ENERGY SPREAD DECREASING IN LINEAR MODE OPERATING LASER PLASMA WAKEFIELD ACCELERATOR

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Abstract
Laser plasma wakefield acceleration (LPWA) [1] is one of the most popular novel methods of acceleration. The LPWA is very perceptively because the accelerating gradient in plasma channel can be a number of orders larger than in metal structures. But the LPWA has two serious disadvantages as very high energy spread and low part of electrons trapped into acceleration. The energy spectrum better than 10 % does not observed anyone in simulations or experiments. Bunching before injection into plasma channel will discuss to decrease the energy spread and to enlarge the electron trapping efficiency.

INTRODUCTION
A number of ideas for increasing the rate of the energy gain have been discussed in the last few decades. Among others, the acceleration of electrons in a modulated plasma channel was proposed by Ya.B. Feinberg in the 1950’s [2]. Possible schemes for the plasma wakefield acceleration (PWA) differing in ways of modulating the plasma channel were developed later. The first one uses a high energy (tens of GeV) beam of particles to form a plasma wave and accelerate a fraction of the injected particles or a probe beam [3]. Another method is the laser plasma wakefield acceleration (LPWA) [1], in which a laser pulse is used to create a plasma wave. The modulation period of the accelerating field (the wakefield) is \( \omega \approx \frac{\lambda}{2} = \frac{c}{2 \omega_p} \), where \( c \) is the speed of light in vacuum, \( \omega_p = (4 \pi n_0 m)^{1/2} \) is the plasma frequency, \( e \) and \( m \) are the elementary charge and mass, and \( n_0 \) is the electron density in plasma. Using two lasers with close frequencies \( \Delta \omega = \omega_p \) was also suggested for enhancing the accelerating gradient even further. The advantage of the LPWA technique vs. conventional accelerators is obvious: the accelerating gradient in a plasma channel can reach hundreds of GeV/m and hence the accelerator can be very compact. This statement does not at present include high power lasers with powers up to \( 10^{12} \) W/cm\(^2\), which are necessary for LPWA, although significant progress is happening in this area with the introduction of fiber lasers. The idea is very popular at present and a number of international collaborations are working on analytical and experimental demonstration of PWA. Large scale projects based on PWA are being discussed now. These include electron-positron colliders, X-FELs and medical facilities. However, the step from a novel acceleration technique to routinely operating facilities has not been made yet. LPWA has two serious disadvantages: a very high energy spread of the accelerated electrons and only a small fraction of electrons is captured into the process of acceleration. An energy spectrum better than 10 % has not yet been demonstrated either in simulations or experimentally. A beam with such a wide energy spread can not be used for the majority of applications including medical and particle physics as the beam can not be transported efficiently.

BEAM DYNAMICS IN LPWA
Considering LPWA, two regimes are distinguished: the underdense plasma, in which \( \pi r_i^2/\lambda_p >> a_0^2/2 \gamma_i \), (quasi linear regime) and the non-linear regime with \( \pi r_i^2/\lambda_p << a_0^2/2 \gamma_i \). Here \( r_i \) is the laser spot size, \( a_0 = e A/\omega_0 \) normalized laser intensity, \( \gamma_i = (1 + a_0^2/2)^{1/2} \). The electron beam dynamics is different in the two regimes. Both regimes, however, experience the high energy spread and low capturing. Conventional accelerators experienced similar problems in the past, where they were solved by bunching the beams using klystron or waveguide type bunchers, and later by producing short bunches with photocathodes. Making a bunch shorter than the accelerating field modulation period \( L_m \) in a plasma channel does not seem to be viable. However, pre-modulation (bunching) of the electron beam can still be used as discussed below.

