I. INTRODUCTION

It is well known that large colliders, linear or circular, with small beam sizes are prone to ground motion effects. The effect of seismic perturbations on machine performance has been the subject of several studies [1,2,3,4].

The object of this paper is to report on the results obtained with a dedicated beam position monitor, concerning ground motion effects on the LEP beams. Since the ultimate objective of this study is the LHC it is appropriate to say that LEP is used as a test bench for LHC. The organisation of the report is as follows. It starts with some properties of ground vibrations which are relevant to the observation of the phenomenon in LEP. Then the transformation of ground motion power into beam motion power is calculated. Based on the properties of the transverse monitor the magnitude of the phenomenon can be measured with sufficient precision. The results of the observations will be presented both at the betatron frequencies and at very low frequencies centred around the common mode. They are compared with published data on direct ground motion to check the plausibility of the hypothesis that ground motion is indeed the primary cause of the observed signals. Finally a discussion on the influence of the effect on the LHC beams is presented.

II. GROUND MOTION

From correlation measurements between two probes for varying distances as a function of frequency it is possible to derive the velocity of the ground waves. Indeed, the correlation between two probes at distance $l$ drops to zero for a frequency such that this distance is a quarter wavelength:

$$ l = \frac{\lambda}{4} = \frac{v}{4f}. $$

It is interesting to note that in the TT2A tunnel an average speed of 1500 m/s is found [6], while in two points in the LEP tunnel this speed has increased to 4000 m/s [7]. This is due to a different quality of rock in which the tunnels have been excavated.

The effect of ground motion in a large accelerator is vehicled by the uncorrelated motion of quadrupoles of a focusing family F or D. It follows that the effect will decrease quickly for frequencies below the coherence limit:

$$ f_m = \frac{v}{4l}, $$

where $l$ is the cell length. For LEP $l=79$ m so that the lower limit of the coherence is reached for a frequency of $12.6 \text{ Hz}$ (upper limit) or $4.7 \text{ Hz}$ (lower limit). In LHC $l=90$ m and the coherent limits are $11$ Hz and $4$ Hz. The absolute cut-off frequency is reached when the whole machine fits within a quarter wave length, that is for $l=$machine diameter. This limit for LEP/LHC is $1/8$ Hz. The famous ’7 s hum’ will be next to invisible.

The spectral power density $S_{gm}$ has been measured at many places. A common characteristic is the fact that it falls off by about three orders of magnitude per decade [5,6,7,8]. The dispersion in the results spans over several decades.

Assuming a logarithmic frequency slope of -2.5, the expected power at 2 kHz (close to betatron frequency in LEP) is $S_{gm} = 110^{-14} \mu^2/\text{Hz}$ and at 100Hz $S_{gm} = 110^{-11} \mu^2/\text{Hz}$.

III. FROM GROUND TO BEAM MOTION

Ground vibrations at frequencies higher than $12.6 \text{ Hz}$ will cause uncorrelated motions of the quadrupoles in LEP. A quadrupole displacement provokes a displacement of the beam. If the frequency of the vibration lies in the betatron frequency band of the beam, then the beam will oscillate with an amplitude depending on the power of the exciter (ground motion) and on the frequency spread in the beam.

A quadrupole that is displaced by an amount $\varepsilon$ will cause a beam displacement with amplitude $x$:

$$ x = \sqrt{\beta_0 Q K'} \varepsilon, $$

where $K'$ is the integrated normalised quadrupole strength, $\beta_0$ and $\beta_O$ respectively are the $\beta$ function at the quadrupole and at the observation point. The contributions of many quadrupoles add quadratically:

$$ x^2 = \beta_0 \sum \beta_0 (K')^2 \varepsilon^2. $$

Response at $\beta$ frequencies for beam with tune spread can be computed as follows. Consider a beam with tune spread $\delta Q$. The frequency of the external excitation $x$ lies in the frequency band of the $\beta$ oscillation of the beam. From the study of the transverse stability diagram and for a reasonable distribution function, following expression holds for the oscillation amplitude of the ensemble:

$$ \bar{x} = \frac{x}{2\delta Q}. $$

This can be applied to the previous result to give:

$$ \bar{x}^2 = \frac{\beta_0 \sum \beta_0 (K')^2}{4\delta Q^2} \varepsilon^2. $$

To compute the effect on the closed orbit it is sufficient to replace the optic amplification factor $1/\delta Q$ by the optic orbit amplification factor $1/|\sin(\pi q)|$, yielding:

$$ x_{co}^2 = \frac{\beta_0 \sum \beta_0 (K')^2}{4\sin^2(\pi q)} \varepsilon^2. $$

