

# A RING LATTICE FOR A 2-TEV MUON COLLIDER

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## 1 INTRODUCTION

A 2 TeV on 2 TeV muon collider has been proposed [1] with a design luminosity of  $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ . The ring has a circumference of about 8 km, and the muon lifetime at 2 TeV corresponds to approximately 1000 turns before the beam luminosity degrades substantially. The cooling process produces a round beam with a normalized emittance of  $50 \times 10^{-5} \text{ mrad}$  and  $2 \times 10^{12}$  muons per bunch. The beam's large emittance requires that  $\beta^*$  at the interaction point (IP) must be only  $3 \times 10^{-3} \text{ m}$ , in order to reach the design luminosity. Due to the hourglass effect, a very short bunch length equal to  $\beta^*$  must be maintained to reach the design luminosity. To prevent the bunch from spreading in time, the momentum compaction factor  $\alpha$  must be  $10^{-5}$  [2], or less; that is, the ring must be nearly isochronous.

The small value of  $\beta^*$  leads to very large peak beta values in the final-focus quadrupoles, and unprecedented chromaticities. Correction of the large chromaticities makes for a highly nonlinear Interaction Region (IR). This IR, combined with the isochronicity condition, makes design of a muon collider lattice very challenging. The lattice presented here builds on a base ring design described in a feasibility report, [1], and, more recently, in several muon-collider workshop proceedings [3].

## 2 LATTICE

The collider lattice must satisfy three major design constraints. The first is an Interaction Region (IR) with an extremely low value of  $\beta^*$  ( $\sim 3 \text{ mm}$ ) at the IP, consistent with an acceptable dynamic aperture. This requirement is difficult because the the final-focus quadrupoles need shielding from the high muon-decay backgrounds [4], which reduces their gradients, leading to increased peak  $\beta$ -function values and nonlinear beam dynamics. Second, the ring must be nearly isochronous in order to preserve short 3 mm long bunches with a modest rf system. Third, the corrected chromaticity must be small, so that the momentum dependent dynamic aperture is sufficient. Following is a description of a lattice which meets these requirements.

### 2.1 Overview

The ring has a roughly oval shape, with reflection symmetry about the vertical axis. The lattice has two circular arcs, separated by the experimental insertion and a

utility insertion for injection, extraction, and beam scraping. The two arcs are identical; each contains 14 periodic cells or modules. One additional arc module located at the experimental-insertion end of each arc can be perturbed and rematched to allow adjustment of the machine tunes without impacting lattice functions in the rest of the ring. Consequently the ring structure, both geometrically and optically, has a single superperiod, and reflection symmetry about the line joining the centers of the two insertions.

**Arc module** In order to have very short 3 mm bunches in the 2-TeV muon collider, the storage ring must be quasi-isochronous, which requires that the momentum compaction  $\alpha$  be very close to zero. Furthermore, the lattice must be designed so that over the required momentum range, the momentum compaction remains small. Since the experimental insertion has bending regions with positive contributions to  $\alpha$ , the contributions of the arcs must be negative.

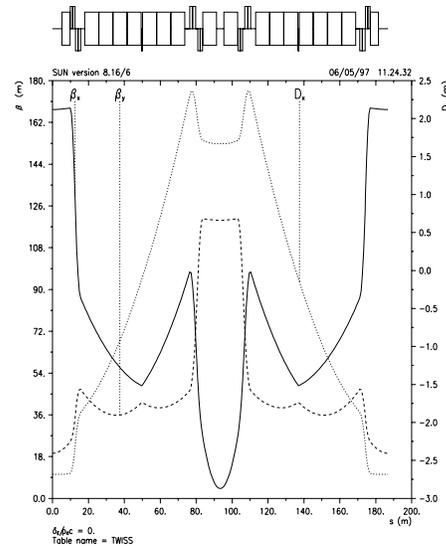


Figure 1: Lattice functions of an arc module ( $\beta_x$ : solid,  $\beta_y$ : dashed,  $\eta$ : dot-dash).

