Impedance Measurements with Strongly Cooled Beams at LEAR

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Abstract

At the CERN Low Energy Antiproton Ring (LEAR) high-density beams are obtained with electron and stochastic cooling. We have tried to determine the characteristics of the beam and its environment in a regime where the cooling force is present and where the impedance is space-charge dominated. Methods used include beam-transfer function measurements and Schottky scans.

INTRODUCTION

During recent years the LEAR users have asked for ever higher fluxes of antiprotons. This has led to the need for a large number of circulating particles, typically 6 to 30 \times 10^9 to date. The improved stochastic cooling reduces the beam emittances (defined here as containing 95% of the particles) to \( \Delta p/p \approx 2 \times 10^{-3}, \epsilon_H = \epsilon_V \approx 5 \pi \text{ mm mrad at the momentum range of } 200 \text{ MeV/c to } 2 \text{ GeV/c.} \) With this density space-charge effects are important at low momentum and the requirements on small impedance of the beam environment made during the LEAR design \((\text{Re}(Z_S/n) < 50 \Omega)) \) must be fulfilled. To counteract instabilities an experimental active feedback system (the "damper") working in both transverse planes was built and tested at strategic momenta (200 and 105 MeV/c).

Electron cooling provides very dense beams \((\Delta p/p \approx 2 \times 10^{-3}, \epsilon_H = \epsilon_V \approx 2 \pi \text{ mm mrad at } 200 \text{ MeV/c}) \) with a cooling time of the order of 1-2 s. In these conditions transverse instabilities occur which limit the number of particles maintained in LEAR to less than \( \times 10^9 \). The damper allows an increase of this limit by at least a factor 10. No longitudinal instabilities are observed. However, the Schottky noise bands of the beam subject to electron cooling develop the humps which are indicative of the strong coupling impedance due to space charge as already shown in 1979 by the Novosibirsk group [1].

To determine the beam characteristics we have combined Beam Transfer Function (BTF) and Schottky noise measurements. In this paper we describe the result of measurements in the transverse plane which include the effect of the damper. We then discuss the longitudinal BTF and Schottky noise measurements which when taken together make it possible to determine the true momentum distribution of a very cold beam and the impedances.

TRANSVERSE PLANE

Since many years it has been shown that coherent beam instabilities can be analyzed in terms of dispersion relations [2, 3] which contain the frequency distribution of the particles and the coupling impedance of the beam environment. The use of the beam response for the determination of these ingredients entering into the stability considerations was first investigated in Ref. [4]. The technique was brought to perfection at the ISR [5], where it was widely used to measure the tunes, tune spreads and coupling impedances. New interest was stimulated in connection with cooled beams, where the response to external and internal excitation is strongly enhanced [1, 6, 7]. Although the theory of the BTF is well known, we recall the essential features here to elucidate the application to cooled beams: the beam response to a transverse excitation is

\[
\frac{\frac{\text{<}x\text{>exp}(\text{j}\omega t)}{B}}{1 + \text{j}DZ_T^*} = \frac{D}{1 + \text{j}DZ_T^*} 
\]

(1)

Here \( <x>\text{exp}(\text{j}\omega t) \) is the beam displacement induced by a sinusoidal excitation \( B\text{exp}(\text{j}\omega t) \) applied on a transverse kicker. The normalized impedance \( Z_T^* \) is related to the transverse coupling impedance \( Z_T \) [4], the circulating beam current \( I = Ne \omega Z_T^* \) and the beam energy \( E = m_0c^2 \gamma \) by \( Z_T^* = Z_T[(c\Omega/2\pi)(E/c)] \). The complex function \( D(\omega) \) is the familiar dispersion integral:

\[
D = \frac{1}{2\Omega} \int \frac{\psi(\omega_B)}{\omega_B - \omega} d\omega_B 
\]

(2)

This integral splits into a real principal value (PV) and an imaginary part (-jPV) due to the residue at the pole. There is a large response when the excitation \( \omega \) is close to a fast (+) or a slow (-) wave frequency \( \omega_B = (n \pm Q)\Omega \). Further \( \psi(\omega_B) \) is the distribution function (normalized to have unit integral) with respect to these frequencies, \( Q \) the tune and \( \Omega/2\pi \) the revolution frequency.

The denominator in Eq. (1) can be interpreted as a shielding - or "Plasma dielectric" - function: due to the presence of the coupling impedance the coherent beam response to external and internal excitation is enhanced by this function. The action of the feedback system appears as a contribution to the real part of the impedance, with appropriate sign to lead to damping. The friction force due to electron cooling can be included as a damping decrement in the single particle frequencies \( \omega_B \) in Eq. (2).

In cooled beams the shielding function \( 1 + \text{j}DZ_T^* \) can reach values very different from 1 for two reasons: the frequency spreads are small, leading to a sharply resonant behavior of \( D(\omega) \) and the density is high, leading to a large space charge contribution to the impedance. In this situation the beam response is strongly distorted and at least two independent...
measurements are necessary to determine the shielding function (i.e., the impedance) and the "true response" \( D(\omega) \). The Schottky signals, where the external excitation can be thought of as being replaced by the beam noise, are modified by the same shielding factor \([1, 6, 7]\). We have therefore chosen to combine BTF and Schottky measurements as discussed in Ref. \([7]\). The data are taken from the same coasting beam, in contrast to the usual practice of combining BTFs from coasts of different intensity. A code has been written which acquires BTF and Schottky scan and deduces the impedance and the distribution function. An example is given for the longitudinal plane below. For the transverse BTF the situation is complicated by the dependence of the betatron frequency on the amplitude.

