Lattice Studies for KAON Factory Accumulator and Booster Rings

A. I. Iliev *, A.V. Budzko *
INR, 7-A 60th October Anniversary Prospect, Moscow, Russia 117312
R. V. Servranckx
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

Abstract
Some results of systematic studies of alternative lattices for the TRIUMF KAON Factory Accumulator and Booster rings are presented. The aim has been to seek lattices of a compatible form for both rings which have the following features: simple structure, dispersion-free straight sections, high transition energy for the Booster and a straight section of about 7 m with finite dispersion for \( \Pi^- \) injection into Accumulator.

I. INTRODUCTION
The development of the racetrack Booster lattice [1] stimulated a search for a compatible racetrack design for the Accumulator ring. The lattices for the Booster and for the Accumulator, sharing the same tunnel, have different requirements which prevent to designing Accumulator as an emptier variant of the Booster ring. The main constraints for A-ring optics are induced by the requirements for \( \Pi^- \) painting [2], i.e. providing about \( \approx 7 \text{ m} \) long drift unobstructed by quadrupoles for placement of the injection septum, bump magnets and centrally located stripping foil; achieving momentum resolution \( \eta_z \equiv \eta / \sqrt{\beta_z} \) of about 1.3 \( \sim \) 1.5 \( m^{1/2} \) and \( \eta_h = 0 \) at the foil location, depending on the size of painted area; and minimizing \( \beta_{x,y} \) at stripping foil.

A number of alternative lattices for the Booster were considered with the aim of overcoming the weaker features of the standard [1] lattice, i.e. a rather large number of quadrupole families and lack of space in the arcs for momentum collimation.

II. ACCUMULATOR
Two approaches have been investigated: 1) lattices with injection in the long straight sections, 2) lattices with injection into the arc. Our study [3] indicates that the lattice with injection in a straight section is flexible for altering the momentum resolution, though it requires retuning their straight section and/or arc, and exhibits good incoherent properties w.r.t. linear space charge. However, the finite dispersion required in both straights makes the transfer from Accumulator to Booster more difficult, involving both horizontal and vertical bending. It looks better to provide \( \Pi^- \) injection in an arc rather in a long straight.

The sample lattice investigated \( (ARS_4.v22) \) has the same number of cells and superperiods per arc as the standard Booster racetrack lattice \( (S = 4, N = 3) \) but has a different structure in the superperiod: the first half-cell of each superperiod being empty creates a double waist for injection. The tune of each superperiod is 0.5, and hence the dispersion vanishes after passing two superperiods. The overall tunes are just above 5 in both planes. Fig. 1 shows the lattice functions for this lattice. The difference in shape compared to the B-ring seems to be acceptable \( (< 1 \text{ m}) \) and has a bonus of leaving more room for the A-to-B transfer line. The momentum resolution at

\[ \eta_z \approx 1.58, \text{ that is near the optimum value for the 40 Hz Booster scenario.} \]

For potential chromaticity correction four sextupoles are arranged in each arc in non-interleaved 180° pairs and placed in high dispersion regions with extreme \( \beta_{x,y} \) to reduce coupling between the pairs. This scheme perfectly cancels sextupole-induced geometric aberrations, resulting in a quite large dynamic aperture for on-momentum particles \( (\text{d.a.} > 1000 \pi \text{ mm mrad}) \), though the chromatic aberrations are noticeable: off-momentum distortion in the \( \beta \)-functions is about 2 m in both planes.

During accumulation the peak current will increase from 0 to design value of \( \approx 6.5 \text{ A} \) (depending on the beam distribution). To study the dependence of the lattice functions on linear space charge we have developed code LinSp [4] which is able to find a matched solution of envelope equations and then calculates Twiss parameters, dispersion, transition energy and chromaticity, as well as providing the stability analysis by computing eigenvalues \( \lambda_i \) of linearized envelope equations. We have found that the lattice is quite sensitive to integer resonances, leading to blow

---

*On leave at TRIUMF, Vancouver, B.C., Canada
up of either amplitude function and/or dispersion. Thus the bare tunes were chosen far from integers (5.28, 5.32), so crossing of an integer occurs only at higher intensity (about 16 A); for lower currents (up to 14 A) dispersion and amplitude functions change only a little (see Fig. 2).

The lattices considered so far for the Accumulator and Booster have a relatively large number of quadrupole families (four families in the arcs). G. Rees [5] has proposed an alternative racetrack lattice for the Accumulator ring, which has a remarkable feature of having only two quadrupole families. Unfortunately the divergence from the geometry of the racetrack Booster is unacceptable. Rees's attempt at designing a racetrack lattice without matched insertions has, however, prompted us to look at alternative designs for Accumulator and Booster.

The alternative lattice for the A-ring has a triplet focusing structure. Each arc is made of 3 superperiods. Each superperiod consists of 3 cells with the central cell being empty to provide an ≈ 8.7 m-long drift for injection. Due to this structure and the phase advance per cell of 90°, the dispersion vanishes over one superperiod with a local peak of ≈ 3 m, and the corresponding momentum resolution is near the optimum ≈ 1.3 m/1/2. Advantages include the need for only two quadrupole types, and the same focusing structure in the long straight sections and the arcs, making the lattice easier to commission and run. Disadvantages include the shorter straights, which result in a potential problem of lack of space for the A-B transfer line, and larger divergence of the ring shape from the current standard racetrack Booster.

