PROPERTIES AND POSSIBLE USE OF BEAM-BEAM SYNCHROTRON RADIATION
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Summary

In large electron-positron colliding beam facilities the fields created by one beam in the crossing point are quite large and cause the particles in the other beam to radiate synchrotron radiation. The total power emitted in the form of this beam-beam synchrotron radiation is calculated for beams with a bi-Gaussian cross-section colliding head-on, and its dependence on beam separation is estimated. The radiation emitted in the forward direction is quite hard and has a complicated spectrum. However the radiation emitted at angles much larger than both 1/π and the deflecting angle, is softer and has the properties of "short magnet" radiation. Its spectrum observed at a fixed angle is directly given by the Fourier transform of the longitudinal dependence of the deflecting field, i.e. of the longitudinal distribution of the particles in the other beam. The polarization has a simple azimuthal dependence. This radiation can be used for beam diagnostics, i.e. centring the two beams suffering a natural separation, optimizing the luminosity, measuring the bunch length.

1. Energy Radiated by the Beam-beam Synchrotron Radiation

The two colliding bunches have the same particle distribution

\[ \phi(x,y,s,t) = \lambda(s,t)G(x,y) \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp \left( - \frac{x^2}{2 \sigma_x^2} - \frac{y^2}{2 \sigma_y^2} - \frac{z^2}{2 \sigma_z^2} - \frac{(st)^2}{2} \right) \]  

where \( \lambda(s,t) \) is the line density \( G(x,y) \) transverse distribution and \( N \) the number of particles per bunch. The probe beam 1 moves with \( s = ct \) against beam 2 creating the field and moving with \( s = -ct \) in the opposite direction. The bunch length \( \sigma_z \) is assumed to be considerably larger than the transverse beam dimensions \( \sigma_x \) and \( \sigma_y \). Furthermore the angles \( x' \) and \( y' \) of the particle trajectories should be small, so that \( x' \sigma_z < \sigma_x \) and \( y' \sigma_z < \sigma_y \) and the transverse dimensions \( \sigma_x \) and \( \sigma_y \) should not change significantly over the length of interaction.

The electric field \( \mathbf{E} \) created by such a bunch (1) is

\[ E_y + iE_x = \frac{e\lambda(s,t)}{4\pi \sigma_x \sigma_y \sigma_z} \sqrt{\frac{2 \pi}{1-\epsilon^2}} \left( \frac{x + iy}{\sigma_x \sqrt{2(1-\epsilon^2)}} - \exp \left( - \frac{x^2}{2 \sigma_x^2} - \frac{y^2}{2 \sigma_y^2} - \frac{z^2}{2 \sigma_z^2} - \frac{(st)^2}{2} \right) \right) \]  

where \( \epsilon = \sigma_y / \sigma_x \) is the aspect ratio. The complex function \( w(z) \) is defined as

\[ w(z) = e^{-z^2(1 - \text{erf}(-iz))}, \quad \text{erf}(v) = \frac{2}{\sqrt{\pi}} \int_0^v e^{-t^2} dt \]  

The magnetic field \( \mathbf{B} \) seen together with \( \mathbf{E} \) by beam 1 is

\[ B_x = -E_y/c, \quad B_y = E_x/c. \]  

Both fields together produce a force \( \mathbf{F} \) on a particle with charge \( e \) of beam 1 moving with \( c = vt \) against beam 2

\[ F(x,y,s) = 2e\lambda \left( F_x(x,y), F_y(x,y) \right) \exp \left( - \frac{2s^2}{c^2 \sigma_z^2} \right). \]  

Here \( F_x(s), F_y(s) \) is the maximum force along a trajectory \( F_x(x,y) = F_x(x,y,0) \). Since the particle and the bunch move against each other the length of the interaction is only half the bunch length. This force results in a curvature \( 1/\rho \) of the trajectory and a deflecting angle \( \alpha \) for the particle

\[ \alpha(x,y) = \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right)^{1/2} \frac{1}{\sqrt{\pi}} \frac{\sigma_z}{\sigma_y} \]  

where \( \rho \) is the classical particle radius and \( \sigma_x, \sigma_y, \sigma_z \) are the second synchrotron radiation integrals due to the horizontal, vertical and total deflection.