A negative value of  $\alpha$  in the arcs is obtained by invok-

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ing the approach used in flexible momentum compaction (FMC) modules[5]; where two FODO cells are connected by a matching region with horizontal phase advance of nearly  $\pi$  (low  $\beta_x$ ). This drives the dispersion  $\eta$  to a negative value at the ends of the module and makes  $\eta_{bends} \sim 0$ .

The specific module used for this muon collider lattice (Fig. 1) has been modified by including an insertion with a vertical phase advance of nearly  $\pi$  (low  $\beta_y$ ) at the end of the module. This modification has several advantages compared to the standard FMC module. Firstly, the negative dispersion region has been flattened and lengthened so that the value of  $\alpha$  is mainly determined by the bends placed in the dispersion plateaus—making it much less sensitive to  $\eta'$ , and thus providing a wider range of quadrupole settings and corresponding lattice functions which satisfy the negative or low- $\alpha$  criterion. In fact,  $\alpha$  can be tuned with a sensitivity of  $10^{-5}$  to  $10^{-7}$  through the use of paired trim dipoles in these two regions alone; thereby preserving both lattice functions and closure. Over a range in  $\alpha$  of  $-10^{-4}$  to  $+10^{-4}$  the impact on the lattice functions in the rest of the ring is negligible.

Secondly, the vertical phase advance of the module is no longer nearly an integer, but is identical to the horizontal phase advance,  $1.5\pi$ . This is particularly important, because when the corresponding sextupoles in successive modules are separated by odd multiples of  $\pi$  in both planes, one obtains partial cancellation of their geometrical effects. (The near  $\pi$  vertical phase advance caused off-momentum particles to experience integer-related resonances in the long arc strings when sextupole correctors were on.)

Thirdly, the regions of high- $\beta_x$ , low- $\beta_y$  and low- $\beta_x$ , high- $\beta_y$  in this arc module are ideal locations for sextupoles for chromatic correction of the arcs and reduction of the variations of  $\alpha$  with momentum. A pair of horizontal sextupoles adjacent to the center doublets are especially effective in eliminating the dependence of  $\alpha$  on momentum offset. With these two sextupoles the variation of  $\alpha$  over the momentum range of  $\pm .004$  can be restricted to less than  $10^{-6}$ , cancelling the dependence to approximately third order.

**Experimental insertion** The design of an insertion with an extremely low-beta interaction region for a muon collider presents a challenge similar to that encountered for the Next Linear Collider (NLC)[6]. The design used here for each half of the symmetric low-beta insertion follows the prescription proposed by Brown[7]; it consists of two telescopes with a chromatic correction section between. Therefore, the experimental insertion consists of three parts: the Final Focus Telescope (FFT) or IR, a Chromatic Correction Section (CCS), and a Matching Telescope (MT). Fig.2 shows the right half of the insertion beginning at the IP. From left to right, the figure shows the FFT, CCS, and MT.

The low beta-function values at the IP are mainly produced by four strong superconducting quadrupoles in the FFT with NbSn coils. Their poletip fields range from 9.5

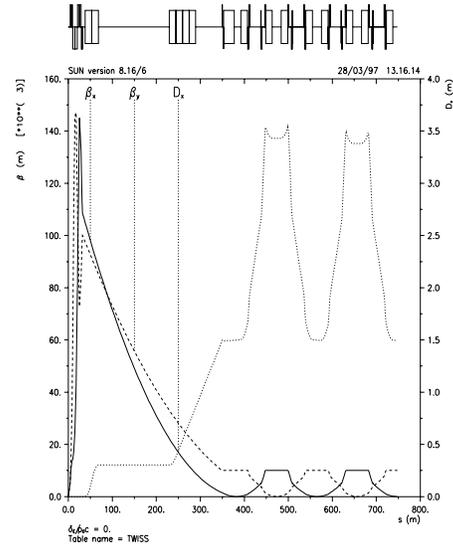


Figure 2: Experimental insertion (half) with 3 mm  $\beta^*$  values at the IP ( $\sqrt{\beta_x}$ : solid,  $\sqrt{\beta_y}$ : dashed,  $\eta$ : dot-dash)

to 12 T depending on the apertures, which determines the size of the coils and sustainable currents [8]. The first of the four quadrupoles begins 4 m away from the IP, they all have 2-cm thick tungsten liners and are interleaved with tungsten collimators to protect them from the intense backgrounds from muon decay [4].

