Experimental Laboratory for Study of Nonlinear Quantum/Classical Dynamics in an R.F. Bucket

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Abstract

Several advantages of using single Penning trapped particles (electron or proton) for long term dynamical studies of chaos are described. Such studies would complement, for instance, several recent speculations on the transition from quantum like to classical like motion of such systems. Since the Penning rap dynamics are formally equivalent to familiar accelerator (synchrocylotron) longitudinal motion these studies would be relevant to understanding aspects of machine stability.

Introduction

Consider longitudinal motion in an ideal synchrocylcotron. We wish to point out that such motion may be studied experimentally to a degree of precision and without extraneous perturbation (e.g. gas scattering or "random" guide field imperfection) heretofore impossible to attain in the usual accelerators. The experimental tool is the geonium Penning trap.[1,2] The importance of such clean dynamical experiments could be in elucidating the long term dynamics of high energy accelerators.[3] For this situation Penning trapped protons (i.e. with negligible radiative decay) can be studied.[4] Possible cyclotron frequencies of > 10 MHz would allow extremely large orbit number studies of nonlinear Arnold diffusion and related conservative Hamiltonian dynamics phenomena.[5]

The same type experiments on trapped electrons represent a complementarily rich laboratory for nonlinear dynamics. In this case radiation damping is signficiant[1,2] (with $|B| \simeq 6$ Tesla). However the damping is still weak in the sense that $\Gamma_{rad}/\Omega_{cycle} < 10^{-10}$. A great deal of interest in the dynamics of such a nonlinear system has been theoretically generated recently.[6,7,8] The interesting point being the transition from QM to classical - like motions as a function of weak damping. Here we discuss this damped system because of its current topical interest, but also due to the existence already of some experimental results.[2,3] In order to study a nontrivial spectrum of equilibrium points the RF driven electron system is studied. A peculiarity is stability properties with respect to RF noise, a problem of interest in high energy machines.[9]

Anharmonic Stability Diagram

The similarity (fundamentally an equivalence) of particle dynamics in a Penning trap to that in a synchrocyclotron has been pointed out.[7] "Vertical" focusing is accomplished electrostatically however. Here we concern ourselves only with the cyclotron motion, which is equivalent to the longitudinal motion in accelerator language. Thus the equations of motion we consider are:

$$\dot{\mathbf{V}} = \gamma^{-1}\Omega \times \mathbf{V} - \Gamma/2\ \mathbf{V} + \frac{e}{m}[E_D(\omega t) + E_F(t)] \qquad (1)$$

*Operated by the Universities Research Association under contract with the U.S. Department of Energy.

where γ is the usual Lorentz factor; Ω the cyclotron frequency; Γ the radiative damping; $E_D(\omega t)$ the rf drive field at frequency ω; and $E_F(t)$ a fluctuating "noise" field. Typical single trapped electron experiments involve $\Omega \sim 10^{12}$ Hz (B ~ 6T) and $\Gamma \approx 10$ Hz, the small ratio Γ/Ω allowing for extreme precision resonance measurements.[1]

We examine (1) in a velocity frame rotating at ω, where stationary solutions $\beta = |v/c|$ are given by (with $E_F = 0$):

$$(\Delta\omega - \Omega\,\frac{1-\gamma}{\gamma})^2 = f^2/\beta^2 - (\Gamma/2)^2 \qquad (2)$$

where $\Delta\omega = \omega - \Omega$; and $f = e/m\ E_D$. Solutions to (2) are along the nonlinear hysterisis curve, Figure 1.[10] Only the largest $\beta(\Delta\omega)$ branch is stable (Bucket center). Next below this is the usual vestable fixed point branch. Less familiar is the single valved region near $\Delta\omega=0$. The unique feature of the Penning electron is that this region is accessible since $\hbar\Omega >> KT$.[1] With $E_D=0$ the electron settles into a Landau ground state, $\Delta\omega \sim 0$. Accelerator along the stable branch must proceed from the origin.

Such acceleration experiments have been performed, the resolution of the stable branch from baseline noise being < 20 $\hbar\Omega$ at present and a goal of one quanta expected.[2,11] A thorough description of the experimental miliev and theoretical analysis may be found in the review of Brown and Gabrielsc, from the point of view of application to precision measurements of g_e-2.