The energy radiated by this particle on one crossing is

\[ U(x,y) = \frac{1}{2} \left( \frac{\pi}{4} \right)^{3/2} \frac{e^2}{c^2 \sigma_z} \left( \frac{1}{\sigma_x} + \frac{1}{\sigma_y} \right) \rho \]  

where \( \rho \) is the classical particle radius and \( \sigma_x, \sigma_y, \sigma_z \) are the second synchrotron radiation integrals due to the horizontal, vertical and total deflection.

\[ \langle I_x \rangle = \langle I_y \rangle + \left( \frac{\pi}{4} \right)^{3/2} \frac{e^2}{c^2 \sigma_z} \left( \frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} \right) \rho \]  

The first case to be considered is that where the two beams collide head-on without crossing angle and without transverse separation between the bunch axes. To obtain the average energy loss per particle in beam 1 the synchrotron radiation integral (6) is integrated over the transverse distribution \( G(x,y) \)

\[ \langle I_x \rangle = \langle I_y \rangle + \left( \frac{\pi}{4} \right)^{3/2} \frac{e^2}{c^2 \sigma_z} \left( \frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} \right) \rho \]  

where \( N \) is the number of particles per bunch in beam 2 and \( g(r,v) \) is a form factor which depends on the aspect ratio \( \epsilon = \sigma_y / \sigma_x \) and beam separation \( v \) which is considered to vanish for the moment \( v = 0 \)

\[ g(r,v) = \frac{2\pi \sigma_y^2 g(r,0)}{2\pi \sigma_y^2 g(r,0)} \]  

where \( \sigma_y \) is the number of particles in beam 2 and \( g(r,v) \) is a form factor which depends on the aspect ratio \( \epsilon = \sigma_y / \sigma_x \) and beam separation \( v \) which is considered to vanish for the moment \( v = 0 \)

\[ g(r,0) = \frac{16\pi}{\sqrt{\pi} \sqrt{2}} \arctan \left( \frac{\sigma_x}{\sigma_y} \right) \]  

with \( D = 2\pi r^2 - 10r^2 + 3, \) \( Q = 3\pi^2 + 8\pi^2 \). This form factor (9) is valid for any positive value of \( \epsilon \) and has the property \( g(1/r,0) = g(r,0) \). For \( 1/\sqrt{3} < r < 1/\sqrt{3} \) the function \( D \) becomes negative and (9) is more conveniently expressed with the inverse hyperbolic function or with a logarithm. The form factor \( g \) goes to 0 for \( r = 0 \) or \( c = 0 \) and has a maximum for a round beam \( r = 1 \) namely \( g(1,0) = 4\ln(4/3) = 1.1575 \).

The integral (7) gives another interesting result

\[ \langle I_x \rangle = \frac{1}{\sqrt{\sigma_z}} \langle x^2 \rangle = \frac{1}{\sqrt{\sigma_z}} \langle x^2 \rangle \]  

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which means that the total radiated energy due to the horizontal deflection is the same as that caused by the vertical deflection.

The average energy lost per particle in one crossing is now
\[
\langle U \rangle = \frac{2}{3} \left( \frac{\gamma M^2}{\alpha} \right)^2 N_c^2 v^2 g(r,0)
\]

The average power radiated in one intersection region by the whole beam 1 consisting of \(k_1\) bunches with \(N_1\) particles colliding with beam 2 having an equal number of bunches with \(N_2\) particles per bunch is

\[
P_1 = \frac{k_1^2 \gamma r^2 M^2 \alpha^2 \gamma N_2^2 \gamma^2 g(r,0)}{3\pi \sigma_x \sigma_y}
\]

where \(f_0\) is the revolution frequency and \(L\) the luminosity.

Next the case of flat Gaussian beams colliding head-on but with a small vertical separation \(y_0\) is considered. Using a normalized separation \(v = y_0/2\gamma\), assuming flat beams \(r \ll 1\) and a separation small compared to the horizontal beam dimension \(y_0 \ll \sigma_y\), a new form factor is obtained:

\[
g(r,v) = g_x(r,v) + g_y(r,v)
\]

with

\[
g_x(r,v) = g_x(v) = \left(1 - R_x(v)^2\right)/2
\]

The components \(g_x\) and \(g_y\) due to horizontal and vertical deflection are no longer equal. The functions appearing in (13) are

\[
R_x = \frac{2\sqrt{3}}{\pi} \left(1 - R_x(v)^2\right)
\]

\[
R_y = \frac{2\sqrt{3}}{\pi} \left(1 - R_y(v)^2\right)
\]

\[
S(v) = \frac{3}{\gamma^2} \left(1 - S(v)^2\right)
\]

where \(S(1) = 1\) and \(S(0) = 0\). These functions are listed in Table 1. An example of the dependence of \(S(v)\) on \(v\) is shown in Fig. 2. It can be seen from (13) and (14) that at least for small \(v\) the radiated power has a minimum for vanishing beam separation.

