

# SOME PROBLEMS OF BEAM SLOW EXTRACTION\*

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## Abstract

In this paper we discuss some problems of modeling of beam slow extraction systems. It is known that similar type of beam extraction is used for different kind of circular accelerators. Among the most important requirements for such systems is necessary to mention the time uniformity of the extracted beam. There exist the following two key causes. The first of them is induced by time discontinuity of the corresponding steering electrical currents, and the second cause is induced by an beam inertia which usually connected with beam feedback mechanism, which is usually used for temporal smoothing of the corresponding magnetic (and electrical) fields. In the base of our approach we put the matrix formalism for Lie algebraic tools, which allows us to analyze different kind of the time discontinuity cause.

## INTRODUCTION

The objectives of modeling long-term (multi-turn) evolution of particles in circular accelerators various purposes to make demands not only the adequacy of mathematical tools and related software, but taking into account the possibility of control actions necessary for the implementation of corresponding scenarios. For example, in the problems of beam particles extraction from the accelerator it is necessary not only to ensure particles extraction satisfying to some certain criteria are met but also to provide quality control for extraction system. Consideration of real (not ideal) conditions should provide an existence of instability of control systems on one side and not the “ideal” in the extraction system itself. As to the quality of the beam in modern accelerators makes more and more demanding, it is very important to develop adequate mathematical models, effective software products. Besides it is very important also to develop an uniform data-processing system, which provides support for a given extraction mode. Obviously, a required step for the creation of such a complex must be provided a predictive modeling of possible deviations as control beam and the beam characteristics from the program scenario. Under the program characteristics we mean a set of control parameters (characterized, first of all, the beam transport system itself and provides stability of the beam during an extraction process) and the extraction system, which provide the required extraction characteristics.

It is well known that the energy and intensity rise of modern accelerators also requires increasing of the slow

extraction duration. Just this increasing demands much of hard requirements on the uniformity of the temporal structure of the extracted beam. These demands are caused by, first of all, a beam inertia (see further), and the feedback systems limitations. That is why the methods are needed that allow to realize effective numerical experiments to determine the degree of influence of various factors on the temporal structure of the extracted beam, and propose suppression mechanisms for unwanted effects using global optimization methods and symbolic computation that allow for parametric optimization of beam systems.

## A THEORETICAL MODEL

To track the dynamics commonly used the well-known Poincaré sections method. In the case of periodic motion the one turn map generates a discrete map – a Poincaré map. This generated map can be written in the form of an operator equation for  $k$ -th turn –  $\mathcal{M}_k$ :

$$\mathfrak{X}_{k+1} = \mathcal{M}_k \circ \mathfrak{X}_k, \tag{1}$$

where  $\mathfrak{X}_k$  is a phase set occupied by beam particles on the  $k$ -th turn. In the case of  $\mathcal{M}_k = \mathcal{M}_{k+1}, \forall k \geq 1$  we will speak about a periodical map. We should note that the support of periodic motion is a standard problem for most circular accelerators. In the case of the slow or fast beam extraction this periodic evolution is disturbed by introducing an additional magnetic (or electric) field, which is a time-varying field and provide the required extraction process. In this paper, the operator equations (1) is replaced by the matrix equations according to the matrix formalism [1]

$$\mathbb{X}_{k+1} = \sum_{i=1}^{\infty} \mathbb{M}_k^{1i} \mathbb{X}_k^j,$$

where  $\mathbb{M}_k^{1i}$  are matrices corresponding to aberrations of  $i$ -th order, and  $\mathbb{X}_k^j$  is a matrix constructed from all Kronecker degrees of all phase vectors  $\mathbf{X}_k$  of  $j$ -th order ( $k = \overline{1, N}$ , where  $N$  is particles number). It should be noted that in the case of an infinite series equation preserves the very important properties of symplectic and energy conservation law (for stationary fields).

However, these are two very important properties violated in termination of the series, which leads to disruption qualitative and quantitative behavior of the beam. Therefore, in this paper we use the process of symplectification matrices [1] until the desired order of nonlinearity and the law energy conservation [3]. It should be noted that the property of symplecticity is universal, which is satisfied for all Hamiltonian systems, while the law energy conservation is connected with the Hamiltonian function, which is

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a complete energy of the particle. In other words, the corresponding conditions depend on the phase coordinates of the particles and the type of the Hamilton function. Following to methods used in classical mechanics (see, eg, [2]), in beam physics we need every time to adjust to phase coordinates of the particle. That is why this approach can be applied only search of the reference trajectory. In this paper, we investigate the influence of the nonconservation of energy on the characteristics of the extracted beam in the case of well-known problem of slow extraction. It should be noted that the transfer of the described technology to the modern system does not cause fundamental difficulties because physics of the extraction process is preserved.

