A NONLINEAR LENS SYSTEM TO SMOOTH THE INTENSITY DISTRIBUTION OF A GAUSSIAN BEAM

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The intensity distribution of a typical ion beam is quite nonuniform with a bright center and a diffuse edge; in many cases, the actual distribution is adequately approximated by a Gaussian distribution. However, many uses for such beams require uniform illumination from the center of the target to its edges. It is shown here that a system of controlled third-order aberrations produced by a pair of octupole magnets is able to substantially smooth the intensity distribution by spreading the center and folding in the edges of the beam such that the intensity is nearly uniform over the entire target.

Introduction

Many applications using particle beams require illumination of an extended target with beams of uniform brightness. For example, local bright spots on targets for high-power beams create difficult cooling problems. Targets in radiation-effect and radiation-damage studies are expected to properly represent the radiation fields they simulate. Here, achieving uniformity by scanning the beam across the target induces discrepancies due to relaxation times in the target material. Finally, the use of ion implantation on large wafers comprising many integrated circuits requires uniform illumination. Beams emanating from accelerators generally have intense centers and extended, diffuse tails. Such beams are well characterized by a Gaussian distribution which clearly makes them unsuitable for the application.

An innovative solution to this problem for low energy beams was reported by Cleland H. Johnson of Oak Ridge National Laboratory. His lens does not affect the portion of the beam near the axis but rather folds in the tails of the beam. This is accomplished by using a transparent ground electrode.

We here show how octupole lenses may be used to even the distribution of a high-power beam.

Thin Lens Description

We assume that the target is large compared to the emittance of the beam so it is necessary to direct a strongly diverging beam to the target in order to cover it. This beam can be regarded as coming from a virtual source which is formed by other elements in the transport system. For the purpose of this description we shall assume the beam is rotationally symmetric. With this assumption, the displacement of a portion of the beam at the target is highly correlated with the divergence of that portion. Thus we can write

\[ x_t = D x_0' \]

where \( D \) is the distance from the virtual source to the target. The intensity distribution is given by

\[ I(x_0') = I_0 \exp \left(-x_0'^2/(2s^2)\right) \]

The distribution at the target is thus also a Gaussian.

Nonlinear Transformation

We now add a nonlinear lens between the virtual source and the target, at a distance \( T \) from the virtual source. This lens is assumed to deflect the beam in proportion to the cube of the trajectory's displacement at the lens:

\[ x_t' = k x_0'^3 = k T^3 x_0'^3 \]

The displacement at the target is then given by:

\[ x_t = D x_0' - (D-T)^3 x_0'^3 \]

Next we invert this expression to give \( x_0' \) entering the lens in terms of \( x_t \) at the target. In doing this we assume that the effect of the nonlinear lens is a perturbation. As we will see later, this is not true, but it does allow us to express results in simple terms and adequately show the effects. We have

\[ x_0' = \frac{x_t}{D} \left[ 1 + (D-T) k (\frac{T}{D})^3 x_t^2 \right] \]

This arrangement is shown in the following sketch.

Intensity Distribution With Cubic Term

We now have

\[ I(x) = I_0 \exp \left[-x^2 \left(1-x^2/x_1^2\right)^2\right] \]

where \( x = x_0/\sqrt{2}D \) and \( x_1 = \left[(D-T)2kT^3s^2/D\right]^{-1} \).

This distribution has two maxima: \( x=0 \) and \( x=x_1 \). There is a relative minimum at \( x = x_1/\sqrt{3} \). Further out this same minimum value is reached at \( x = 2x_1/\sqrt{3} \), and beyond this, the intensity falls rapidly. In the figure at the top of the next page, we show intensity curves for two different values of \( x_1 \) along with the original Gaussian curve.

Optimizing the Distribution

We can choose \( x_1 \) to optimize the distribution. We ignore the portion of the beam lying beyond \( x = 2x_1/\sqrt{3} \), assuming it is scraped off.
We will adjust the linear parameters of the system so this value lies at the edge of the target. Then as we increase $x_1$, we increase the fraction of the beam retained, but we also increase the nonuniformity of the beam. We show in the next figure how the half width of the resulting distribution depends upon the fraction retained (solid curve) and the value of $x_1$ used to obtain that half width (dotted curve). We note that if one is willing to discard (or ignore) 20% of the beam, this nonlinear lens will produce a distribution that is uniform within ±2%. If 90% of the beam is to be retained, then the nonuniformity will increase to ±9%. Of course, we can keep all of the beam if we are willing to accept a variation of ±50%.