The IR quadrupoles are followed by a pair of 15-m long bucked superconducting dipoles which sweep background particles produced by muon decays away from the IR. A long space follows without quadrupoles but with a substantial length of bending magnets that function to match  $\eta$  and  $\eta'$  from their zero values at the IP into the CCS and to give additional protection of the detector from muon decay products. In this IR design,  $\beta_{max}$  in both planes is 145 km. A more detailed description of the experimental insertion, especially the FFT, can be found in the Snow-mass proceedings[9].

The extremely high beta values in the FFT quadrupoles produce large chromaticities, which must be corrected locally with sextupoles. The natural chromaticity of the FFT is -1500 in the horizontal and -2200 in the vertical. The purpose of the CCS is to correct these large first-order chromaticities locally, relatively close to the IP, by using interleaved sextupole pairs. These sextupole pairs are located at positions with large values of the dispersion and of the beta function corresponding to the chromaticity to be corrected by that pair.

In this design, these  $\beta$  values are 10 km in the plane being corrected and .5 – .7 m in the opposite plane. The dis-

persion is 3.4 m at the horizontal sextupoles and 1.5 m at the vertical ones. The sextupoles which comprise each pair are separated by betatron-phase intervals of  $\phi = \pi$ . Additionally, they are located at positions where the phase interval from the IP is an odd multiple of  $\pi/2$ . The vertical-correction sextupole is closest to the IP, since the vertical chromaticity is the largest.

This sextupole arrangement cancels the second-order geometric aberrations of the sextupoles, which reduces the second order tune shift by several orders of magnitude. The large ratio between the beta functions allows the sextupoles to be interleaved and still maintain the delicate higher-order cancellation. Shortening the chromatic correction section—especially with respect to the number of maxima and minima included for sextupoles—proves to be very important in improving the dynamic aperture. Placement of sextupoles at minima in the plane not being corrected reduces significantly the aberrations arising from sextupole length and cross-plane correlations.

**Utility insertion** The utility insertion has been specifically designed with high-beta regions to facilitate beam scraping and extraction of unwanted beam, and injection and extraction. A detailed discussion of this insertion can be found in a paper submitted to this conference [11].

### 3 SUMMARY

Work on improving the experimental insertion at first concentrated on reducing its chromaticity by altering the FFT, but it proved necessary also to optimize the CCS and the global phase advance to improve the dynamic aperture. When the peak beta functions in the CCS were lowered from 100 to 50 km, the dispersion raised in the insertion sextupoles, and the working point optimized using the phase trombones, the dynamic aperture increases from 1 to 5 sigma. Further studies indicated that a 10 km version of the CCS with the same final focus structure had an even more improved dynamic aperture due to a much reduced tuneshift with amplitude created by the strong chromatic correction sextupoles.

In a sextupole-dominated ring, as is the muon collider, a global tune of just below the integer or half integer preserves the near  $\pi$  phase advance required on successive passes of beam through strong sextupoles. There appears to be a delicate cancellation of residual nonlinearities on a turn-by-turn basis even when sextupoles are paired with the required  $\pi$  phase difference. The large (negative) tuneshift with amplitude counters the near integer/half integer tune so that particles off the closed orbit are displaced away from and do not cross resonances. Therefore, despite a near integer tune, the working point is stable and appears to be a characteristic of a sextupole-dominated ring.

The chromaticity must be corrected in order to provide a reasonable momentum aperture; if it is not the sign of the momentum which crosses the integer tune (as determined by the residual chromaticity) is severely clipped.

The overall momentum bandwidth of the system is limited by third-order aberrations and residual second-order amplitude-dependent tune shifts. These aberrations arise from small phase errors between the sextupoles and the final quadruplet, and from the finite lengths of the sextupoles. Presently, the entire ring has a dynamic aperture of greater than 5 sigma and a momentum acceptance larger than  $\pm 0.15\%$ . The base working design stands to be improved further by raising the dispersion in the utility sections (from .6 m at the sextupoles) and by implementing higher-order correction schemes; as was done in another collider lattice design by K. Oide [10], which has many good features and performance.

### 4 REFERENCES

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