The data on ground motion are given in the form of spectral densities. Previous formulae can be rewritten taking this aspect into account. Indeed,

$$ S_{gm}(f) = \frac{\varepsilon^2}{df} $$

and

$$ \frac{\bar{x}^2}{df} = \frac{\beta_0 \sum \beta_0 (K')^2}{4\delta Q^2} S_{gm}(f). $$

(1)

$$ \frac{x_{co}^2}{df} = \frac{\beta_0 \sum \beta_0 (K')^2}{4\sin^2(\pi q)} S_{gm}(f) $$

(2)

It is worth pointing out that $\bar{x}$ and $x_{co}$ are quantities that are measurable by a beam position monitor. The next step is to estimate the various constants in the expressions.
The parameter $\beta_o \sum \beta_q (Kt)^2$ depends on the machine optics. It has been computed for the 1993 Pretzl optics used in LEP during physics runs. The result for the vertical plane is 18700 and for the horizontal one 16600.

As far as the tune spread $\delta Q$ is concerned a pragmatic attitude is adopted in the sense that it is taken to be 0.015 which is some average value based on the actual observations. It is realised that this is in contradiction with the expectations for beams influenced by a beam-beam tune spread generated by four interaction points.

The fractional tune $q$ in LEP varies between 0.1 and 0.3. Hence the orbit amplification factor $\sin(\pi q)$ varies between 0.3 and 0.8, say an average of 0.5.

This then leads finally to a value for the expected spectral density of the beam motion at the betatron frequencies of:

$$\frac{\delta}{\sqrt{df}} = 0.43 \text{ nm/} \sqrt{\text{Hz}}$$

while the orbit motion around 100 Hz will have a spectral density of:

$$\frac{\delta_o}{\sqrt{df}} = 0.55 \text{ nm/} \sqrt{\text{Hz}}$$

### IV. PROPERTIES OF THE BEAM POSITION MONITOR SYSTEM

A beam position monitor of the directional coupler type had been installed in LEP for general purpose use. The properties of this monitor and its associated acquisition system can be analysed in great detail for a single lepton bunch coasting in LEP. Special care has been taken to maximise the sensitivity. A resolution of $0.16 \text{ nm/} \sqrt{\text{Hz}}$ is obtained for a bunch intensity of 300$\mu$A, sufficient to observe the expected excitation by ground motion.

### V. OBSERVATIONS

A typical FFT plot of the $\beta$ oscillations is shown in Figure 1.

![Typical FFT plot of beam signals excited by ground motion](Figure 1)

This and similar plots can be analysed using previous expressions to yield a value of $S_{gm}$. The results are presented in next table.

<table>
<thead>
<tr>
<th>$i_b$</th>
<th>frequency</th>
<th>$\delta Q$</th>
<th>peak signal</th>
<th>$S_{gm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$A</td>
<td>Hz</td>
<td>bit</td>
<td>$10^{-14} \mu^2$/Hz</td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>1720</td>
<td>0.016</td>
<td>540</td>
<td>550</td>
</tr>
<tr>
<td>240</td>
<td>1760</td>
<td>0.012</td>
<td>84</td>
<td>8.8</td>
</tr>
<tr>
<td>300</td>
<td>1800</td>
<td>0.014</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>117</td>
<td>1870</td>
<td>0.006</td>
<td>40</td>
<td>2.1</td>
</tr>
<tr>
<td>289</td>
<td>1900</td>
<td>0.014</td>
<td>67</td>
<td>5.3</td>
</tr>
<tr>
<td>300</td>
<td>1940</td>
<td>0.02</td>
<td>45</td>
<td>4.5</td>
</tr>
<tr>
<td>230</td>
<td>1940</td>
<td>0.015</td>
<td>93</td>
<td>18.4</td>
</tr>
<tr>
<td>324</td>
<td>2210</td>
<td>0.018</td>
<td>65</td>
<td>6.5</td>
</tr>
<tr>
<td>190</td>
<td>1090</td>
<td>0.004</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>240</td>
<td>3150</td>
<td>0.006</td>
<td>44</td>
<td>6.8</td>
</tr>
<tr>
<td>260</td>
<td>3160</td>
<td>0.015</td>
<td>140</td>
<td>370</td>
</tr>
<tr>
<td>300</td>
<td>3200</td>
<td>0.008</td>
<td>43</td>
<td>7.4</td>
</tr>
<tr>
<td>340</td>
<td>3300</td>
<td>0.01</td>
<td>32</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>3300</td>
<td>0.008</td>
<td>30</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Table 1: Results at $\beta$ oscillation frequencies

A typical FFT plot of the closed orbit spectrum is shown in Figure 2.

