Summary

In conventional theory, steering or bending an ion beam of high energy and high current requires very intense magnetic fields, which are both uneconomical and bulky. This problem is even more severe for singly charged ion beam with very high atomic number, which require large magnetic field energy both to bend and also to focus the beam against its self electric field. In this paper we present a new and simple technique, which will substantially alleviate these problems.

Introduction

We consider here the possibility of mixing the ion beam with an electron beam moving at the same speed, but with a much higher number density. We require

\[ \frac{m_i}{m_e} \gg \frac{N_i}{N_e} \gg 1, \]  

(1)

where \( N_i \) and \( N_e \) are line number densities of electrons and ions, and \( m_i \) and \( m_e \) are electron and ion rest mass, respectively. The transverse applied magnetic field in this case is required only to bend the electron beam, which in turn traps the ion beam by its own attractive electrostatic potential. Therefore the center of mass radius of curvature and applied magnetic field energy can be reduced substantially.

Although these two beam axes are slightly separated due to polarizing effect of the magnetic field on their opposite charge, under certain conditions they will however remain together as a stable configuration. The required magnetic field strength and field energy for bending the comoving beams are reduced by the ratios \( N_i/N_e \) and \( (N_i/N_e)^2 \), respectively, from their values for bending the ion beam alone. This substantial savings in bending field energy coupled with similar reductions in the field energy for beam focussing (see below) make this technique especially attractive.

It is well known that a non-neutral particle beam will rapidly expand and be lost due to its self electrostatic field, if external focussing force is not being applied. Several techniques have been developed in order to accomplish this objective. These techniques include an externally applied magnetic field along the beam direction of propagation; discrete magnetic quadrupole lenses; and continuous magnetic quadrupole field generated by four helically wound conducting wires of alternatively opposing current, superimposed on an axial magnetic field. In principle, all of these techniques can be used to confine an ion beam, but in general they are more efficient in confining electrons due to the light electron mass. In fact, for comoving beams where relation (1) holds, the magnetic field energy needed for confinement is reduced by the ratio \( m_e N_i/m_i N_e \) from its value in the absence of electrons.

Thus in this paper we will explore the possibility of bending the ion beam by mixing it with a comoving electron beam.

Analysis

To investigate the properties of comoving electron and ion beams in the presence of an external transverse magnetic field, \( B_z \), perpendicular to the propagation direction of the beams, we treat the electron and ion beams as rigid body, i.e. the beam maintains fixed density and current profiles with respect to its axes. This is a reasonable assumption, since the self fields do not directly affect the lateral displacement of the beams axes. Therefore, the equations of motions for lateral displacements of the ion and electron beam axes \( (x_i, x_e) \), respectively relative to their original propagation direction can be expressed as:

\[ \frac{d^2 x_i}{dt^2} = \frac{ze \beta}{y \mu_i} + F \left( x_e - x_i \right)/y \mu_i N_i \]  

(2)

\[ \frac{d^2 x_e}{dt^2} = -\frac{e \beta}{y \mu_e} + F \left( x_i - x_e \right)/y \mu_i N_e \]

where \( z \) is the charged state of ions, \( \gamma \) is the relativistic factor, \( \beta = \frac{v}{c} \), and \( F(\alpha) \) is the transverse (electric and magnetic) static force exerted on a column of species \( \alpha \) by a column of species \( \delta \).

This force can be expressed as:

\[ F_{\alpha \delta} = e_{\delta} \int n_{\delta} \left[ E^\alpha - (\rho_{\delta} B^\alpha) \right] dS_{\delta} \]

(3)

\[ = \frac{e_{\delta}}{y} \int n_{\delta} E^\alpha dS_{\delta}, \]

where \( n_{\delta} \) is the number density, \( E^\alpha \) and \( B^\alpha \) are self electric and magnetic field of the species \( \alpha \), the subscript \( \alpha \) denotes the direction of the force, and the integral is to be carried out over the cross section of column of species \( \delta \).

