INTENSE ION RING ACCELERATION IN A FLUX COMPRESSION LINER

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Summary

Circular accelerators based upon increasing magnetic flux comprise one of the oldest classes of machines, the Betatron being a prime example. We have found that intense, charged neutralized ion rings can be accelerated to interesting energies by imploding a conducting liner on the field/ring configuration. Because the ring radius is not fixed, we find that the energy scaling varies from linear in the field ($p_B^2 \propto B_z$) at low energies to square root scaling ($E \propto B_z^{1/2}$) at relativistic energies. Simple modeling has confirmed many of the characteristics we have seen in 2+D fully electromagnetic particle simulations.

Introduction

It has been suggested a number of times over the last decade that an intense ring of charged particle imbedded in a guide field can be efficiently accelerated by adiabatically compressing that flux. Such concepts are generically similar to the classic betatron, in that the acceleration gradient is created by monotonically varying a magnetic field in time. They differ from the betatron by permitting non-constant orbit radii in some cases and by employing a different set of focusing fields. In this paper we present analysis and simulations of an intense, charge neutralized ion ring in a flux compressing geometry.

The principle of generating inductive fields by increasing a magnetic field in time is the same as for a common transformer. Steinbeck first conceived of using this field to accelerate charged particles in 1936. By 1939 Kerst had succeeded in demonstrating successful electron acceleration in such a device, the betatron. He exploited this with ever more powerful betatrons until the 300-MeV betatron was constructed in 1950. At that point, limitations in magnetic field generation technology plus intrinsic radiation losses for small radius electron orbits indicated that future betatron development would not be profitable.

From a first principles analysis, the radiation loss problem is still a major obstacle for electron devices, although it can be ameliorated with larger radius machines. Ions, however, are far from synchrotron loss limited at comparable energies. Magnetic flux compression technology, moreover, can routinely produce pulsed fields on the order of mega-gauss. Since the final particle energy depends on the ratio of initial to final field strength, such compression offers great potential for this type of accelerator. On the other hand, space charge limitations are far more severe for ions. To quantify these questions, we next present a simple model analysis for the acceleration process.

Theory

A useful and tractable approach for following the collective behavior of intense beams is the cold fluid equations for a species of mass, m, and charge, q.

$$\frac{\partial n}{\partial t} = -\nabla \cdot n\mathbf{v} \quad (1a)$$

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{1}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (1b)$$

Equation (1b) resembles the single particle equation of motion, but $\mathbf{p} = m\mathbf{v}$, $[\gamma - (1 - \mathbf{v} \cdot \mathbf{c})^2]^{-1/2}$ is a fluid quantity and $\partial \mathbf{p}/\partial t \equiv \partial \mathbf{p}/\partial t + \mathbf{v} \cdot \nabla \mathbf{p}$. Also, $\mathbf{p} = \sqrt{\mathbf{v} / c}$ in (1b). In cylindrical coordinates, we can quite generally derive the condition for negligible radial motion, that is $\mathbf{v}_r = 0$. The solution for the rotational momentum $p_\theta$ in terms of external and self-fields is

$$p_{\theta} = \pm \frac{\Omega}{2} \left[ \sqrt{1 + \frac{4m^2 \gamma E_r (\mathbf{B}_r - \mathbf{B}_z \mathbf{B}_\theta)}{r q B_r^2}} \right] \quad (2a)$$

where $\Omega = q B/m c$, and if axial gradients and time variations are small,

$$E_r \equiv 4\pi e \int_0^r r' dr' (zn_i - n_e) \quad (2b)$$

$$B_\theta \equiv 4\pi e \int_0^r r' dr' (\mathbf{B}_r z_i - \mathbf{B}_z n_e) \quad (2c)$$

The two signs in Eq. (2a) correspond to slow and fast rotational modes. For an idealized ring, we neglect axial motion, so that $B_z = B_\theta = 0$.

It is obvious from Eq. (2a) that a necessary condition for a nonexpanding ring state is that the quantity under the square root be positive

$$\frac{4m^2 \gamma E_r}{q} < r B_r^2 \quad (3)$$

The presence of the mass in the numerator means that this is a much more stringent condition on ions than on electrons. For unneutralized ion rings, therefore, the maximum density at a given magnetic field
is smaller by roughly \( \frac{\gamma_i}{\gamma_e} (m_i/m_e) \) than for electrons of the same energy.

A "simple" way around the limitation of Eq. (3) is by reducing \( E_z \). This can be accomplished by introducing a neutralizing electron background, \( n_e \equiv n_i \). This was the motivation for the plasma betatron in the 1950's and early 1960's.\(^9,10\) While this achieved some acceleration, it was beset by instability problems. This method of overcoming space charge in intense ion beams is still sufficiently attractive, however, to warrant further study.

If we assume that problems with electron neutralization are not insoluble, then Eq. (3) will be trivially satisfied. In fact as long as \( E_z \geq 0 \), Eq. (2a) for a rotating ion ring is just the minus root

\[
P_{\Theta i} = -r_0^2 E_z/B_z\]

Equation (4) is, therefore, a fixed relationship between rotational momentum, \( P_{\Theta} \), radius and magnetic field. As long as the compression is adiabatic, we expect \( v_r \) to be small compared to \( v_0 \) and that Eq. (4) will remain approximately valid.

