Abstract

In the present work the mathematical model of the multipole system is presented. The multipole system is composed of arbitrary even number of the uniform electrodes. Each of the electrodes is a part of the plane. The potentials of the electrodes are the same modulus and opposite sign for neighboring electrodes. The variable separation method is used to solve the electrostatic problem. The potential distribution is represented as the eigen functions expansions. The boundary conditions and the normal derivative continuity conditions lead to the linear algebraic equations system relative to the series coefficients.

INTRODUCTION

Electrostatic multipole systems are widely used in the accelerator technology for the charged particle beams transport [1]–[3]. In this paper the mathematical modeling of the electrostatic multipole system is presented. The multipole system consist of the even number uniform plate electrodes of the same shape and size. Fig. 1 shows a schematic representation of the multipole system. The similar system was investigated in [4]. A quadrature expression was obtained for the field potential and the constraints imposed on the electrode potentials, under which such a solution is possible, were determined. In our work a system with an arbitrary even number $2N$ of electrodes is modeled. The variable separation method [5]–[7] is used in plane polar coordinates $(r, \alpha)$ to solve the boundary-value problem for the Laplace equation [8].

The multipole potential distribution has the planes of symmetry $\alpha = (\pi k)/N$ and planes of antisymmetry $\alpha = \pi/(2N) + (\pi k)/N$, $k = 0, N - 1$. An additional plane $r = R_2$ can be introduce to limit the area of the problem under consideration without loss of generality. Thus it suffices to consider sector $0 \leq \alpha \leq \pi/2N$, $0 \leq r \leq R_2$ to find the electrostatic field. Schematic diagram of the multipole system sector is presented on Fig. 2 ($\alpha_1 = \pi/2N$).

The problem parameters are:

- $(R_1, 0)$ — the coordinate position of the multipole electrode’s edge,
- $R_2$ — the radius of the area,
- $\alpha_1 = \pi/2N$ — the boundary of the area (the plane of antisymmetry).

\[ U_0 \] — the multipole electrode potential ($\alpha = 0$, $R_1 \leq r \leq R_2$).

MATHEMATICAL MODEL

The electrostatic potential distribution $U(r, \alpha)$ in the area $(0 \leq r \leq R_2, 0 \leq \alpha \leq \alpha_1)$ satisfies the Laplace equation and the boundary conditions.

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The potential distribution functions (2)–(5) are the Laplace equation solutions for each subdomain \((i = 1, 2)\). The boundary conditions (1) are satisfied for \(\alpha = \alpha_1\) \((0 \leq r \leq R_2, r = R_2 (0 \leq \alpha \leq \alpha_1), \alpha = 0 (0 \leq r \leq R_1)\).

The coefficients \(c_m\) in expansion (4) can be calculated in an explicit form with the boundary conditions (1) \(\alpha = 0 (R_1 \leq r \leq R_2)\):

\[
c_m = \frac{U_0}{\pi m} (1 - (-1)^m).
\]

The continuity condition for the potential distribution \(U_1(R_1, \alpha) = U_2(R_1, \alpha)\) for \(0 \leq \alpha \leq \alpha_1\) leads to the relation between the coefficients \(a_n\) and \(b_k\):

\[
a_n = \frac{2}{\alpha_1} \frac{\mu_n}{\pi} \times \left[ \sum_{k=1}^{\infty} \frac{b_k}{\nu_k} \left( \frac{R_2}{R_1} \right)^\nu_k + \left( \frac{R_1}{R_2} \right)^\nu_k \right] + \frac{U_0}{\ln \frac{R_1}{R_2}} \sum_{m=1}^{\infty} \left( 1 - (-1)^m \right) \frac{\lambda_m}{\lambda_m} \frac{\alpha_1}{\alpha_1} \coth \lambda_m \alpha_1.
\]

Then, in consequence of the formulas (7) and (9) the coefficients \(b_k\) are the solutions of the linear algebraic equations system:

\[
\sum_{k=1}^{\infty} \frac{b_k}{\nu_k} \frac{\nu_k}{\nu_k} \left( \frac{R_2}{R_1} \right)^\nu_k + \left( \frac{R_1}{R_2} \right)^\nu_k \times \left[ \frac{\nu_k}{\mu_n} \left( \frac{R_2}{R_1} \right)^\nu_k + \left( \frac{R_1}{R_2} \right)^\nu_k \right] + \frac{U_0}{\ln \frac{R_1}{R_2}} \sum_{m=1}^{\infty} \left( 1 - (-1)^m \right) \frac{\lambda_m}{\lambda_m} \frac{\alpha_1}{\alpha_1} \coth \lambda_m \alpha_1.
\]
CONCLUSION

In this article a mathematical model of the electrostatic multipole system is considered. The multipole system electrodes are the uniform plate electrodes of the same shape and size. The variable separation method is applied to calculate the electrostatic potential distribution for the boundary-value problem (1) in plane polar coordinates. The potential distribution function is represented as Fourier series (3)–(5). Some of the coefficients (6) are determined from the boundary conditions (1) in an explicit form. The continuity conditions for the potential distribution and normal derivative of the electric displacement vector continuity conditions makes it possible to reduce the original boundary value problem (1) to the system of the linear algebraic equations with the constant coefficients (7), (10). All geometric dimensions and potential of the multipole system electrodes are the parameters of the problem.

REFERENCES