### COMPUTATIONAL EXPERIMENTS

For computational experiments we use both analytical and numerical presentation of the matrices  $M^{11}$  and  $M^{12}$  (we confine ourselves to by second order of nonlinearities). The symplecticity and energy conservation properties is embedded the corresponding algorithm for correction of elements for  $M^{12}$  (see, i.e. [1]). The matrices  $M^{11}$  and  $M^{12}$  are considered as sums of two additional matrices  $M_1^{1k}$  and  $M_2^{1k}$  ( $k = 1, 2$ ). The matrices  $M_1^{11}$ ,  $M_2^{11}$  fit to linear effects of regular evolution and the shift to the resonance frequency correspondingly. The matrices  $M_1^{12}$  and  $M_2^{12}$  correspond to nonlinearities of the basic field and to nonlinear effects generated by sextupoles properly (see, i. e. [4]).

The computational experiments are provided in several steps. The first step corresponds to resonance progress. This corresponds to phase ellipse transformation to some more complex figures (see on the fig. 1 a–d)).

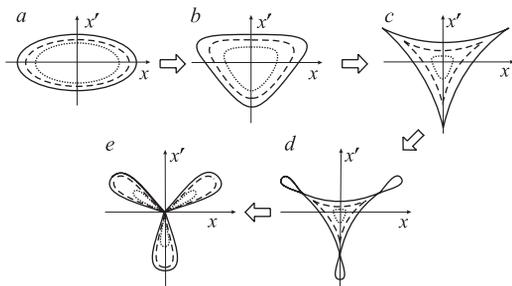


Figure 1: The sequences of phase ellipse in the process of slow phase portrait deformations in the process of an approach the resonance.

Some series of computational experiments were realized for different velocities of resonance progress. The corresponding results demonstrate some very interesting results. For some fixed location Belonging to different layers of the initial phase ellipse extracted particles reach the septum-magnet at the same time (see fig. 2) – one can see the effect of layers mixing. The corresponding analysis of this effect demonstrates also that there is a central part of the initial phase beam portrait, the particles of which are not extracted (see fig. 3). We also investigated the time variations of ex-

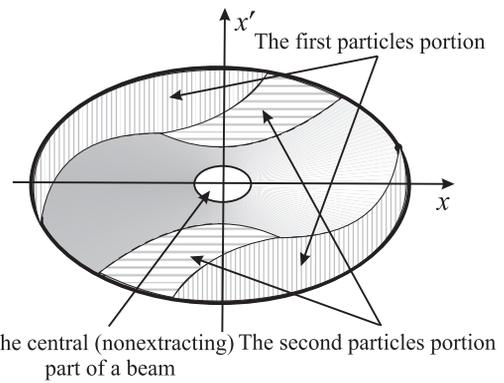


Figure 2: Initial phase coordinates of particles in  $\{x, p_x\}$  extracted from the accelerator.

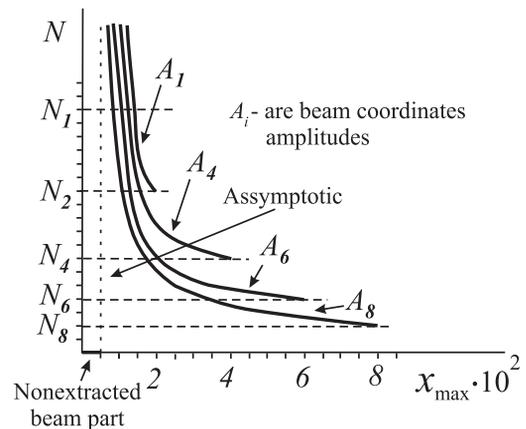


Figure 3: The dynamics of the particles extraction for the case of a fixed magnetic index at the resonance value.

tracted beams in the presence of additional oscillations in additional terms in the linear and quadratic terms of the equations of motion. The corresponding computational experiments were realized for different frequency and amplitude characteristics. As a correction action we considered some feedback pulses supplied with reverse polarity with respect to the sign of the derivative of rise/fall of the amplitude of oscillations of the beam generated by periodic interference in the power system. In this study we also obtained some interesting results, demonstrating beam inertia depending on the amplitude and action time of the feedback impulse (see, e. g, Fig. 4,5).

The amplitude of the peak intensity of the extracted beam from the impulse amplitude are clearly defined nonlinear character (see, Fig. 6 and 7).