The electrons must also be nonexpanding if they are to fulfill the role of neutralization. They, therefore, satisfy Eq. (2a). Here we must examine both roots of Eq. (2a). If the electrons are in the fast rotational ring mode, we find

\[
P_{\Theta e} = +r_0^2 E_z/B_z\]

Since the electrons are at the same radius as the ions and in the same field, they must have a much greater momentum, \( P_{\Theta e} \equiv (m_i/m_e) P_{\Theta i} \). The rotation is, furthermore, in the opposite sense as the ions. This large velocity differential is probably unstable with respect to the rotating ion-electron two stream instability. Instability plus the very high injection energy needed to establish such an electron ring makes the fast mode undesirable for electrons.

The slow rotational mode for the electrons is approximately

\[
P_{\Theta e} = -\frac{cE_r}{B_z} \leq 0\]

In other words, the electrons are nearly stationary. From an experimental standpoint, such a configuration is very attractive and could probably be created by any of a number of methods. A small velocity difference between ions and electrons still exists, but a small amount of temperature would be strongly stabilizing in this case.

Finally, it is simple to calculate the ion orbits when a highly conducting liner is compressing the flux. Flux is conserved within such a conductor to a high degree. If we concentrate on ring currents that are far from field reversing, diamagnetic effects can be neglected, and the flux enclosed within an individual ion orbit will be nearly conserved,

\[
r^2(t_2)B_z(t_2) = r^2(t_1)B_z(t_1), \]

where \( t_1 \) and \( t_2 \) are different times. As a function of field strength, we find

\[
r = r_0 \sqrt{B_z(t)/B_z(t_0)}(6a)\]

and

\[
P_{\Theta i} = \left( \frac{m_i c}{m_e} \right) \sqrt{\frac{B_0 B_z(t)}{B_z(t_0)}}(6b)\]

Equations (6) and (7) define the acceleration of the ion ring within this simple model. Because of the many approximations, we checked this model with fully electromagnetic, 2Ax-D particle-in-cell (PIC) simulations.

Simulations

The methodology in the simulations was to initialize a finite, annular ring of ions, usually protons, exactly charge neutralized with electrons. One or more conducting liners with finite radial and axial extent was initialized at radius larger than that of the ring. These liners could be initialized either at the asymptotic implosion velocity or accelerated smoothly from rest. For most conductivities and implosion times employed, field diffusion through the liners was negligible. The liner/ring system then evolved self-consistently through the induced currents and their associated fields.

An important constraint on the model is that the compression should occur adiabatically. What we mean here is that \( v_r \) is small, or intuitively that the change in the magnetic field is small over an ion orbit period,

\[
B_z \frac{\partial B_z}{\partial t} \gg \frac{2\pi}{\Omega_i}
\]

For realistic electron-to-ion mass ratios, this condition leads to unacceptable long simulation times. As a test, the calculation was conducted with \( m_i/m_e = 250 \). For a constant velocity liner, \( v_0 \), at an initial radius, \( r_0 \), the characteristic field variation time is \( t_B = r_0/2v_0 \).

A simulation was performed with \( v_0/c = -0.01 \), \( m_i/m_e = 250 \) and \( r_0 = 4.8 \). The equal electron and ion densities were \( 2.8 \times 10^{11} \text{ cm}^{-3} \), and the initial ion momentum was \( P_{\Theta i}/c = -0.10 \). The radial extent of the uniform ring was from 2.28 to 2.72 in units of \( c/\omega_p \). The axial extent was from 5.0 to 25.0; the ring ran from 2.0 to 28.0. The initial ion ring configuration and azimuthal phase space are shown in Fig. 1. For these conditions \( t_B = 250 \) and \( 2\pi/\Omega_i = 157 \), in units of \( \omega_p \). This does not strongly satisfy the requirement of adiabaticity. Figure 2, however, shows that
Fig. 1. Initial ion ring rotates with $P_0 = -0.1$.

Fig. 2. Adiabatic Compression of a long layer leads to uniform acceleration.

It is clear in the above simulation that electron cyclotron waves led to excessive nonadiabatic electron heating. In a mirror configuration, hot electrons would be better confined than cold ones. The simple liner compression configuration, however, intrinsically establishes deconfining magnetic fields. It is, therefore, important to determine the source of those large amplitude cyclotron waves.

Analysis of the cold fluid electron equations shows that cyclotron waves can be excited by time-dependent $E_z$ and $E_\theta$ fields, time varying space-charge fields and by rotation coupled into axial $B_z$ gradients. We conducted periodic simulations without axial gradients to isolate the factors. The parameters were the same as for the nonadiabatic calculation discussed above. Coupled $r$-$\theta$ oscillations were again observed in the electron phase space, but this did not excite finite $k$ waves. With $k = 0$, the phase velocity of any disturbance cannot interact directly with the particles. The conclusion is that axial gradients are critical for this mode of wave excitation.

Overall, we have shown that charge neutralized ion rings can be successfully accelerated to multi-MeV energies. It is important to compress adiabatically. Loss of adiabaticity can result in electron heating and consequent loss of neutralization. Only a small fraction of the original ion ring can then be confined.

Acknowledgments

Useful discussion with Jeffrey Golden and Thomas Hussey are gratefully acknowledged.

This work was performed under the auspices of the U.S. Department of Energy.

References