A few methods for improving the energy spread in the non-linear regime have been proposed. The first is to use two plasma stages with constant but not equal plasma densities and a transient stage with exponentially varying plasma density between them for the beam modulation [4, 5]. The second is so-called ponderomotive injection using two synchronized laser pulses [6]. Two lasers can also excite a beat wave in the plasma, which is then used for bunching in the third method [7]. These methods improve the energy spread to about 3 % for a 1 GeV beam. Still, this number is too high for many applications. The electron capturing efficiency also remains problematic. All the methods described above apply to the non-linear regime. However, the linear LPWA mode is also interesting for practical use. The rate of the energy gain can still be very high, while the laser power requirements are comparatively moderate, meaning that compact, laboratory scale facilities could be designed for accelerating electron beams to hundreds of MeV. Studies of the linear LPWA regime have been conducted at LBNL and INFN LNF and showed that electrons can be accelerated to 1 GeV with an energy spread of 6-10 %. Below two possible bunching schemes can be proposed to decrease the energy spread and improving the number of electrons captured by the plasma wave in the linear LPWA mode.
BEAM PRE-MODULATION SCHEMES

The code for electron beam dynamics simulation in LPWA channel was designed and two possible beam pre-modulation schemes were studied. The plasma channel is divided into two stages in the first scheme. The plasma density is varying in the first, pre-modulation stage, and is constant in the second, the main accelerating stage. The following assumptions are made for simulation: the beam is injected externally, the amplitude of the electric field does not vary on the scale of the time of flight, the plasma is cold, linear and collisionless, the space charge field of the injected electrons is much lower than the plasma wakefield, the beam motion is 1D. The beam dynamics can be studied analytically and numerically in a way similar to how it is done for electron RF linacs. Functions $\omega_p(\xi)$ and $E(\xi)$ describe dependencies of the plasma frequency and accelerating field on the longitudinal coordinate $\xi=2\pi z/\lambda_i$. A variable similar to the wave velocity in a conventional accelerator is introduced $\beta_+(\xi) = (1 - \omega^2_p(\xi))^1/2$, where $\omega_p(\xi) = \omega_p(\xi)\lambda_i/2\pi c$ is the normalized plasma frequency and $\lambda_i$ is the laser wavelength. The equations of motion for an electron in a plasma channel in Cauchy form then are:

$$\frac{d\gamma}{d\xi} = \epsilon(\xi)\sin\varphi,$$

$$\frac{d\varphi}{d\xi} = (1 - \omega^2_p(\xi))^{1/2} - (1 - \gamma^2)^{1/2},$$

where $\epsilon(\xi) = eE(\xi)\lambda_i/2\pi W_0$ is the normalized amplitude of the longitudinal accelerating field in the plasma channel and $\gamma$ is Lorentz factor. Hamiltonian formalism can be applied to the above equations for studying the beam-wave system and the standard energy balance equation written. Injection conditions can thus be analyzed analytically. In contrast to conventional accelerators, the phase velocity and amplitude of the accelerating field are not independent variables, but functions of the plasma electron density $n_0(\xi)$ and are related as $E = me\omega_p/e$. Therefore, optimizing the parameters of the plasma channel is a complex problem in LPWA. The linearity condition for the plasma wave can be expressed as $E(\xi)\sqrt{k(\xi)} = E(\xi = 0)\sqrt{k(\xi = 0)}$, and hence the amplitude of the accelerating field only depends on the longitudinal coordinate. Here $k(\xi)$ describes the plasma wave number in the longitudinal direction.

The beam dynamics in the main accelerating stage of the plasma channel is considered to be similar to the beam dynamics in waveguide or resonator accelerators with a phase velocity $\beta_\parallel = 1$. Strictly speaking, this is only true for LPWA with a zero-field channel in which $n_0 = 0$ (in case the plasma is absent). However, the phase velocity must be close to 1 ($\beta_\parallel \rightarrow 1$) for an efficient acceleration, and at least for underdense plasma we can approximate $\beta_\parallel = 1$. Simulations were first done to find the optimal phase size and energy spread for the injection into the accelerating stage. Electrons previously bunched in the modulating stage can be accelerated to an energy of about 200 MeV with an energy spread $\Delta\gamma/\gamma = 4\%$ providing the initial phase size and energy spread are $\Delta\varphi = \pi/2$ and $\Delta\gamma/\gamma < 2.5\%$ respectively. The choice of plasma (and accelerating field respectively) distribution in a modulation stage was the following step of simulation.

