A SIDE-INJECTED-LASER PLASMA ACCELERATOR

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ABSTRACT:

A new method for driving relativistic plasma waves capable of ultra-high acceleration gradients (order 1 GeV/cm) is presented. By injecting a single laser frequency from the side, rather than collinearly with the accelerated particles, both pump depletion and particle dephasing may be avoided. The coupling of the side injected laser to the relativistic plasma wave via a pre-formed density ripple in the plasma is modelled analytically and with computer simulation.

INTRODUCTION:

Present laser schemes for realizing the ultra-high gradients possible in plasma space charge waves (order 1 GeV/cm) are generally collinear and suffer from the problem of pump depletion. That is, the laser continually feeds its energy to plasma waves which, although having high phase velocity, have low group velocity and hence leave their energy behind.

In this paper, we present a side-injected-laser scheme which enables the laser energy to be resupplied along the length of the accelerator. Only one laser frequency is needed for this scheme, and the phase velocity of the excited plasma waves can be controlled along the accelerator. Thus, both pump depletion and particle dephasing can be avoided.

The basic ideas are illustrated in Fig. 1. Laser radiation of frequency \( \omega_0 \) is incident approximately perpendicular to the axis of a long column of plasma. The plasma is of average density \( n_0 \), such that the plasma frequency \( \omega_p = \sqrt{4 \pi n_e e^2 / m} \) is slightly below \( \omega_0 \), and contains a neutral density ripple of wavelength \( \lambda_r \) along the plasma axis. Such a ripple might be produced by launching an ion acoustic wave in the plasma or by ionizing a grating to form the plasma. If the laser is polarized along the plasma axis, a quasiresonant coupling between the laser and the ripple drives a plasma space charge wave with phase velocity \( \omega_0 / k_p \) along the axis.

This scheme somewhat resembles the so-called near field laser acceleration schemes which use a grating to couple laser energy to accelerated particles. Here the role of the grating is played by the plasma density ripple (with the advantage that the plasma is already ionized and cannot be destroyed by the laser fields). The plasma dynamics are similar to the quasiresonant mode coupling between long wavelength plasma waves and ion acoustic waves, and also to parametric decay.

The coupling of laser energy to longitudinal wave energy via the ion acoustic or density ripple can be viewed as a three-wave process. The vector addition of the wave energy-momentum 4-vector of the laser (\( \omega_0, k_0 \)) plus that of the ripple (\( \omega_r, k_r \)) equals that of the plasma wave (\( \omega_p, k_p \)):

\[
\begin{align*}
\omega_0 + \omega_r &= \omega_p \\
k_0 + k_r &= k_p
\end{align*}
\]

Since \( \omega_0 \) must be greater than \( \omega_p \) in order to propagate in the plasma (\( k_p^2 = (\omega_0^2 - \omega_p^2) / c^2 \) is the dispersion relation for light in a plasma), these equations can only be satisfied approximately (hence the term quasiresonant coupling). If we take \( \omega_0 \) as close to \( \omega_p \) as possible while still allowing propagation, the dispersion relation for light waves indicates that \( k_0 \) will be small and \( k_p \) will lie approximately along the \( k_r \) axis. The gradient in plasma density which often occurs in the \( y \) direction will further facilitate propagating the laser into the ripple.

The plasma velocity of the resulting plasma wave is evidently from (1)

\[
V_{ph} = \frac{\omega_p}{k_p} = \frac{\omega_0 + \omega_r}{k_0 + k_r} = \frac{\omega_0}{k_r}
\]

where we have assumed \( \omega_0 \) near \( \omega_p \) so that \( k_0 = 0 \).

By slightly varying the ripple wavelength (\( \lambda_r \)) is such a way that \( \lambda_r(x) = 2\pi v(x)/\omega_0 \) where \( v(x) \) is the velocity of an accelerating particle (e.e), the phase velocity can be adjusted to match the accelerating particles. Alternatively, the particles might be surfed (phase locked by an imposed DC magnetic field).

The small amount of angle in \( k_r \) relative to the plasma axis can be compensated for by angling the wavefronts of the density ripple as shown in Fig. 2. Alternatively, one might inject lasers from both sides of the plasma and accelerate down the symmetry axis of the converging plasma waves that result.

Figure 1. A laser polarized along a preformed plasma density ripple wiggles electrons to produce a relativistic space charge wave.

Figure 2. Angled ripples enable \( k_p \) along axis.

