BUNCH LENGTHENING IN SOR

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Summary

Bunch lengthening in SOR, a 408 MeV electron storage ring dedicated for synchrotron radiation experiments, was found to be induced by a longitudinal coupled bunch instability. The bunch length which increases as a 1/5.8 power of the beam current, can be explained by the balance between the growth of the instability and Landau damping due to the spread of synchrotron frequency, and by introducing a weak frequency dependence of coupling impedance. In connection with this phenomenon, slow fluctuation of synchrotron radiation was found to come from a detailed unbalance of the equilibrium electron distribution on the longitudinal phase space. Touschek life time of the expanded beam and Landau cavity to suppress the instability are described.

Introduction

Bunch lengthening of stored beam current has been observed in many electron storage rings. In most cases the bunch length increases as one third power of the beam current and accompanies coherent synchrotron oscillations as well as bunch widening. This phenomenon has been believed to be induced by a single bunch effect and explained with longitudinal instability of bunched beam or with stability criterion of coasting beam. Bunch lengthening observed in SOR shows several characteristic features similar to or different from those in other rings. In this paper we give the experimental results of the lengthening and related phenomena as well as the analyses which give satisfactory explanation of the results.

Bunch lengthening

As shown in Fig.1 the bunch length in SOR increases as a 1/5.8 power of the beam current above a threshold current. The lengthening, observed in multi bunch operation but not in single bunch operation, is thought to be induced by inter-bunch interaction. Several side bands of synchrotron oscillation were observed around every harmonics of revolution frequency, which indicate the presence of coherent modes of synchrotron oscillation. Figure 2 compares the observed side bands with those of bunch shape mode around coupled modes expected from the theory of longitudinal coupled bunch instability.

It is well known that bunch lengthening due to longitudinal instability obeys scaling law in regard to the parameter \( \frac{I}{E \delta E} \), where \( I \) is the beam current, \( E \) is beam energy and \( \delta E \) is synchrotron oscillation frequency divided by revolution frequency. Experimental data of the bunch length in SOR follow the scaling law as shown in Fig.3.

Together with the observations of slow fluctuation of the coherent modes described later, we imagine that a longitudinal coupled bunch instability expands the electron distribution on the longitudinal phase space and a damping mechanism (Landau damping) suppresses this expansion to a balanced state resulting in the increase of the electron distribution on the phase space. Usually in the longitudinal instability theories the mode \( m = 0 \) is thought to be stationary. But we imagine that the increase of coherent modes \( m \neq 0 \) and continuous smearing of these modes by Landau damping bring about the expansion of zero mode distribution.

With the use of the growth rate of the coupled bunch instability and stability criterion for bunched beam (derived from Landau damping), the bunch length is found to increase as one fifth power of the beam current. Assuming the coupling impedance \( \frac{Z}{p} = 40 \text{ } \Omega \) we can obtain a nice fitting to experimental data as indicated with solid lines in Fig.1. A better fitting is obtained by introducing the following weak frequency dependence of the impedance:

\[ Z = 3.4 \times 10^6 \frac{\text{freq.}}{\text{GHz}} \]

as shown with broken lines in the figure. Observed half bunch length is 0.4~1.5 nsec in time corresponding to the frequency range 0.27~1.1 GHz with respect to the impedance.

Bunch widening

Because of the expansion of the electron distribution on the longitudinal phase space, the energy spread of the beam is expected to increase above the threshold current as follows,

\[ \frac{\sigma_E}{E} = \left( \frac{\sigma_E}{E} \right)_0 \left( \frac{1}{I_{th}} \right)^{1/5} \]

where \( \left( \frac{\sigma_E}{E} \right)_0 \) is the energy spread below the threshold. Then the following relation is to be satisfied in the horizontal direction:

\[ F(\sigma_x) = \frac{\sigma_x^2}{\sigma_x} - \frac{\sigma_x^2}{\sigma_x} = \left( \frac{1}{I_{th}} \right)^{2/5} \]

where \( \sigma_x \) is the beam size, \( \sigma_x \) is that due to betatron oscillation and \( \sigma_x \) is dispersion of magnet. Experimental data fit very well to the above relation as shown in Fig.4.

Vertically we do not expect any increase of the beam size so long as the coupling coefficient of the betatron oscillations in both directions is independent of the oscillation amplitude. Experimentally the vertical size increases considerably above 40 mA (much above the threshold current).

