Abstract
The performance of the Fermilab Tevatron Collider at the commencement of collider run Ib (Jan. 1994) was far below expectations. The poor performance was found to be due to a rolled low-β quadrupole downstream of CDF. This rolled quadrupole coupled the horizontal and vertical motion of the Tevatron beams and significantly reduced the Tevatron luminosity. When the roll in the quadrupole was corrected the performance of the Tevatron improved dramatically. This note will discuss the experimental data indicating the presence of coupling, and subsequent calculations which show how coupling can affect the luminosity.

I. OBSERVATION OF COUPLING

Introduction
Two different kinds of measurements made on the Tevatron beam showed that the Tevatron was coupled. These measurements were of the change in the closed orbit when the strength of a dipole corrector is changed, and the variation in tune as the strength of a quadrupole is changed. [1]
In the case of the closed orbit measurements the coupling manifested itself as the large change in the closed orbit in the crossed plane; the plane orthogonal to the plane of the dipole whose strength was varied. This is shown in figure 1, where the bump was generated using the horizontal correction dipole HE13. (The figure was generated assuming a kick of 30µr.) The rms size of the change in the closed orbit in the vertical plane is ~40% of the rms slope in the horizontal plane. It is possible to understand these data, as well as the data on the tune shift, with a model of the Tevatron lattice which includes a rolled quadrupole downstream of the CDF detector.

With the tune shift measurements the evidence for coupling comes from the anomalously small values of the tune shift measured for quadrupoles outside of the interaction region (IR). The β values that would be imputed from these values would be comparable to those at the interaction point and make no sense with our lattice.

Closed Orbit Measurements
Measurements were made of the change in the closed orbits in both the horizontal and the vertical planes, after applying to the beam a single, known dipole bump. To increase the precision of the resulting data, the measurements, of the changed closed orbit are repeated with a number of different values for the dipole bump, and the slope of the closed orbit value with respect to the size of the dipole bump was calculated. The correction dipoles located at HE11, HE13, VE11, and VE14 are used to produce the bumps. These pairs of correctors are chosen because the phase advance between the locations is large (Δφ_x ~69° and Δφ_y ~95°) and yet is not near a multiple of 180°.

When the measurements were made the horizontal and vertical tunes had been brought together by adjusting the strengths of the skew correction elements in the lattice. The large change in the closed orbit in the crossed plane, which must be due to coupling, was therefore surprising.

The change in the closed orbit in the crossed plane, due to a coupling element of a given magnitude, will be greatest if the coupling element is located at a place in the lattice where the product (β_x β_y) is large. For the Tevatron lattice we find that the maximum values of (β_x β_y) occurs at the outside end of the interaction region triplet. This, together with the fact that the low-β quads are very strong, makes these elements natural ones to look to as the source of the coupling. The closed orbit data unfortunately do not allow the identification of which element is producing the coupling, or even on which side of the IR the coupling occurs.

The normal computer model of the Tevatron contains no skew elements except for the skew quadrupole and skew sextupole elements located in the Tevatron spool packages, and the high order skew moments in the Tevatron magnets.

*This work was performed at the Fermi National Accelerator Laboratory, under contract DE-AC02-76CHO3000 with the U.S. Department of Energy.
The high order moments are not strong enough to account for the magnitude of the observed closed orbits in the crossed planes. The values in the skew quadrupoles are adjusted to bring the tunes in the two planes together, i.e., to reduce the coupling, and therefore should not be the source of the coupling. In order to study the source of the coupling we have to introduce coupling into the description of the Tevatron lattice. This has been done by allowing, in the description of the Tevatron lattice, for a roll, about the beam direction, of the low-β quads.

This modified lattice has been used with a version of the tracking code Tevlat [2], which allows fitting experimental data with MINUIT, varying the parameters of lattice elements, such as the roll, during the fit. While this analysis of the data was under way, a limited survey of the low-β magnets was performed which revealed that the outer member of the triplet downstream of CDF (Q2 in the Fermilab nomenclature) was rolled by ~7 mr.

Since a large roll had been found in a particular magnet I tried to fit simultaneously, the closed orbit data at all four bump locations, by varying the size of the roll of this magnet and the roll of the corresponding magnet at D0. The differences between the measured change and the fitted change in the closed orbit, assuming a kick of 30 µr, are plotted in figure 2 for a bump at HE13. With the other bump locations the fits are of comparable quality.

The fit gives a value of +7.0 mr for the roll in the downstream Q2 at B0 and -1.4 mr for the downstream Q2 at D0. I would estimate that the uncertainty in these values is ~1 mr. The agreement with the survey result is striking and reassuring, but should not be overemphasized. We have represented all the skew effects by the roll in only two magnets which is certainly not the case. We get the correct value in this instance because of the large roll in a single magnet. When the skew effects are distributed among several magnets we cannot expect to so easily find the individual rolls.

