STIMULATED RADIATION COOLING *

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Abstract
In this paper the Stimulated radiation cooling (SRC) of ion beams and methods of formation of the broadband laser beams with given spectral distribution are discussed.

INTRODUCTION
Beam cooling is the increase of six dimensional (6D) phase space density and reduction of the 6D emittance of the beam. The brightness of the particle beam (the current density per unit solid angle and per unit energy spread) is proportional to its 6D phase space density. Being an important parameter of the beam quality, it determines the luminosity of the colliding beams and brilliance of the light sources based on particle beams. Thus the cooling of the beams is of high importance for many applications.

The rate of the 6D beam density change in circular machines is determined by the damping increment

$$\alpha_{6D} = \text{div} \dot{F}_r = \frac{2}{\beta^2} \frac{\dot{P}_r(p)}{\varepsilon} \bigg| + \frac{d\dot{P}_r(p)}{d\varepsilon} \bigg|,$$  \hspace{1cm} (1)

where $\dot{F}_r = -\alpha(\tilde{r}, p, t) \dot{\tilde{n}}_p$ is the friction force, $\alpha(\tilde{r}, p, t) = \frac{P_r(p, t)}{c \beta}$ is the frictional coefficient, $\dot{\tilde{n}}_p = \frac{\tilde{p}}{\|\tilde{p}\|}$ is the unit vector directed along the particle velocity, $\tilde{p}$ is the particle momentum, $p = \|\tilde{p}\|$, $\frac{d\dot{P}_r(p)}{d\varepsilon}$ is the averaged rate of the particle energy loss due to friction, $\beta = v/c$ is the relative velocity of the particle, $\varepsilon$ is the energy of the particle, and index $s$ refer to the synchronous values. The equation (1) follows from the Robinson damping criterion [1], [2].

The value $\alpha_{6D} = 2 \sum_{i} \alpha_i$, where $\alpha_i$ are the damping coefficients for the corresponding variables. From the equations of motion and for uncoupled vertical oscillations in a curvilinear coordinate system used in the particle accelerator theory the coefficients are

$$\alpha_s = \frac{1}{2} \frac{\dot{P}_s}{\varepsilon_s} \frac{d\dot{P}_s}{d\varepsilon}, \quad \alpha_s = \frac{1}{2} \frac{\dot{P}_s}{\varepsilon_s} \frac{d\dot{P}_s}{d\varepsilon}, \quad \alpha_{6D} = 4 \frac{\dot{P}_s}{\varepsilon_s}.$$  \hspace{1cm} (2)

The energy loss function must increase with the energy for longitudinal cooling to occur. In the case of the synchrotron radiation cooling if $\frac{d\dot{P}_s}{d\varepsilon} = \frac{d\dot{P}}{d\varepsilon}$, $\frac{\dot{P}}{\varepsilon} = k \varepsilon^2$, where $k$ is a constant, then $\frac{d\dot{P}_s}{d\varepsilon} = 2 \frac{\dot{P}}{\varepsilon}$, $\alpha_s = \frac{\dot{P}}{\varepsilon_s}$, $\alpha_{6D} = 4 \frac{\dot{P}}{\varepsilon_s}$.

Fast (enhanced) 6D cooling will occur if the ratio $\frac{d\dot{P}_s}{d\varepsilon} \bigg|_{\frac{\dot{P}}{\varepsilon} = \gamma} > 1$. One of the examples of the fast cooling is stimulated radiation ion cooling by broadband laser beams [3]. In this case the term “SRC” means a fast cooling of ion beam in the bucket by a broadband laser beam based on the backward Rayleigh scattering [4]. It does not refer to the stimulated emission. In [3] the nonlinear regime of cooling ($\dot{P} = 0$ at $\varepsilon < \varepsilon_c$ and $\dot{P} > 0$ at $\varepsilon > \varepsilon_c$) was considered. Non-fast cooling of ion beam in the bucket by a broadband laser beam based on the backward Rayleigh scattering was named radiative ion cooling [4].

Note, that the beam density obeys exponential law only for the system described by linear differential equations. In this case all parts of the beam are cooled identically.

ENHANCED COOLING OF ION BEAMS
Below we will consider stimulated radiation cooling of ion beams by a broadband laser beam interacting with electrons in the straight section of a storage ring with zero dispersion function. The radial betatron and longitudinal dispersion function. The radial betatron and longitudinal oscillations in a curvilinear coordinate system used in the equations of motion and for uncoupled vertical coefficients for the corresponding variables. From the equations of motion and for uncoupled vertical oscillations in a curvilinear coordinate system used in the particle accelerator theory the coefficients are

$$\alpha_s = \frac{1}{2} \frac{\dot{P}_s}{\varepsilon_s} \frac{d\dot{P}_s}{d\varepsilon}, \quad \alpha_s = \frac{1}{2} \frac{\dot{P}_s}{\varepsilon_s} \frac{d\dot{P}_s}{d\varepsilon}, \quad \alpha_{6D} = 4 \frac{\dot{P}_s}{\varepsilon_s}.$$  \hspace{1cm} (3)

For the enhanced cooling $\alpha_s >> \alpha_s$, $\alpha_s$, which is the reason we consider cooling in the longitudinal plane only and determine the damping time for the energy variable $\tau_s = 2 \alpha_s$. Below linear and nonlinear versions of enhanced ion cooling will be considered.