### CONCLUSION

As a result of numerical experiments we obtained some interesting results on assessing the impact of various factors on some the extracted beam characteristics. These results allow us to do some conclusions concerning to some procedures correction of undesirable effects. On the basis of these data we deduced some conclusions for the possible

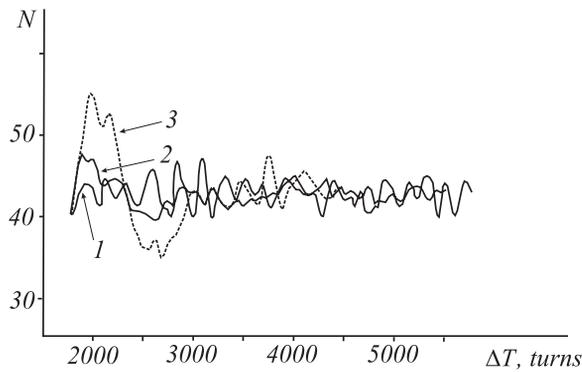


Figure 4: Inertial properties of the beam for different amplitudes of the feedback impulse with the amplitude  $k_s = 10^{-5}$ : 1 -  $\Delta T_1 = 0$ , 2 -  $\Delta T_2 = 7 \cdot 10^{-5}$ , 3 -  $\Delta T_3 = 2 \cdot 10^{-4}$ .

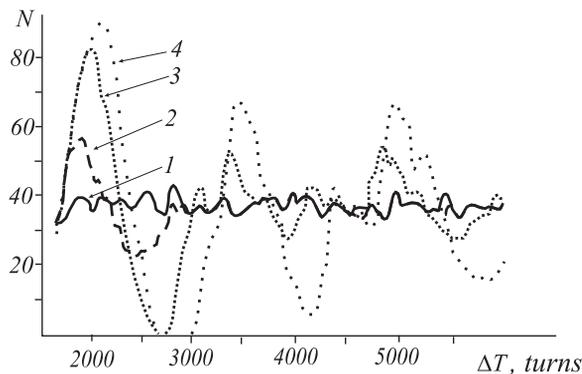


Figure 5: Inertial properties of the beam for different amplitudes of the feedback impulse with the amplitude  $k_s = 3 \cdot 10^{-5}$ : 1 -  $\Delta T_1 = 0$ , 2 -  $\Delta T_2 = 7 \cdot 10^{-5}$ , 3 -  $\Delta T_2 = 2 \cdot 10^{-4}$ , 4 -  $\Delta T_3 = 3.5 \cdot 10^{-4}$ .

correction procedures for unwanted effects. The effectiveness of the computational process is ensured by used matrix formalism. Indeed this mathematical methods allows us to use of parallel and distributed computational resources in the frame of proposed concept of "Virtual Accelerator" [5]. All necessary computational processes are realized using some parallel and distributed technologies on the base of the cluster of virtual machines in Saint Petersburg State University.

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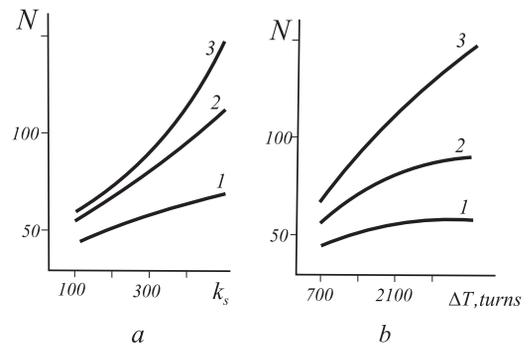


Figure 6: The dependence of amplitude of the extracted beam intensity peaks of the amplitude of the impulse – a) and the length of the impulse – b).

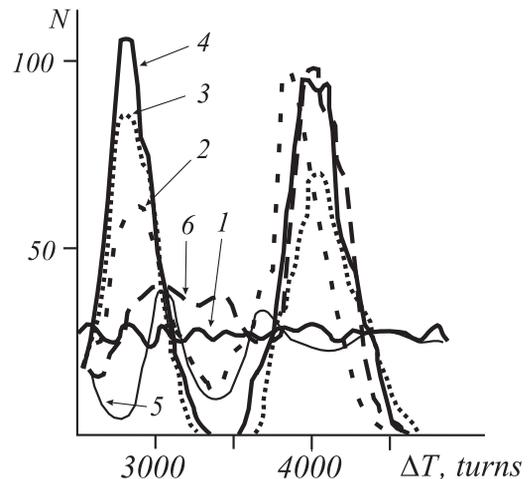


Figure 7: Influence of feedback on the impulse intensity of the extracted beam.

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