The roll in the surveyed magnet was corrected. An immediate result was increase in the luminosity by a factor of two. The closed orbit measurements were repeated. Though the coupling from the interaction regions is considerably decreased there was a fair amount of coupling left in the Tevatron (figure 3). Comparing crossed plane data taken after correcting the quad roll to the original data we find that the crossed plane amplitudes have been reduced by approximately a factor of 2. These data have been fitted as before. The fitted value of the roll in the quadrupole at B0 is now 0.3 mr consistent with our having removed the roll. The fitted value of the roll in the downstream Q2 at D0 did not change from the value found earlier. That is reassuring since there was no change made to that magnet and it gives me some confidence that our fitting procedure has some connection with reality.

**Tune Shift**

The traditional way of measuring the value of β at a location in an accelerator is to vary the gradient of a quadrupole, ΔB', of length l, at that location and measure the resulting tune shift. In the absence of coupling the tune shift $\delta \nu$ is related to the value of β by:

$$\delta \nu = \left(\frac{1}{4\pi}\right) \frac{β l ΔB'}{B ρ}$$

The only data available were taken before the unroll of the downstream Q2 at B0. The current in the different correction quadrupoles at B0 was varied and the tunes measured.

In a coupled machine the simple relationship above is no longer true. It is however, possible to fit the measured tune shift, $\delta \nu$, as a function of quadrupole strength, ΔB', using the model of the Tevatron and MINUIT. The parameters used were the same as the ones used to fit the closed orbit data. The value of the roll in the downstream quad at D0 was fixed at the value coming from our fit to the closed orbit data. It is used because we have no tune shift data taken by varying the quads near the D0 interaction region.

For the case with the large quad roll, the only case for which we have data, the solution gives +6.5 mr for the roll at B0. This agrees very well with the values found from fitting the closed orbit data. It is reassuring that two independent methods find the same roll for the quadrupole at B0 and that both results agree with the survey measurements.

**II. LUMINOSITY**

**Introduction**

With a given intensity in the proton and pbar bunches the
luminosity depends on the sizes of the bunches at the interaction point. The smaller the size of the beams the higher the luminosity. In an uncoupled machine the size, $\sigma$, of the beam is given, when either the dispersion $\eta$ or the momentum spread $dp/p$ is zero, by $\sigma_{rms} = (\beta \varepsilon_{rms})^{1/2}$. The luminosity $L$ is then proportional to $1/(\beta_x \varepsilon_{x rms} \beta_y \varepsilon_{y rms})^{1/2}$.

In the case where we have a coupled machine it is not possible to use the normal horizontal and vertical $\beta$ functions to calculate the beam size. In order to see the effect of the coupling on the beam size, and hence on the luminosity Tevlat was used to track a distribution of 1000 particles for a thousand turns. The initial particle distribution was generated assuming the design lattice with no coupling. The values $\sigma_x$ and $\sigma_y$ for the distribution were chosen so that the particles had a 95% normalized emittance, $\varepsilon_n = 25 \pi$ mm mrad.

During the tracking, for each turn, the value of $\sigma$ for the particle distribution was calculated at D0 and B0 for both planes. The $\sigma$s vary from turn to turn since the original distribution was not a stationary distribution. The values of $\sigma$ were therefore averaged over 100 turns. The resulting average values for $\sigma$, which go into the calculation of $L$ are very constant over the several thousand turns for which we have tracked.

Calculations and Results
The computed values of $(\sigma_x \sigma_y)^{1/2}$ at B0 and D0 are plotted in figure 3 for four configurations of the Tevatron:
- The design lattice with $\beta \sim 0.35 m$.
- The solution found for data taken with the downstream Q2 at B0 rolled.
- The solution found for data taken after the roll of the downstream Q2 at B0 was corrected.
- The solution found for data taken after the shutdown in 9/94 during which additional low-$\beta$ magnets were surveyed and their rolls corrected.

The calculated radius of the beam, $\sim(\sigma_x \sigma_y)^{1/2}$ at B0 after the roll of the Q2 was removed, $\sim 46 \mu$, is $\sim 74\%$ of the calculated radius using the fit to data where the Q2 was rolled. This decrease in radius should translate into an $\sim 80\%$ increase in the luminosity. There is a much smaller change in the beam radius at D0 with the changes in the quad rolls, the unrolled value is $\sim 93\%$ of the value with the rolled Q2.

Using the measured luminosity and bunch intensities it is possible to calculate an equivalent beam radius at the interaction point. Comparing the data from the stores before the unroll of the magnet to the data from stores after the unroll we find that the equivalent beam radius at B0 decreased by $\sim 63 \pm 13\%$ after the unroll. The beam size at D0 did not change significantly. This is in good agreement with the change predicted by the calculation in Tevlat of the beam size.

III. Conclusion
The data on the change in the closed orbit with a dipole bump and the change in tune with a change in the strength of a quadrupole can be used to study coupling in the Tevatron. From the measurements it seems possible to construct a reasonable model of the Tevatron which, together with tracking can be used to calculate the Tevatron luminosity.

IV. Acknowledgments
The data analyzed here were obtained by a number of members of the Accelerator Division as part of an extensive series of measurements performed to better understand and improve the Tevatron. Among those responsible for the data used in this note are G. Annala, S. Assadi, G. Goderre, G. Jackson and W. Marsh. I wish to express my gratitude to them first for taking the data with such care, and then for making it available in a form convenient for analysis.