THEORY:

We may describe the growth and saturation of the plasma wave by a simple cold fluid model. Consider a neutral, rippled plasma of density \( n(x) = n_0 + n_1(x,t) \) and velocity \( \mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1(x,t) \), where \( \mathbf{v}_0 = (-eE_0 / m)\mathbf{b} \cos \omega_0 t \). We obtain for the momentum and continuity equations:

\[
\frac{\partial \mathbf{v}_1}{\partial t} + \nabla \cdot (\mathbf{v}_0 \mathbf{v}_1) = -\frac{e}{m} E_1
\]
\[
\frac{3n_1}{\varepsilon} + x^2 \left( a_0 + \frac{\sin a_1 x + n_1}{\sin a_1 x - n_1} \right) (v_1 - v_2 a_0 \cos a_2 x) = 0
\]

\( x \)

Subtracting the spatial derivative of (2) from the time derivative of (3), we obtain a wave equation for the electron plasma wave density:

\[
\frac{\partial^2}{\partial t^2} \left( \frac{n_1}{\varepsilon} \right) + \omega_p^2 \left( \frac{n_1}{\varepsilon} \right) = 0
\]

(4)

\( \frac{1}{2} \left( v_2^2 \left( \frac{n_1}{\varepsilon} \right) (v_{th}/c) \right) \left[ \sin (v_2 c + \kappa x) \cos (v_2 c + \kappa x) \right]
\]

where we have kept only the lowest order terms (assuming \( \delta n/n \ll 1 \), \( v_\text{th}/c \ll 1 \)), and we have used Poisson's equation for \( \varepsilon \).

Each term on the right-hand side of (4) represents a driver for the plasma wave which is analogous to the pondermotive force term in the beat wave excitation scheme. In fact, equation (4) should be just like that of the laser beat excited plasma waves with the replacement of \( \delta n/n \) with \( v_\text{th}/c \) of the second laser and the inclusion of the relativistic correction to \( \omega_p^2 [v_\text{th}^2 + (1-3/8)(\delta n/n)^2] \). The present case differs in that here both left and right going plasma waves are excited.

Initially, since \( a_0 = v_0 \), each wave exhibits the secular growth of a resonantly driven harmonic oscillator:

\[
e = \left( \frac{v_{th}/c}{\sqrt{2}} \right) \left( \frac{\delta n}{n_0} \right) \omega_p t \]

(5)

where \( e \) is the amplitude of \( \delta n/n_0 \). The corresponding electric field amplitude associated with the plasma space charge waves is from Poisson's equation:

\[
E = 4\pi n_0 / k_x = \varepsilon \sqrt{\varepsilon_0} \ v/c
\]

where \( n_0 \) is in units of \( \text{cm}^{-3} \).

The growth of the plasma wave will be limited by the onset of any of the following three mechanisms.

First, the plasma wave cannot grow beyond the amplitude corresponding to wavebreaking or trapping of the background plasma, namely:

\[
e \leq 1 - \sqrt{3} v_\text{th}/c \leq 1/\gamma_\text{ph}
\]

(5a)

where \( v_\text{th} \) is the plasma thermal velocity and \( \gamma_\text{ph} = \sqrt{\gamma_\text{ph}^2 + (v_\text{th}/c)^2} > 1 \).

Second, the detuning between the laser at \( v_0 \) and the plasma at \( v_0 \) will stop the wave growth after a time \( (v_0 - v_p)t = \pi \) at which time

\[
e \leq \left( \frac{v_{th}/c}{\sqrt{2}} \right) \left( \frac{\delta n}{n_0} \right) \omega_p \frac{1}{2}(v_0 - v_p)\pi \]

(6b)

Finally, if \( a_0 \) is very close to \( v_0 \), then the relativistic frequency shift of the plasma wave may dominate the detuning. In analogy with the beat wave, the relativistic detuning will cause saturation at

\[
e \leq \left( \frac{16 \ v_{th}/c}{\sqrt{2}} \right) \left( \frac{\delta n}{n_0} \right) \omega_p\]

(5c)

The actual saturation amplitude will then be governed by the smaller of Eqs. (5a-c).

**SIMULATION:**

The mechanisms described in the previous sections have been studied with one-and-three-halves-dimensional (i.e., \( x, v_y, v_x, v_z \)) particle-in-cell simulation codes. The simulations illustrate the growth and saturation of the plasma waves and demonstrate subsequent acceleration of particles injected along the ripple direction. In all simulations, the density ripple is initially sinusoidal, ions are injected, the imposed electric field is of the form \( E = E_{\text{peak}} \sin (k_y x) \) and periodic boundary conditions are used.

