Coherent Beam-Beam Interaction in DAΦNE

Kohji Hirata
KEK, National Laboratory for High Energy Physics, Tsukuba, Ibaraki 305, Japan
and Eberhard Keil
CERN, European Laboratory for Particle Physics, CH-1211 Geneva 23

Abstract We study the coherent beam-beam interaction in a collider with two rings of equal circumference for the counter-rotating beams, and in particular the case of DAΦNE. There may be one or two active interaction points. The rings may have errors on the phase advances, etc. The bunch population may vary from bunch to bunch. We show that DAΦNE can use the two interaction points at the same time, although care should be taken in the choice of the tunes, because of the existence of a complicated resonance structure in the tune diagram.

1 Introduction

The coherent beam-beam interaction has been studied extensively, cf. [1] for a list of references. It makes it almost impossible to operate a two-ring collider with different circumferences [1, 2]. Although the problem allows a linear analysis [3], one usually needs a numerical calculation, except for very simple cases where the symmetry simplifies the mathematical structure.

Here we study the case of DAΦNE[4]. Its configuration is different from the geometries which have been studied previously. This paper is organized as follows: Section 2 contains a detailed description of the DAΦNE geometry needed for our calculation. Section 3 shows the mathematical model. Section 4 presents computed results, and Section 5 is a discussion of our results and conclusions.

2 Collision Geometry

DAΦNE[4] is a two-ring e−e+ collider. Its configuration is illustrated in Fig. 1. At most 120 bunches circulate in each ring. A pair of e+ and e− bunches collides at the first interaction point IP1. The two bunches then pass through their respective arcs (1− and 1+), and collide at the second interaction point IP2. Whatever is the total number of bunches in each ring, one e+ bunch always collides with the same e− bunch. This problem is quite similar to the case of a one-ring collider with one bunch in each beam and two interaction points. The main difference arises because the phase advances through the arcs are not necessarily close to each other, and because the tunes of the two rings may be different.

3 Mathematical Model

Similar problems were studied earlier [1]. We shall adapt this method and the computer program [3] to our problem. The coordinate and slope of each particle in the e± bunch are denoted by z± and z±, where z refers to either horizontal (x) or vertical (y) coordinates. The barycentre and its slope of each bunch are denoted by Z± and z±, respectively. We assume that the density distributions of both bunches are Gaussian. We calculate the beam-beam kick which acts on each particle according to Bassetti and Erskine[5]. Assuming that the beams always remain Gaussian with fixed rms beam radii σx and σy at the interaction points it was shown[6] that the kick for the barycentre coordinate can be expressed by the standard expression when the σx are replaced by

\[ \sigma_x' \rightarrow \Sigma_x \equiv \sqrt{(\sigma_x^+)^2 + (\sigma_x^-)^2}. \]  (1)

When the separations \( \bar{z}_x \) and \( \bar{y}_x \) are small compared to \( \Sigma_x \) and \( \Sigma_y \), respectively, we can linearize the beam-beam force and get

\[ \delta z' = -\frac{4\pi \Xi_x}{\beta_x^*} (\bar{z}_x - \bar{z}_+), \]  (2)

where \( \Xi_x \) is the effective beam-beam strength parameter

\[ \Xi_x = \frac{r_e N_x \beta_x^*}{2\pi \gamma_x (\Sigma_x + \Sigma_y)}. \]  (3)

Here \( r_e \) is the classical electron radius. \( \gamma_x \) is the relativistic Lorentz factor, and the \( \beta_x^* \) are the value of the amplitude...
functions at the IP. They may all be different for $e^+$ and $e^-$ beams. The usual symmetric colliders are a special case. Note the unconventional definition of $\Xi$: when the beam sizes of both bunches are the same, we get $\Xi = \xi/2$, where $\xi$ is the usual beam-beam strength parameter. On the other hand, in the weak-strong case, we get $\Xi = \xi$.

We employ the following set of canonical variables for the barycentres:

$$Z_{\pm} = \sqrt{N_{\pm}} \gamma_{\pm} \frac{z_{\pm}}{L_{\pm}}, \quad P_{\pm} = \sqrt{N_{\pm}} \gamma_{\pm} \frac{p_{\pm}}{L_{\pm}},$$

(4)

Here $\sqrt{N_{\gamma}}$ plays the role of the bunch mass. A bunch with more particles and/or more energy is heavier in the sense that it is less kicked by the same force: one usually uses momentum (mass $\times$ velocity) rather than the velocity when one studies interaction between particles with different masses.