An important application of the transverse BTF has been the surveillance of the damper adjustment \([5]\). To this end the inverse response \( 1/r_{l,0}(\omega) = -jB/\langle x \rangle \) is routinely displayed (c.f. Fig. 1) using a network analyzer connected to a kicker and a pickup system. Instability for the \( n-Q \) mode under consideration can occur, when the inverse response curve touches the origin of the complex plane \( \langle x \rangle /B \rightarrow \infty \). The shift of the curve with increasing gain of the damping system (Fig. 1) indicates the stabilizing action. Combining this measurement with Schottky noise scans, the threshold conditions can be determined with good accuracy.

\[
E_{ll}(\omega) = 1 + jZ_{n}^{*}D(\omega)
\]

where \( Z_{n}^{*} \) is related to the coupling impedance, the circulating current I and the beam energy E by

\[
Z_{n}^{*} = Z_{o} - \frac{\eta l}{\beta E/e} = Z_{0} \cdot A
\]

Here \( \eta = (1/\gamma^{2}) - (1/\gamma^{2}) \) is the off-energy function and \( \beta = \gamma/c \) the relativistic factor. The longitudinal dispersion integral is given by

\[
D = \frac{\beta}{\rho \Omega - \omega} \int \frac{d\Omega}{p \Omega - \omega} - j \frac{\pi}{p} \psi_{\omega}(p/\rho)
\]

where \( \psi_{\omega}(p/\rho) \) describes the distribution of particle revolution frequencies. The mode number \( p \) is the harmonic of \( \Omega \), with \( \rho \) close to \( n_{l} \), the frequency of the excitation. The BTF response (defined as the beam current modulation at frequency \( \omega \) referred to the voltage on the excitation kicker) and the inverse response function are:

\[
r_{l} = \frac{r_{l,0}}{1 + r_{l,0}Z_{n}^{*}} ; \quad \frac{1}{r_{l}} = \frac{1}{r_{l,0}} + Z_{n}^{*}
\]

where \( r_{l,0} = jA \cdot D \) is the response in the absence of the impedance, with \( A \) defined in Eq. (4). The Schottky power spectrum is modified \([5, 6]\) via the dielectric function to

\[
P_{l}(\omega) = \frac{e^{2}N}{\pi \rho E_{l}^{2}} \psi_{\omega}(\rho/\rho)
\]

In a strongly cooled beam, the space-charge impedance leads to a dielectric function which introduces a peak at the two borders of each Schottky band. To reconstruct the momentum distribution, the method discussed in Ref. \([7]\) is used: the computer program estimates the impedance vector from the inverse response diagram, then computes the corresponding dielectric function and the "true response" \( r_{l,0} \). The Schottky spectrum is then corrected, using the dielectric function, to obtain the "momentum distribution" \( \psi_{\omega}(\rho) \). This curve is compared to the distribution \( \psi_{\omega}(\rho) \) obtained from the real part of the response \( r_{l,0} \). Ideally these functions are identical, each of them giving the momentum distribution. Using an iterating algorithm the evaluation usually converges rapidly. This method was used to estimate the impedance and the momentum distribution of the 200 MeV/c beam subject to electron cooling (Fig. 2). The impedance obtained as an average from a number of measurements is \( Z_{n}/p = (300 \pm 150) \Omega - j(3500 \pm 10000) \Omega \). The reactive part is about 3 times the "Keil-Schnell threshold" which is an ellipse in the response diagram. The resistive part, which is difficult to determine due to noise on the signal and due to the presence of the large reactive component, seems to exceed the 50 \( \Omega \) of the specification. More work is necessary to improve the accuracy.

To identify equipment contributing to the impedance, a program of measurements using the wire method was launched during the last shutdown. A number of critical components were found: \( Z_{n}/p = 50 \Omega \) around 50 MHz for injection kickers \([8]\) and 200 \( \Omega \) for rf cavities \([9]\). Corrections were made for improving the continuity of the image current in the kickers and in ceramic chamber sections. In addition, the rf cavities are now short circuited by "gap relays" during coasting beam operation. Another important contribution to the impedance of

LONGITUDINAL PLANE

The longitudinal shielding function can be expressed as:

\[
E_{ll}(\omega) = 1 + jZ_{n}^{*}D(\omega)
\]

Fig. 1. Transverse stability diagram obtained with electron cooling and damper working. Trajectories in the complex plane of the inverse response \( -jB/\langle x \rangle \) are displayed as a function of frequency. In this example the \( (n-Q) \) band near \( f = 5 \text{ MHz} \) is scanned. The shift of the curves with increasing gain of the damper exhibits the stabilizing effect.

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LEAR is probably due to the large number of pickups and kickers for stochastic cooling.

CONCLUSIONS

When the beam is strongly cooled, Landau damping is lost. A powerful active feedback is necessary to recover beam stability and increase the possible circulating current. Transverse BTF's are a convenient tool to adjust and monitor the action of the damper. By combination of longitudinal BTF and Schottky measurements, the momentum distribution of very cold beams can be determined despite the strong signal distortion, and the beam coupling impedance can be estimated.

REFERENCES


Fig. 2. 2a) Longitudinal response diagram \( l/(\theta l/\omega) \) of a dense beam obtained with electron cooling \((N = 7.7 \times 10^9 \) protons circulating in LEAR). The harmonic near \( f = 8 \) MHz is scanned.
2b) Corresponding Schottky spectrum: notice the two peaks and the depression of the signal in the middle of the band.
2c) True momentum distributions. The noisy curve is deduced from the Schottky measurement; the smoother one from the BTF. In 2a) the impedance vector and the "Keil-Schnell threshold" ellipse are shown. The results of this measurement are:
\[ Z_{//}/p = (140-j3200)\Omega, \quad \text{Im}(Z) = 4 \times \text{K-S threshold}, \quad (\Delta p/p)_{\text{FWHM}} = 1.3 \times 10^{-4}. \]