Figure 1 does not describe the whole dynamical phase space. In particular the classical picture of the bucket separatrices is not clear near the origin. This familiar accelerator point of view is thus extended in Figure 2. The novel feature in this regime is that the separaticies impinge along the zero kinetic energy ordinate for a finite phase interval. As the multibranched region is entered a second (non accelerating) bucket appears with its separatrix at first tangent to the upper branch's.

Quantum vs. Classical Dynamics

The dynamics represented above is purely classical although we showed that experiments down to a quantum mechanical energy scale are possible. RF excitation (acceleration) along the stable branch of figure 1 is a very complex (unsolved[12]) quantum mechanical problem.

Huberman and Crutchfield pointed out, quite some time ago, the rich dynamical structure of classical systems (i.e. driven damped anharmonic oscillators) essentially equivalent to (1).[8] To aide comparsion with their work, we note that (1) (or at least its rotating frame equivalent) is really a one dimensional problem.[13] This is perhaps not surprising since the classical cyclotron orbit problem (even with damping) clearly is equivalent to a 1° SHO. Simple use of the gauge freedom expressing the canonical angular momentum yields (equivalent to (1)):

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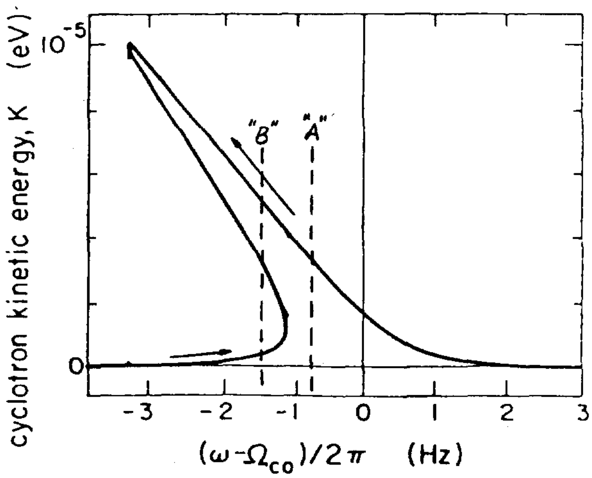

Figure 1. Anharmonic stationary solution curve for eqns. 1-3. This may be viewed as a resonance curve which is tilted by the nonlinearity. Thus its "width" (spacing between upper branches) is determined by (damping) and f (power broadening). The scales pertain to an explicit electron experiment. Synchrocyclotron acceleration corresponds to sweeping RF drive up along the top branch (arrow up).