Linear version of the stimulated radiation ion cooling.
In the linear version of cooling the friction power of ions is a linear function of the energy in the limits of the energy spread of the particle beam:

$$\dot{P} = \dot{P}_m - \frac{e - \varepsilon}{\varepsilon_s - \varepsilon + \sigma_s}, \quad \varepsilon_s < \varepsilon < \varepsilon_s + \sigma_s,$$

$$\dot{P} = 0 \quad \text{at} \quad \varepsilon < \varepsilon_s, \quad \varepsilon > \varepsilon_s + \sigma_s \quad \text{at} \quad \varepsilon < \varepsilon_s - \sigma_s, \quad 2 \sigma_s$$

where $\varepsilon < \varepsilon_s - \sigma_s, \quad 2 \sigma_s$ is the energy spread of the beam. In this case, the minimum of the damping time occurs at the boundary of the linear regime corresponding to the energy $\varepsilon = \varepsilon_s - \sigma_s$. According to (3), the damping time is $\tau_s = 4 \sigma_s / \dot{P}_m$.

Nonlinear version of the stimulated radiation ion cooling.
In the nonlinear version of cooling the friction power of ions $\dot{P}$ can be complicated nonlinear function of the energy in the energy band of the beam being cooled. In the simplest case of stimulated radiation cooling
\[
\overline{P} = 2\overline{P}_n \frac{\epsilon - \epsilon_s}{\epsilon_s - \epsilon_s + \sigma_s} \quad (\epsilon_s < \epsilon < \epsilon_s + \sigma_s),
\]
\[
\overline{P} = 0 \quad (\epsilon > \epsilon_s, \quad \epsilon > \epsilon_s + \sigma_s),
\]
where \( \epsilon_s = \epsilon_s \). In this case the Robinson damping criterion in the form (1)-(3) does not work. It works separately at the energies \( \epsilon > \epsilon_s \) (the rate of damping is 2 times less then in the previous case) and \( \epsilon < \epsilon_s \) (no damping) leading to the average damping time equal to that in the linear case if average powers \( \overline{P}_n \) in these cases are equal at the energy edges \( \epsilon = \epsilon_s + \sigma_s \).

Note that in the linear version of ion cooling the synchronous ions interact with the laser beam and emit scattered photons with power \( \overline{P}_n = \overline{P}_n / 2 > 0 \). It means that the ion beam is heated in this process due to the quantum nature of the light scattering. The equilibrium energy spread of the ion beam in this case is

\[
\sigma_{\epsilon, \mathrm{eq}}^\mathrm{lin} = \sqrt{\frac{\hbar \omega \rho \epsilon}{\hbar \omega / \hbar \omega > \hbar \omega}},
\]

where \( \hbar \omega = \hbar \omega / \gamma \) is the average energy of scattered photons, \( \hbar \omega / \gamma \) is the energy of laser photons, \( \gamma = \epsilon / m_c c^2 \) and \( m_c c^2 \) are the relative and the rest energy of the ion.

In the nonlinear version of ion cooling the energy jumps lead to the decrease of ion energy oscillations if \( \epsilon > \epsilon_s \) and the oscillations are not changed if \( \epsilon < \epsilon_s \) as in the last case the emission of photons is absent. Limiting energy spread in this version of ion cooling is about \( \sigma_{\epsilon, \mathrm{eq}}^\mathrm{non-lin} \leq \frac{\hbar \omega}{\epsilon \sigma_{\epsilon, \mathrm{eq}}^\mathrm{lin}} \).

The nonlinear version of the laser cooling considered above is not optimal one. Faster increase of the power losses at the energies close to the synchronous one like

\[
\overline{P} = \overline{P}_n \frac{\epsilon - \epsilon_s}{\epsilon - \epsilon_s + \sigma_s} \quad \text{at} \quad \epsilon_s < \epsilon < \epsilon_s + \sigma_s,
\]
\[
\overline{P} = 0 \quad \text{at} \quad \epsilon < \epsilon_s, \quad \epsilon > \epsilon_s + \sigma_s,
\]
and at \( \epsilon_s \ll \sigma_s \) is preferable for the decrease of the equilibrium energy spread. Optimization of the nonlinear version is the topic for the future search.

Up to this moment we supposed that the laser beam is homogeneous in the limits of the ion beam area. If the dispersion function of the storage ring differ from zero and the laser beam density decrease in the radial direction at the location of synchronous orbit then the emittance exchange between radial betatron and synchrotron oscillations will take place (wedge shape target) [5]. Fast cooling in the transverse plane will take place in this case as well. The regime of the wedge shape target will occur if the centers of ion and laser beams will be displaced.

**FORMATION OF LASER BEAMS WITH GIVEN SPECTRAL DISTRIBUTION**

The energy dependence of the power \( P(\epsilon) \) is determined by the spectral distribution of the laser intensity. The generation of a laser beam with a broad band frequency spectrum and sharp frequency cutoff is important problem in radiative ion cooling technique. Different schemes of generation can be used. 1) The necessary power can be generated by broadband lasers, filtered (to form necessary spectral distribution) and then amplified by optical parametric amplifiers. 2) An undulator with a deflecting parameter \( K \bot 1 \) together with narrowband laser light located in the interaction region can be used for the ion excitation [3]. In this case a tapered undulator with the magnetic field varying by definite law and monochromatic laser beam will be equivalent to laser with a broad band and a sharp frequency cutoff. 3) Successive frequency shift of a single mode laser by an acusto-optic modulator coupled to a passive ring cavity and other methods can be used [6].

Combination of first or third scheme with second one having much lesser spectral band but higher spectral intensity and more sharp edge can be used for production of the smallest equilibrium beam emittance.

**CONCLUSION**

Fast (enhanced) 6D ion cooling occur in the case of stimulated radiation cooling by broadband laser beams. In this case the equilibrium energy spread and 6D emittance are extremely small. New generations of light sources can be based on ion storage rings if stimulated radiation cooling is used at these rings [7], [8].

**REFERENCES**