At first, using the RF linac analogy again, we borrow a function conventionally used for waveguide type bunchers for the field distribution in the first stage of plasma channel:

$$E(\xi) = E(\xi = \xi_i) + [E(\xi = 0) - E(\xi = \xi_i)] \times \left[1 - \sin^2(\pi \xi/2 \xi_i)\right]$$

where $\xi_i$ is the normalized amplitude of the bunching region. A number of modulation stage configurations were simulated and the optimal parameters chosen. The beam can be modulated using distribution (2). The resulting energy spread is 4 % with the capturing coefficient reaching 40-45 % front-to-end. Higher capturing (up to 70 %) with a wider energy spread, or better energy spectrum with a lower capturing coefficient (about 20-25 %) can be achieved with different injection conditions of the pre-modulated beam. In general though, these results do not agree well with the single-particle simulations: a phase size lower than $(0.7-0.8)\lambda_i$ has not been observed with field distribution (2) after pre-modulation.

For those reasons, a different bunching scheme consisting of a number of short plasma sub-stages (several $\lambda_i$ long each) separated by drift gaps was considered. This configuration resembles a multigap klystron type buncher. The plasma density distribution in the sub-stages can be simulated using standard functions (step, Gauss, etc.). The step function was chosen for the simulation. The accelerating field distribution in the bunching part is shown in Fig. 1a. The $\Delta\varphi$ and $\Delta\gamma/\gamma$ necessary for an efficient acceleration can be achieved with $E(\xi = \xi_i)/E(\xi = 0) = 0.85$ and a low value of the accelerating field in the bunching part $\epsilon(\xi = 0) = 0.009$ for an injection energy $W_0 = 10$ MeV (see Fig. 1b).

The beam is accelerated in the main plasma stage with $\epsilon(\xi = 0) = 0.033$. It has $\Delta\gamma/\gamma \leq 4\%$ at the output while accelerating from 12 to 108 MeV (the channel length $z_{ch} = 1000\lambda_i$, see beam distribution in the $\varphi$, $\gamma$ phase plane in Fig. 2a and energy spectrum in Fig. 2b). Note that the energy spread decreases with energy increase as in the conventional accelerator and it is equal to $\Delta\gamma/\gamma \leq 2.8\%$ at 205 MeV ($z_{ch} = 2000\lambda_i$, Fig. 2c and 2d) and $\Delta\gamma/\gamma \leq 1.3\%$ at 520 MeV ($z_{ch} = 5000\lambda_i$, Fig. 2e and 2f). In Fig. 2 the beam parameters after the pre-modulation stage are shown as red points, after the main stage as blue ones. In Fig. 3 the energy spread (a) and the part of electrons captured by the plasma wave (b) are shown as the function of output energy.
CONCLUSION

Two possible beam pre-modulation schemes discussed to decrease the energy spread for the linear laser plasma wakefield acceleration mode. The bunching scheme consisting of a number of short plasma sub-stages (several $\lambda_i$ long each) separated by drift gaps is preferable. The low energy spread electron bunch can be accelerated to hundreds of MeV. The part of externally injected electrons captured in to acceleration is very high. Note that the part of accelerated electrons decreases with the beam energy increase due to loses from resonant beam-field interaction.

REFERENCES


Fig. 1. The accelerating field distribution in the bunching part (a) and modulated beam after first channel stage (b).

Fig. 2. The beam distribution in the ($\varphi, \gamma$) phase plane and energy spectrum for different output energies.

Fig. 3. The bunch energy spread (a) and the part of captured electrons (b) as the function of output energy.