We assume both ion and electron beams possess Gaussian profile, i.e.

\[ n_{\alpha}(r) = \frac{N_{\alpha}}{2 \pi \alpha^2} \frac{1}{r^2} e^{-r^2/\alpha^2}, \]

(4)

where \( R \) is the beam radius. Then after some simple geometric calculations, we can write Equation (3) as
\[ F_{\alpha\beta}(x) = \frac{2e_{0}e_{0}N_{e}}{R_{0}^{2}} \int_{0}^{x^{2}/R_{0}^{2}} \frac{e^{2} - e^{-2}}{1 - e^{2}/R_{0}^{2}} \, dr \]

\[ = \frac{Z_{e}e_{0}e_{0}N_{e}}{R_{0}^{2}} \left[ e^{2}I_{1/2}(z) - \frac{1}{2}e_{0}N_{e} - 2z I_{1/2}(z) \right] \]

where \( z = x^{2}/2m_{e} \), \( \eta = \frac{R_{0}^{2} + R_{0}^{2}}{m_{e}R_{0}^{2}} \), and \( I_{1/2} \) is modified Bessel function of order one half. By using the identity

\[ I_{1/2}(z) = \left( e^{z} - e^{-z} \right) R_{0}^{2}, \]

we can simplify \( F_{\alpha\beta}(x) \) to:

\[ F_{\alpha\beta}(x) = \frac{2e_{0}e_{0}N_{e}}{R_{0}^{2}} \frac{1 - e^{-2}}{2z} \cdot \]

The equation of motion of the relative separation between the two beam axes \( (r_{1} - r_{2}) \) can be obtained from Equation (2). Using the force form of Equation (7), we can write this equation of motion as

\[ \frac{d^{2}x}{dt^{2}} = A - D \left[ 1 - e^{-2} \right], \]

where we now define \( x = |x_{1} - x_{2}|/(R_{1}^{2} + R_{2}^{2})^{1/2} \), and the constant \( A \) and \( D \) are given by

\[ A = \frac{eB_{z}B_{z}}{\gamma m_{e}R_{1}^{2} + R_{2}^{2}}, \]

\[ D = \frac{2N_{e}N_{e}^{2}}{\gamma m_{e}R_{1}^{2} + R_{2}^{2}}. \]

In obtaining Equation (8), the constraints in Equation (1) have been applied.

Integrating Equation (8) over \( x \) with the initial condition \( \frac{dx}{dt} = 0 \) at \( x = 0 \), we obtain

\[ \frac{dx}{dt}^{2} = x \left[ A - D \frac{G(x)}{2} \right], \]

where

\[ G(x) = \frac{1}{x} \left[ \ln(x^{2})^{2} + E_{1}(x^{2}) + 0.57721 \right]. \]

And \( E_{1} \) denotes the exponential integral. Equation (10) denotes the kinetic energy of electron beam in the frame of reference moving with the ion beam.

We are mainly interested in the maximum separation of the two beams, \( x_{m} \), which occurs when \( dx/dt = 0 \), or

\[ G(x_{m}) = \frac{2A}{D} = \frac{\gamma B_{z}B_{z}^{2} + \gamma^{2} e^{2} e^{2}}{2eN_{e}}. \]

\[ G(x) = \begin{cases} \frac{1}{x} \left[ \ln(x^{2}) + 0.5777 \right], & x \ll 1 \\ \frac{1}{x} \left[ \ln(x^{2}) + 0.5777 \right], & x \gg 1. \end{cases} \]

Therefore a pair of solutions to Eq. (12) occur for \( 2A/D < 1.0 \), one with \( x < 2.1 \) and one with \( x > 2.1 \). Only the solution with \( x < 2.1 \) is physically obtainable and stable. Assuming small relative displacement, \( x_{m} \ll 1 \), Eq. (12) simplifies to

\[ N_{i} = \frac{\gamma B_{z}B_{z}^{2} + \gamma^{2} e^{2} e^{2}}{2eN_{e}}. \]

Equation (14) therefore uniquely determines the lower limit on the ion line density, if we do not want the relative separation of the two beam axes to exceed \( x_{m} \). Note that Eq. (14) applies only when \( N_{i}m_{i} >> N_{e}m_{e} \) (see Eq. (1)). Otherwise, \( N_{i} \) should be replaced by \( N_{i} + m_{e}N_{e}/m_{i} \).