2. Spectral angular distribution of the beam-beam synchrotron radiation

The complete spectral-angular distribution of this radiation is rather complicated but could in principle be computed from a Fourier transformed Lienard equation. However, for many applications an approximate treatment is sufficient. The qualitative behaviour of the beam-beam synchrotron radiation depends on the magnitude of the deflecting angle \(\theta\). If this angle is larger than \(\theta_0\) and larger than the observation angle \(\theta_0\), the radiation can be treated in reasonable approximation as ordinary, or strong magnet synchrotron radiation. For the cases where deflecting angle is smaller than \(\theta_0\) or smaller than the observation angle \(\theta_0\), the radiation can be treated as so-called short magnet radiation. Neglecting for the moment the angular spread of the particles in the beam the spectral-angular distribution of the energy radiated by one particle in one traversal of a short magnet is

\[
\frac{dU}{dQ} = \frac{\sigma_y^2}{\gamma \pi c} \left[ \left( \frac{\gamma}{2c^2} \right)^2 - \left( \frac{\gamma}{2c^2} \right) \right] \frac{dU}{d\theta}
\]

where \(Q = \sin\theta\) and \(\theta\) is the solid angle (Fig. 1) and \(\omega\) the frequency of the emitted radiation; \(F(\theta)\) is the Fourier transform of the deflecting force \(F_\theta\) with respect to \(\theta\):

\[
F(\theta) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{F(\psi)}{\sqrt{1 + (\psi/\theta)^2}} d\psi\]

The angular distribution function \(f_\theta(\psi,\theta)\) is different for the polarization component \(\theta\) with the electric field parallel or antiparallel to the deflecting force \(F_\theta\) than for the \(\pi\) component being perpendicular to \(F_\theta\):

\[
f_\theta(\psi,\theta) = \left(1 + \frac{\psi^2}{\theta^2} \cos^2(\theta)\right)^2, \quad f_\pi(\psi,\theta) = \left(1 + \frac{\psi^2}{\theta^2} \sin^2(\theta)\right)^2
\]

Equation (16) can be interpreted as a decomposition of the deflecting force into undulators with different period lengths having the corresponding contribution to the total spectrum.

Taking the case of the beam-beam force \(F_x = F_y\) given by (3) and substituting \(k = \omega(1 + \gamma^2)/2\gamma\) gives the spectral-angular distribution for the horizontal and vertical polarization component of the average radiated power by one beam in a crossing point:

\[
\frac{d^2P_x}{dQ d\omega} = \frac{2}{\pi c} \left(1 + \gamma^2\right)^2 \frac{d^2P_y}{dQ d\omega}
\]

where \(P_1\) is the total average radiated power (12). Here \(g_x\), \(g_y\), \(g_x\), and \(g_y\) are the form factors (9) or (13). In the case of vanishing separation \(y_0 = 0\) one has \(g_x = g_y = g_x = g_y\) and the radiation from the beam as a whole is not polarized.
3. Possible Application

The main properties of the beam-beam synchrotron radiation in LEP operating at 51.5 GeV with a luminosity of \(1.1 \times 10^{31}\ \text{cm}^{-2}\ \text{s}^{-1}\) are listed in Table 1.

This radiation may be used for beam diagnostics.

The two beams could have a natural separation in the interaction point which is expected to be non-negligible in the vertical direction (6, 17). By moving the beams with respect to each other by means of the separators and measuring the relative intensity (13) or the polarization at a certain angle (18) the correct collision conditions can be found.

The radiated power from unseparated beams is closely related to luminosity (12). The absolute accuracy of a luminosity measurement obtained by means of this radiation is not very good since several parameters involved, such as bunch length, residual separation and fraction of collected radiation, are not well known. However the measurement is fast and could be used to optimize the luminosity.

The spectrum of the radiation observed at large angles has the form of the Fourier transform of longitudinal bunch form (18) and could be used to measure the bunch length.

An image of the interaction point formed with the visible part of this radiation cannot resolve the vertical beam dimension in most cases and is of limited interest.

The radiation power available for these measurements within an accessible solid angle is rather small. The resulting technical problems have to be studied in order to prove the feasibility of the proposed methods.

<table>
<thead>
<tr>
<th>beam energy</th>
<th>GeV</th>
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<tbody>
<tr>
<td>particles/bunch</td>
<td>(N)</td>
</tr>
<tr>
<td>number of bunches</td>
<td>(k_B)</td>
</tr>
<tr>
<td>luminosity</td>
<td>(L)</td>
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<tr>
<td>bunch length</td>
<td>(\sigma_x)</td>
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<tr>
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<td>(\langle g \rangle)</td>
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<td>(\langle E \rangle)</td>
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<tr>
<td>radiated power</td>
<td>(P_1)</td>
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<td>rms deflect. angle</td>
<td>(\alpha_{rms})</td>
</tr>
<tr>
<td>(\chi_{rms}) at (\theta = 4\ \text{mrad})</td>
<td>(dP/d\Omega) at (\theta = 4\ \text{mrad})</td>
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<td>(\mu\text{W/ster.})</td>
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<tr>
<td>Table 1. Properties of the beam-beam synchrotron radiation in LEP Phase 1.</td>
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References