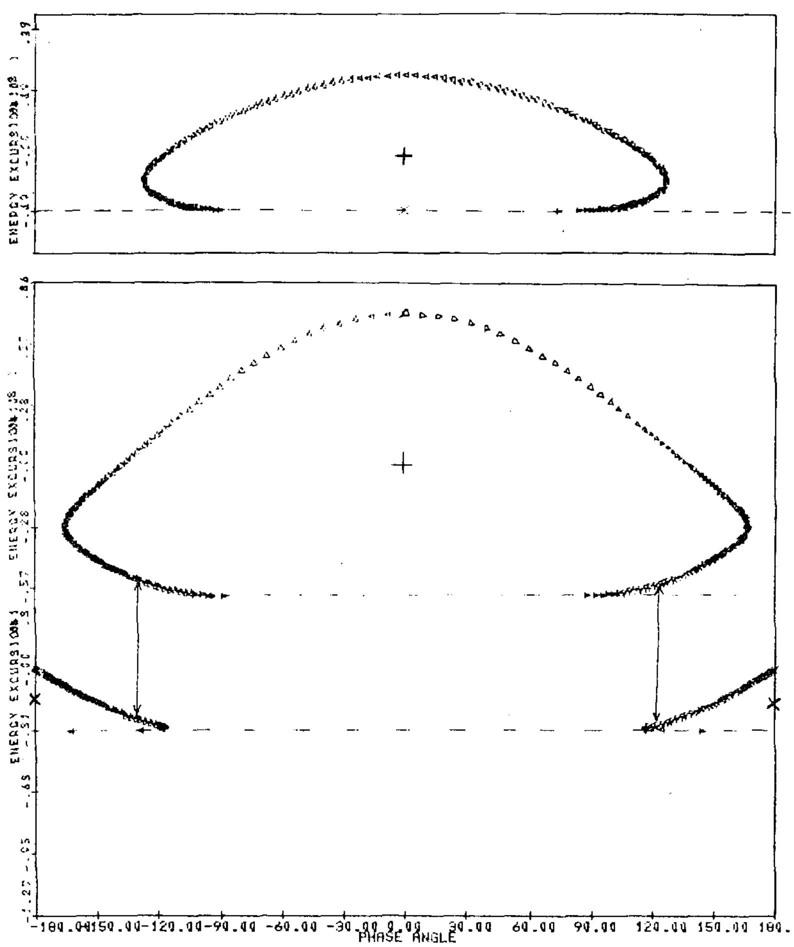

Figure 2. TOP curve: separatrix corresponding to in single valued region of Fig. 1 ("A"). BOTTOM curve: separatrix for multivalued region "B" of Fig. 1. Arrows indicate common stable region boundaries which are separated here for clarity. "X"'s mark the stable regions (bucket centers).

$$\ddot{x} = -\Omega^2/\gamma^2 \ x - \gamma^{-1} f + x^2[f + (\gamma^{-1} - 2)(P_y - \Omega x)] + \gamma^{-1} P_y \qquad (3)$$

where P_y = const. is the second dimension conomical momentum, and $\gamma = \gamma c(x, \dot{x}, P_y)$. A typical pattern of bifunctions leading to chaos about a strange attractor (characteristic of the drive-damping balance) results from these systems.[8] The extreme low noise environment necessary for the Penning experiments is easily ammenable to detection of the predicted bifurcation power system.

The quantum dynamics, that is the real physics of the trapped electron, may be examined via a Hamiltonian equivalent to (1), (3):

$$H_e = [m^2 + P_x^2 + (Py - \Omega x)^2]^{1/2} + e\,\phi + H_I \qquad (4)$$

where ϕ is a static potential generating $E_D \hat{e}_x$ in the rotating frame. H_I represents the electron interaction with the cavity it is in, which will account for the damping. A system essentially equivalent to (4) has been studied numerically, showing a remarkable connection to the classical behavior in the presence of H_I.[6,8] Those authors consider:

$$H = H_o + \alpha\, H_o^2 + H_I \qquad (5)$$

If we view H_e "biased" (renormalized) about some stable fixed point then (5) closely approximates (H_e - $e\phi_{bias}$), by expanding the relativistic form (4) to second order (an excellent approximation for the millier energy trapped e-!).

It has been proposed that (5) also models birefrianent light propagation and that actual experiments on, for example optical fibers could demonstrate various predicted effects.[6,14] Following quantum evolution via typical electron dynamics is potentially a much cleaner experimental test of such predictions. In any case it is a radically different test, reaching the few quanta limit of the theory (one e-; $\hbar\Omega > kT$). Since the crucial parameter $\Gamma/\Omega \propto$ B, the experiments can be tuned to study different regimes.

Noise

Already "noise", that is quantum noise, is included in both the actual electron system and simplicity in the quantized version of (5) referred to above. More significant to accelerator problems[9] and to actual Penning trap experiments[2] is the inevitable sideband noise which accompanies any applied rf note.

A series of experiments has dramatically demonstrated the sensitivity of Penning trapped electrons to random rf sideband noise.[2,15] Unlike in the case of existing accelerators, where various "dirt" effects may confuse studies of rf nose bucket diffusion, these Penning results can be unambiguously attributed to such noise. The absence of perturbations and the extreme high Q of the cyclotron motion ($\Omega/\Gamma >> 1$) allows capture of the electron by broadband noise far more probable than by a carrier unless the noise is exceptionally low. In practice it is extremly difficult to make pure enough rf sources to avoid excitation from being anything but characteristic of the sideband noise amplitude envelope. The immediate goal of such experiments is to resolve the quantum anharmonicity (unequal level spacings) of the relativistic Landau level spectrum starting with ground state excitation.[2,15] Detailed analysis of the noise influence[13] will be necessary since the present resolution is still sideband noise limited.

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