![Vertical spectrum at low frequencies(0 to 112 Hz)](Figure 2)

A clear valley at $q=0.001$, or $f=11.2$ Hz can be seen. It is very likely that this corresponds with the coherence limit that was calculated before and found to be at 12.6 Hz. In table 4 measurements at frequencies 45 and 90 Hz are shown.

<table>
<thead>
<tr>
<th>$i_b$</th>
<th>$f=90$ Hz peak sig.</th>
<th>$S_{gm}$</th>
<th>$f=45$ Hz peak sig.</th>
<th>$S_{gm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$A</td>
<td></td>
<td>$10^{-10} \mu^2$/Hz</td>
<td></td>
<td>$10^{-10} \mu^2$/Hz</td>
</tr>
<tr>
<td>340</td>
<td>170</td>
<td>3.1</td>
<td>1400</td>
<td>210</td>
</tr>
<tr>
<td>200</td>
<td>86</td>
<td>2.3</td>
<td>970</td>
<td>290</td>
</tr>
<tr>
<td>300</td>
<td>120</td>
<td>2.0</td>
<td>1280</td>
<td>230</td>
</tr>
<tr>
<td>80</td>
<td>40</td>
<td>3.1</td>
<td>370</td>
<td>260</td>
</tr>
<tr>
<td>135</td>
<td>70</td>
<td>3.4</td>
<td>450</td>
<td>140</td>
</tr>
</tbody>
</table>

Table 2: Results from low frequency measurement of ground motion effect

This is the moment to make a small digression. It can be shown that Schottky noise is 20 to 30 dB below the noise level of the observation system. Hence, it cannot be at the
origin of the observed signals. A second contender can now be ruled out for the excitation of the beam at a few kHz. That one is power supply noise. Power supplies in LEP are essentially low frequency devices with a bandwidth of a few Hz only. While it is not unreasonable that their effect may be visible at these low frequencies it is to be expected that their strength decreases at a rate of at least two orders of magnitude per decade. At 2kHz the skin effect of the Al chamber will cause a further attenuation in the order of a factor of 10. All this adds up to a signal density which is a lot smaller than the resolution of the system.

In Figure 3 the measurement results with beam are assembled in a comparison plot with direct seismic measurements[7].

![Comparison plot between seismic measurements in LEP and beam motion measurements](image)

Comparison plot between seismic measurements in LEP and beam motion measurements

Figure 3

The correspondence between the direct seismic measurement results and the beam measurements is astonishing. Even the logarithmic slope of 2.5 shows up clearly in the beam measurements. Of course, the vertical scale is very compressed. It is not even sure that the scatter of the results is due to the measurement. Indeed, there is no good reason why the spectral density of the seismic activity should be constant in time. On several occasions, highlighted in Table 2 and 3, a much higher activity was noted.

VI. CONSEQUENCES FOR THE LHC

The transverse excitation source for LEP and LHC is obviously the same $S_{gm}$. The effect of uncorrected ground motion in the LHC is beam blow up since damping is very small. The blow up can be computed with following formula:

$$\tau^{-1} = \sum \beta_j (Kc) \left( \frac{f_{oc}}{2} \right)^2 \gamma \frac{\varepsilon}{\gamma} S_{gm}(f).$$

where $\varepsilon$ is the normal beam emittance and $\gamma$ mass to rest mass ratio.

Assuming that the growth rate should not be less than 40 hours, allowing for a damping time of around 25 hours and an emittance of 3.75 $\mu$radm and $\gamma=8000$ we find an upper limit for $S_{gm}$:

$$S_{gm} \leq 7.010^{-14} \mu^2/Hz.$$ 

In Figure 3 this level is designated by the label 'blow up limit'.

This result shows that most of the time at fractional tunes of 0.15 and above this condition is met. However, seismic activity is not constant and cases were observed where the limit has been exceeded considerably over many hours. The impressive scatter of the results shown in Figure 1 maybe partially due to power variations of seismic vibrations. Moreover, even if the source of the effect is the same in LEP and LHC it is not sure that the motion of the quadrupoles is identical. It is well known that the magnet supports can enhance the motion considerably [1,2]. Therefore, it may be wise to consider a narrow band low power and low noise transverse feedback system that will keep the persistent oscillation amplitude $\tilde{X}$ below a level that corresponds to an acceptable growth rate. The feasibility study of such a system goes beyond the scope of this report.

VII. CONCLUSIONS

The excitation of the beam in LEP by seismic vibrations has been measured rather accurately. Its effect on the blow up of the LHC coasting beams has been estimated. In view of large temporal variations of the power of seismic vibrations it is suggested that a special feedback is envisaged to keep the effect within acceptable bounds at all times.

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IX. REFERENCES