A turn in DA$\Phi$NE is represented by a $4 \times 4$ matrix $M$, acting on the 4-vector $(Z_+, P_+, Z_-, P_-)^t$:

$$M = V_1 R_1 V_2 R_2.$$

Here $V_i$ describes the free betatron oscillations through the $i$-th arc:

$$V_i = \begin{pmatrix} U(\nu_i^+) & 0 \\ 0 & U(\nu_i^-) \end{pmatrix}$$

(5)

with

$$U(\alpha) = \begin{pmatrix} \cos 2\pi \alpha & \sin 2\pi \alpha \\ -\sin 2\pi \alpha & \cos 2\pi \alpha \end{pmatrix}$$

(6)

where $\nu_i^\pm$ is the phase advance of the $\pm$ particle through the $i$-th arc, normalized to $2\pi$, and $R_i$ describes the beam-kick at the $i$-th interaction point:

$$R_i + I + 4\pi \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\Xi_+ & 0 & \sqrt{\Xi_+ \Xi_-} & 0 \\ 0 & 0 & 0 & 0 \\ \sqrt{\Xi_+ \Xi_-} & 0 & -\Xi_- & 0 \end{pmatrix}$$

(7)

$I$ is the $4 \times 4$ unit matrix. Differences in $\gamma, N, \beta$ and $\sigma$ are integrated into $\Xi$'s. Only $\Xi_{\pm}$ and $\nu_{\mp}$ are independent parameters. The mapping $M$ is stable if and only if all its eigenvalues are unity in absolute value.

### 4 Computer Results

We use the BDPHII program for our computer studies. It is a member of the BBMODE [3] family. The independent variables are the normalized design phase advances $\nu_i^\pm$. We adapt the labelling of the arcs and the bunches, shown in Fig. 1, to the BBMODE convention by imposing the following relations on a perfect DA$\Phi$NE:

$$\nu_1^+ = \nu_2^- \quad \text{and} \quad \nu_1^- = \nu_2^+$$

(8)

If beam-beam collisions happen only at one interaction point, IP1, we get the results shown in Fig. 2. The lego plot shows by how much the largest absolute eigenvalue exceeds unity. Three resonances cut diagonally across the figure. They occur when the tunes $\nu_1^+ + \nu_2^+$ are just below an integer or half an integer. These results are identical to those obtained earlier [1]. Inside the stopband, the growth rate reaches 0.35 per turn, corresponding to an e-folding time of about 3 turns. The shape of the stopband is independent of the phase advance.

A richer spectrum of resonances appears when the second interaction point in DA$\Phi$NE is also activated, and
when phase advance errors \( \phi \) are put into the arcs. The recipe is as follows:

\[
\begin{align*}
\nu_1^+ &\rightarrow \nu_1^+ + \phi_1 & \nu_1^- &\rightarrow \nu_1^- - \phi_2 \\
\nu_2^+ &\rightarrow \nu_2^+ + \phi_2 & \nu_2^- &\rightarrow \nu_2^- - \phi_1
\end{align*}
\] (9)

When \( \phi_1 \) and \( \phi_2 \) are equal and opposite, the two rings have the same tunes, otherwise the tunes of the two rings are different. The result for a case with split tunes is shown in Fig. 3. Resonances occur when the tune in one ring, or the tune in the other ring, or their sum is just below an integer or half an integer. The maximum growthrate inside the stopband is about twice the value shown in Fig. 2. The shape of the stopband is modulated with the phase advance.

When the phase advance errors vanish, but the two bunches in collision have population differences of \( \pm 10\% \), a narrow weak stopband parallel to the half-integral stopband appears.

Differences in the \( \beta \)-functions between the two interaction points cause a variation of the effective beam-beam strength parameter \( \Xi_y \) and lead to a narrow weak stopband parallel to the half-integral stopband, as shown in Fig. 4.

### 5 Discussion

When DAΦNE is operated with one interaction point, and the beams are well separated at the other one, the system describing the beam-beam collisions in linear approximation is reduced to the case considered in Ref. [1]. There are three classes of resonances:

- **Half-integral resonances** when one of the vertical tunes \( \nu^\pm \) is just below an odd half integer
- **Integral resonances** when one of the vertical tunes \( \nu^\pm \) is just below an integer
- **Sum resonances** when the sum of the vertical tunes \( \nu^+ + \nu^- \) is just below an integer

This set of resonances is visible in Fig. 2. The stopbands occur along lines where the phase advances through the two arcs add to a constant value. The growth rate is projected almost along the stopband. The same resonances occur in the horizontal plane.

When DAΦNE is operated with two interaction points, and the phase advances in the two rings are the same, the same number of stopbands is found, but the growth rate becomes a function of the phase advances, and reaches a peak which is twice as high as with one interaction point. When phase advance errors are introduced in addition such that the tunes of the two rings become different, the number of stopbands increases, as shown in Fig. 3. However, all stopbands still belong to the set listed above because the half-integral and integral stopbands for the two rings no longer coincide. This leads to the three neighbouring stopbands shown, those for each ring surrounding the sum resonance.

Variations in the bunch populations and in the effective beam-beam strength parameters \( \Xi_y \) do not increase the number of stopbands further.

Clearly, the structure of stopbands does not change much when the number of interaction points changes from one to two. However, the area in phase advance space occupied by stopbands is close to one third. Finding an operating point outside all the stopbands requires a tight control of all phase advances, in both rings and in both the horizontal and vertical plane.

We have studied only the beam-beam collisions at the interaction points, and neglected the peripheral collisions which may occur at multiples of half the bunch spacing away from the interaction points if the bunches are inadequately separated there.

### References