III. Booster

The alternative lattice investigated, NEWBR, has 5 cells and two superperiods per arc, the tune of each superperiod being 1.5. Two empty cells are provided in each superperiod: one at the ends and one in the centre. Such an arrangement is required in order to push up γι. Transition is at γι ≈ 13 and the peak dispersion is about 5.3 m. The straight sections are built from the same FODO cells as the arc's superperiods. The nominal working point is (7.83, 6.74).

The lattice functions for this lattice are shown in Fig. 3. Since canceling the dispersion at the ends of the arcs together with raising γι required a rather high phase advance per cell (108 degrees) the lattice has a higher peak beta function in the arcs (≈ 20 m) than the standard racetrack design. Since the arcs are not second-order achromats chromaticity correction is more difficult. A comparative study of closed orbit corrections indicates that both lattices (alternative and standard) have similar good stability against misalignments and field setting errors: the maximum residual orbit excursion after correction does not exceed 3 mm. However, the variation of the peak dispersion in the arcs and the amount of spurious dispersion in the straight sections were found to be less in the standard lattice: σ<sub>θ</sub> < 1% against σ<sub>θ</sub> < 4.5%, η<sub>ss</sub> < 0.015 m against η<sub>ss</sub> < 0.034 m.

Particle tracking for the chromaticity-uncorrected lattices, with misalignments, field and systematic multipolar errors, have shown that d.a. for the standard lattice BRS4 is in the range of about 750-950 μm mrad with dips near Δp/p = 0 and 0.3%. The dynamic aperture for the alternative lattice lies in the range of about 350-750 μm mrad with two dips in acceptance near δp/p = 0.2% and 0.4%. For low-amplitude motion the dominating resonances in both lattices are the coupling resonances ν<sub>ν</sub> ± ν<sub>γ</sub>, which can be corrected by skew quadrupoles, but as the amplitude of the particles is increased, odd order coupling resonances (mainly ν<sub>ν</sub> - 2ν<sub>γ</sub> and 3ν<sub>ν</sub> - 2ν<sub>γ</sub>) become stronger, resulting in a quick loss of stability in the NEWBR lattice. We found that for both lattices the d.a. dropped very fast with increasing multipolar harmonics B4 and B6 up to about 3 · 10⁻⁴.

Both lattices exhibit similar momentum dependence of tunes and beta-functions, but the higher order off-momentum dispersion is much larger for the NEWBR lattice: ν<sub>eff</sub> = dz<sub>co</sub>/dδ ≈ -0.018 m, also leading to a noticeable momentum dependence of the γ<sub>ι</sub>. The effect on beam stability will have to be carefully evaluated.

To reduce number of quadrupole families a modified standard lattice has been considered ( b-febl0 ). The lattice has the same 8 x 3 FODO structure of arcs, but the empty cell in each superperiod is not shortened anymore. Each long straight is simply 3 regular empty cells which are similar to those in the arcs. Hence only 2 quadrupole families are required, while space for momentum collima-
tion is created, though at the expense of shortening each straight by \( \approx 10.5 \) m. The available space in the straights is still sufficient for rf cavities and beam transfer systems. Some of the features of the lattice: 1) slightly higher peak vertical beta-function in the arcs (18.5 m) due to \( \beta_y \) mismatch in the empty cells (this can be eliminated by trim quads); 2) working points either below half-integer \( \gamma_t \approx 9 \), or above half-integer \( \gamma_t \approx 11 - 12 \) (in both cases dispersion in the straight differs only slightly from zero); 3) good stability under space-charge conditions; 4) less room available for the A-B transfer line.

This lattice seems to be an interesting alternative to the standard design and has to be investigated in detail for placement of all the necessary hardware.

IV. Space-charge effects in the Booster

Self-consistent space-charge effects of the unbunched beam in the standard racetrack Booster have been studied by using a particle-in-cell computer code developed at INR [6]. Several runs have been performed for 2-D initially K-V and Gaussian distributions at a bare tune close to the half-integer (7.65, 5.6). LinSp's prediction of envelope instability for beam with the phase-space density \( J = \gamma / m_{\text{rms}} \) of \( 0.5 - 1.25 \) A/\( \pi \) mm mrad has been confirmed by observing emittance blow-up for an initially K-V beam in the ideal lattice, while an initially Gaussian distribution has shown a smaller growth of rms emittance. After introducing rather strong quadrupole imperfections \( (\Delta G/G = 0.3\%) \) we have observed large rms and full size emittance growth in the x-plane in the range \( J \approx 0.72 - 1.3 \) (see Fig. 5, where the core width, i.e. 38% of all particles, widths of middle (68%) and tails (95%) are plotted versus phase-space density \( J/e_0 \)). This can be compared with the results of a LinSp run for a lattice with quadrupole errors (Fig. 6). Here we clearly see a point of bifurcation \( (J \approx 0.853) \), where a unique (unstable) equilibrium splits into two branches. Bifurcation of equilibrium produces changes in the topological structure of the phase space distribution, which have been observed in computer simulations.

V. Acknowledgments

We would like to thank Prof. M. Craddock for hospitality during our (A.I., A.B.) stay at TRIUMF. We are grateful to him for his support and encouragement during this work. We have benefited from discussions with U. Wienands, G. Mackenzie, R. Baartman, S. Koscielniak, Yu. Senichev. Some of the work in the lattice designs has been aided by the suggestions and insight of G. Rees.

VI. References

[5] G. Rees (private communication)