In order to fully characterize the behavior of the beams motion, we define the center of mass of the two beams as

\[ x_{c} = \frac{N_{i}m_{i}x_{1} + N_{e}m_{e}x}{N_{i}m_{i} + N_{e}m_{e}}. \]

This equation together with Equation (2) give us the equation of motion for the center of mass

\[ \frac{d^{2}x_{c}}{dt^{2}} = \frac{eB_{z}B_{z}}{\gamma(\alpha N_{i}m_{i} + N_{e}m_{e})} \left( 2N_{i} - N_{e} \right) \]

\[ = \frac{eB_{z}B_{z}}{\gamma N_{e}m_{i}}. \]

Where we have used the constraints in Eq. (1). Since \( \alpha \) and \( \gamma \) remain unchanged, Eq. (16) implies that the acceleration of the center of mass is constant and perpendicular to the direction of motion. Thus the center of mass motion bends with radius of curvature \( R_{c} \),

\[ R_{c} = \frac{\gamma m_{e}c^{2} N_{e}}{eB_{z}^{2} N_{e}}. \]

Equations (14) and (17) constitute the main results of this paper, and can be used together with the constraints in Eq. (1) to calculate the design parameters for various applications, when steering or bending a high energy, high current ion beam is necessary. However, in arriving at these results, we have assumed the beams to be rigid, i.e. neglecting the effects of self fields. While the ion beam is trapped and confined by the electron beam space charge potential \( (N_{i}/N_{e} >> 1) \), external field is necessary to confine the electron beam minor cross section from self expansion. Which is especially detrimental for applications, where it is necessary to have the two
beams remain together as long as possible. Part of the reason is obvious from Eq. (14), in which the critical current scale as $R^2 + R_e^2$. To confine the electron beam minor cross section from self expansion, we can adopt the techniques used in various cyclic accelerators, i.e., magnetic quadrupole field focussing as in the quadrupole accelerator or in the stellatron, or an externally applied magnetic field along the direction of propagation as in the modified betatron.

In this paper, we consider the use of an applied magnetic field along the trajectory of the center of mass, which can be readily calculated from Eq. (16), in order to focus the electron beam. The effect of this axial magnetic field on the center of mass trajectory is negligible, provided the condition $x_m < 1$ is satisfied. At any rate, any possible electron drifts due to the bending magnetic field error or the presence of the axial magnetic field can be effectively prevented by introducing a strong focusing field, such as the continuous quadrupole field in the Stellatron. A charged particle beam can be transported along an axial magnetic field, if the following condition is satisfied:

$$n < \frac{\gamma B_0^2}{8\mu_0 c^2},$$  
where $n$ is the beam density and $B_0$ is the axial magnetic field strength. It is obvious from the above equation that substantial solenoidal magnetic field energy can be saved by employing the comoving electron beam, due to the fact that $N_m \gg N_e$. The axial magnetic field $B_0$ can be obtained by solenoidal winding around the center of mass trajectory.

In order to bend the beam, a uniform bending magnetic field must be applied transversely to the plane of the beam propagation; this can be achieved either by conventional permanent dipole magnets or by a new winding technique we recently developed. In this technique, parallel conducting wires are formed into a tube (the tube does not have to be straight), so that the cross section of this tube is as shown in Figure 1. In Cartesian coordinates, when the wires are packed close to one another and the current in each wire are given by:

$$I(z,x) = \frac{I_{\text{max}}}{a} \frac{x}{a},$$  
where $a$ is the radius of the cross section of the tube, we find that the magnetic field inside the tube can be expressed as

$$B(z,x) = \frac{\mu_0 I_{\text{max}}}{2a} \frac{z}{2a}.$$

Since this field is uniform, it can effectively be used as a bending field for charged particle beams.

In order to illustrate some of the points discussed here, we consider the bending of a proton beam with a current of 200 Ampere, energy of 20 MeV and $R_p = 1$ cm, by employing a comoving electron with a current of 5kA and $R_e = 5$ cm. If we demand $x_m = 0.1$, then from Eq. (14) the bending magnetic field must be $B_2 \approx 16.2$ Gauss. Thus the radius of curvature is $r_c = 10.6$ meters from Eq. (17). The axial field strength required is $B_0 = 1.3$ kG from Eq. (18). For the same configuration without the presence of the electron beam, the magnetic field required for bending and focussing the ion beam would be 405 Gauss and 5.6 Tesla, respectively. Obviously, considerable magnetic energy must be expended to confine and to bend an ion beam without the presence of the electron beam.

**Conclusion**

We have presented here a new technique to bend and confine a high energy and high current ion beam. It should be noted, however, that the feasibility and usefulness of this technique depends on the objectives one wants to accomplish and the availability of technology. For certain applications, electron cooling of high current ion beam, for example, this technique is very advantageous in reducing the physical space required.

**References**

1. R. B. Miller, An Introduction to the Physics of Intense Charged Particle Beams, (Plenum Press, 1982).