The phase space distributions are sketched in the next two figures: that at the lens (top) and that at the target (bottom). The dotted curves show the distribution in the absence of the nonlinearity whereas the solid curve shows the effect of the nonlinear lens. We can now imagine using the direction normal to the plane of the paper to represent the intensity axis. It is then easy to see how the twisting of the phase space distribution by the nonlinear lens results in a nearly uniform distribution in real space.

Real Systems

We now replace the idealized thin nonlinear lens with a system of octupoles. However, we retain for now the assumption for the remainder of the transport system: that it produces a rotationally symmetric virtual source upstream from the target.
Aberration Coefficients

The lowest order effect of a properly constructed and aligned octupole magnet is third order in the displacements and slopes of the beam (two planes). We expand the displacements in the two planes at the target in a Taylor series in terms of the displacements and slopes at some reference point, which we will take to be at the virtual source. The derivation of the coefficients is contained in the author's thesis. The same notation is used here; it differs slightly from that used in TRANSPORT. We have

\[ x_t = D x_0' + C_{333} x_0 y_0 + C_{444} x_0 y_0^2 + \ldots \]

\[ y_t = D y_0' + C_{444} x_0 y_0 + C_{444} y_0 x_0 + \ldots \]

We have retained only the terms in generalized spherical aberration because of our assumption of the nature of the virtual source. The other four possible spherical aberration terms are precluded by symmetry arguments. It is shown in the thesis that the two mixed coefficients are not independent. Having assumed a beam with rotation symmetry at the virtual source, we are assured that the behavior in the two transverse planes will be identical because octupoles are symmetric with respect to reflection through the bisecting planes. Thus \( C_{333} = C_{444} \), and \( C_{443} = C_{433} \). There would thus appear to be two independent coefficients. In order to have a beam that is rotationally symmetric to the target, we must have all four of these coefficients the same: equal to \( k \) in the above thin lens expression. One would expect that this can be achieved with two octupole magnets, one of which is converging in the principal transverse planes and is diverging in the bisecting planes whereas the other is diverging in the principal planes and converging in the bisecting planes. This is similar to the well-known focusing principal in linear optics—that a system comprising a diverging lens and a converging lens is generally converging overall.

It is here that I had planned to conclude this paper; that this was a straightforward problem and could be easily solved for any particular system. However, I decided to add a derivation for these coefficients, and in the process proved that it is impossible to have these coefficients equal and nonzero in a rotationally symmetric system no matter how many octupoles were used. C'est la vie! The proof follows.

An Impossible Condition. The expressions for the two aberration coefficients are

\[ C_{333} = \frac{1}{3} x_e \int \psi x_0^2 \, dz - \frac{1}{3} x_0 \int \psi y_0^2 \, dz \]

\[ C_{443} = -x_e \int \psi x_0 y_0 \, dz + x_0 \int \psi y_0 x_0 \, dz \]

where \( \psi(z) \) is proportional to the strength of the octupole field component at \( z \) and \( x_0 , y_0 \), \( x_e \) and \( y_e \) are linearly independent solutions of the linear equations with \( x_0 \) and \( y_0 \) having zero displacement and unit slope at the reference point whereas \( x_e \) and \( y_e \) have unit displacement and zero slope at that point.

Now if the \( x \) and \( y \) solutions within the octupoles are everywhere identical, we see that

\[ 3 C_{333} + C_{443} = 0 \]

regardless of the number or disposition of the octupoles. Thus we cannot have \( C_{333} = C_{444} \) and nonzero, and therefore a correcting system with octupoles cannot have a "round" beam in the octupoles. There will thus be three independent coefficients to set.

Generalizations. It is not necessary to have restrictions on the linear part of the system. The beam need not be symmetrical with respect to exchange of the two planes. Moreover, it is possible to place the octupoles elsewhere in the beam system provided subsequent lenses are sized to accept the much larger effective emittance. If the source cannot be characterized as being dominated by the divergences, then more aberration coefficients must be considered. The correction will then involve optimizing linear elements and may require additional octupole magnets. All of this is best left to a computer. The author's machine-language code, 4P, written as part of the thesis project calculates and optimizes all of these coefficients. However, the code requires an IBM 7094 computer, and the author is not aware of the existence of a current operating model of that machine.

Tolerances

This system obviously requires very strong octupole magnets. The tolerances in building and aligning such magnets will be severe. Failures in meeting the tolerances will induce sextupole and skew sextupole fields as well as quadrupole and dipole fields.

References

2) P. F. Meads, Jr., Lawrence Radiation Laboratory Report UCRL-10807 (1963)