In Fig. 3, we show the growth of the plasma wave \( e \) at time at a fixed \( x \) position for a simulation with \( \delta n/n = 0.2, v_{th}/c = 0.1, v_0 = 1.01 \omega_p \). The theoretical growth rate from Eq. (5) is shown for comparison (actually the growth rate plotted is twice Eq. (5) since \( e \) represents the sum of both left and right going waves). The wave detuning and subsequent decrease in amplitude after time \( 5.1 \) is visible in the figure. The saturation amplitude is about 50% smaller than predicted by Eq. (5c).

**Figure 3.** Plasma wave growth, \( e \) vs. \( t \) at fixed \( x \) in 1-D simulation. Straight lines are linear theory.

In Fig. 4, we show the energy gained by injected electrons in another simulation (\( v_0 = 1.1 \omega_p, v_{th}/c = 0.37, \delta n/n = 0.2, v_0/c = 0.99 \omega_p \)). The electrons have gained energy from \( y = 1.3 \) to \( y = 100 \) in a distance \( 30 \ v/c \omega_p \) (approximately 0.8 cm for CO2 parameters). The particles in Fig. 4 have reached their maximum energy as determined by the length over which they dephase (outrun) from the wave (the RWA limit): \( \Delta y = 2v_\text{th}/c \). The maximum energy gain for such a case without phase locking \( (B_y = 0) \) would be from above: \( \Delta y = 10 \). In Fig. 5, the particles are phase locked and have already gained three times this dephasing limit.

In principal, higher energies can be reached by increasing the ripple spacing to speed up the wave or by rotating the particles across the wavefronts. In Fig. 5, we have applied a DC magnetic field \( B_y \) in the \( z \) direction in order to test the surfatron phase locking mechanism. The ripple and laser parameters were chosen such that \( y_\text{ph} = 1.2, v_\text{th} = 0.12 \), \( \delta n/n = 1.0 \). The wave field amplitude is roughly
\( \varepsilon = 2.1 \) at this time and peaked at \( \varepsilon = 3.8 \); the theoretical values from Eqs. (5) and (6) are \( \varepsilon = 2.9 \) and \( \varepsilon = 4.4 \), respectively.

![Figure 5. \( \gamma \) vs. \( x \) in a surfatron run (\( \gamma_{ph} = 3.2 \)) and energy limit without phase locking (dashed).](image)

Secondary peaks at the trough of each wave are distinctly visible in the figure. These are due to the space charge fields of trapped particles. Although injected uniformly with only \( \pm \) of the background plasma density, it is clear from the phase space figures (5a & 6) that they have become bunched in each of the plasma waves. Note also that the ratio of momenta \( p_y/p_x \) is near the asymptotic surfatron value of \( 1/\gamma_{ph} \) (from \( p_y/p_x = \gamma_y/\gamma_x = (c^2-v_y^2)^{1/2}/(c^2-v_x^2)^{1/2} \)).

![Figure 6. a) \( \gamma \) vs. \( x \) at \( \omega_t = 48 \), b) \( p_x \) vs. \( x \) and c) \( p_y \) vs. \( x \) at \( \omega_t = 120 \) in a surfatron run.](image)

**DISCUSSION**

One readily conceivable way to experimentally verify the wave excitation and acceleration mechanism would be to use a CO2 laser prepulse to ionize a solid target upon which was etched a grating of ten micron periodicity. This would produce a rippled plasma. A second pulse which contained a few millijoules in the first couple of picoseconds of its rise could then accelerate electrons from the background plasma to up to 10 MeV in .5 mm.

The realization of the present scheme as a full-scale accelerator faces several technological hurdles. Requirements on the accuracy of the density ripple spacing are severe. A practical way of sweeping the laser energy along the system must be developed (such as that depicted in Fig. 7). Without sweeping the laser energy, the length of each stage would be limited by the onset of ion motion to be about \( (\lambda_t/m) \) plasma wavelengths (where \( \lambda_t \) is the ion mass).

![Figure 7. 2 possible means of sweeping laser energy.](image)

A number of physics issues remain to be studied. For example, the effect of the pondermotive force caused by the finite width of the laser is of interest, as are ripple decay times and finite \( k_0 \) effects. The potential advantages of this scheme--ultra-high gradients, avoidance of pump depletion and particle dephasing, and the use of a single frequency laser--make these issues worth pursuing.

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