bath temperatures and 704 MHz.

Data from the individual tests that set up the collective data can also be well reproduced by similar fits with

Conclusion

A quantitative relation for the total surface resistance \( R_s \) is established including the \( Q\)-slope/\( Q\)-drop often observed in superconducting cavities. It is based on the entry of magnetic flux, by the action of the RF magnetic field \( B \), starting at normal conducting “defects” at the surface (nucleation centers) and increasing the normal conducting charge carrier density and consequently the surface resistance. A fit without free parameters of the large sample of collective cavity data allows the determination of the model-relevant physical parameters, which agree with accepted values.

REFERENCES