Slow fluctuation

The intensity of the side bands or the coherent modes of synchrotron oscillation fluctuates slowly with the frequency 100

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depending on the beam current. Because of this fluctuation of the coherent mode, the instability has been probably called turbulent microwave instability. The peak intensity of the bunched beam fluctuates with the same frequency. It is observed that when the side band intensity increases, the peak intensity of the bunch decreases. We think that the fluctuation of the coherent mode is due to the detailed unbalance of the equilibrium electron distribution determined by the balance of the growth and damping of the instability. It is expected that the faster the growth rate, then the larger the fluctuation frequency. Figure 5 shows roughly the following relation between the observed fluctuation frequency \( f_F \) and the growth rate \( \gamma_g \):

\[
f_F \approx \frac{\gamma_g}{2\pi}
\]

**Touschek life time**

Although the instability reduces the brightness of synchrotron radiation because of the bunch widening, it gives us a merit of lengthening the bunch life time determined by the Touschek effect. According to this effect the bunch current decay as follows

\[
I(t) = I_0 \left( 1 + \frac{t}{\tau_T} \right)^{-1} \approx I_0 \exp\left( -\frac{t}{\tau_T} \right), \quad \text{for} \ t < \tau_T
\]

where \( \tau_T \) is half life time. Since the life time is inversely proportional to the electron density within the bunch, the lengthening and widening increase the life time. Figure 6 shows the observed life time (instantaneous exponential decay time) and the Touschek life times calculated with the beam size observed or expanded (solid line) or with the beam size expected without instability (broken line). We see in the figure that the Touschek life time is increased much by the instability and longer than the observed one by factor three.

**Landau cavity**

In spite of the merit of the instability that lengthens the life time of the beam, the widening reduces the brightness of the radiation, and the slow fluctuation of the electron distribution accompanies radiation fluctuation, which are unfavorable to radiation users for precise measurements. To overcome these defects, a Landau cavity was constructed to suppress the instability and is now in test operation. The cavity is excited with a second harmonics of the acceleration frequency. With the cooperation of two cavities, the bunch length and the frequency spread of synchrotron oscillation are to be increased while the beam width remains small. Figure 7 is the expected bunch length, frequency spread and threshold current for n-th harmonics.

**References**

6. S. Asacka et al., to be published in Nuclear Instr. and Methods.
10. H. Bruck, "Circular Particle Accelerators" (Institut National des Sciences et Technique 1966) Chap. XXX.

**Fig.1** Current dependence of bunch length \( A \) (FWHM) divided by that \( A_0 \) below the threshold current. Theoretical current dependence is shown with solid lines for the coupling impedance \( \left| 2 \mu_m / m_n \right| = 40 \Omega \), and with broken lines for the impedance given in the text.

(a) coupled mode

(b) bunch shape mode

(c) \( m = 2, 1, 1 \)

**Fig.2** Fourier spectrum of bunched beam current, (a): the spectrum in the frequency range \( 0 \sim 1.2 \text{ GHz} \), (b): The spectrum around revolution frequency, representing the side bands of synchrotron frequency 110 kHz, (c): the spectrum expected for longitudinal coupled bunch instability.
Fig. 3 Bunch length as a function of \(1/E\langle 2\rangle\) with \(\langle 2\rangle = 0.0075\) for \(E = 308\) MeV and with 0.0067 for 380 MeV.

Fig. 6 Current dependence of beam life time, solid circle: experimental value of instantaneous exponential decay, solid line: half life time determined by the Touschek effect which is calculated with the expanded beam size observed, broken line: Touschek half life time expected without the instability.

Fig. 4 Current dependence of bunch widening.
(a): horizontal widening function \(F(x) = \frac{1}{\langle 2\rangle} \cdot \frac{\langle 2\rangle}{1 + \langle 2\rangle \cdot \frac{1}{1 + \langle 2\rangle}}\) is expected to vary as \(F(x) = \left(\frac{1}{1 + \langle 2\rangle} \cdot \frac{1}{1 + \langle 2\rangle}\right)^{x}\) above the threshold current, which is shown with solid line, (b): vertical widening given by the square of FWHM and by coupling coefficient \(k_3\).

Fig. 5 Current dependence of slow fluctuation frequency \(f_0\) and the growth rate \(\alpha_f\) of the instability which depends on the beam current as \(\alpha_f \propto \left(\frac{1}{1 + I_{th}}\right)^{0.45}\).

Fig. 7 Expected frequency spread \(\Delta f_{s}/f_{s}\) of synchrotron oscillation frequency, bunch length \(A_{3H}\) (FWHM) and threshold current \(I_{th}\) as a function of harmonic number \(n\) of RF frequency with the condition that the main cavity voltage is \(n\) times higher than that of the Landau cavity.