INTENSE ANTIPROTON SOURCE FOR A 20-TeV COLLIDER*

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The feasibility of producing, collecting and cooling $\bar{p}$'s at a rate $>3\times 10^{10}$ s$^{-1}$ is demonstrated. This implies a filling time of $\sim 12$ hours to reach a luminosity of $2\times 10^{36}$ cm$^{-2}$s$^{-1}$ in the collider.

Considerations of Collider Parameters

We consider a 20 TeV pp collider operating with bunched beams and zero crossing angle. Bunched beams are preferred because this eliminates the need for exceedingly small emittances, or conversely very large numbers of p's, and because at least for high magnetic fields ($B \geq 10T$) the energy loss due to synchrotron radiation must be compensated. The main attractive feature of using p's is of course the possibility of achieving colliding beam operation with only one ring. It is however essential that the $p$ and $\bar{p}$ bunches are crossing each other only at the design interaction points and are kept apart everywhere else around the ring, e.g. by means of electrostatic deflectors. This imposes a lower limit on the lattice design. If we denote by $N$ the average number of events for a single bunch crossing $\sigma_{\bar{p}} = 100$ mBarn we can rewrite standard expressions for the luminosity $\mathcal{L}$, the beam-beam tune shift $\Delta \theta_{bb}$ and the number of particles per bunch $N_p$:  

\[
\mathcal{L} = 3 \times 10^{37} \frac{8}{8} \frac{\sigma_{\bar{p}}}{8} \quad \text{(1a)} \\
\Delta \theta_{bb} = 1.87 \times 10^{-6} \left( \frac{\sigma_{\bar{p}}}{\epsilon_p} \right)^{1/2} N_p^{1/2} \quad \text{(1b)} \\
N_p = 3.84 \times 10^{12} \left( \frac{\sigma_{\bar{p}}}{\epsilon_p} \right)^{1/2} N_{\bar{p}}^{1/2} \quad \text{(1c)}
\]

where the numerical coefficients apply for $\gamma = 2.13 \times 10^4$ (i.e. 20 TeV), $\sigma_{\bar{p}}$ is the interaction $\sigma_p$= $\epsilon_p$, and $\epsilon_p$ the normalized emittance. Assuming $\sigma_{\bar{p}} = 8 \times 10^{-4}$ and $\epsilon_p = 6.10^{-4}$, we obtain:

\[
\mathcal{L} = 2.09 \times 10^{28} \frac{8}{8} \frac{\sigma_{\bar{p}}}{8} \frac{N_{\bar{p}}}{N_p} \quad \text{(2)}
\]

The controlling importance of $N_{\bar{p}}$ becomes evident from (1a): for $N_{\bar{p}} = 240$ m, corresponding to 250 bunches in a ring of 50 km circumference $\mathcal{L}$ (at 1) is $1.25 \times 10^{34}$ cm$^{-2}$s$^{-1}$.

To produce $\mathcal{L} = 10^{32}$ cm$^{-2}$sec$^{-1}$, we allow $N_{\bar{p}} = 8$ and assume $\sigma_{\bar{p}} = 2$ m, $\epsilon_p = 10 \mu$m to find we need $N_p = 4.86 \times 10^{10}$ and a total $N_{\bar{p}} = 1.2 \times 10^{3}$p's. We shall discuss the feasibility of accumulating this number of antiprotons.

It might be of some interest to comment briefly on the effects of ring size, and therefore magnetic field. With the same emittance and bunch in bunch distance, a larger ring will require more antiprotons to reach the same luminosity. At low luminosity, corresponding to $N = 1$, this can be compensated by decreasing $\epsilon_p$ at higher values of $\epsilon_p$ however, the larger ring with smaller emittance reaches a critical value of $\Delta \theta_{bb}$ earlier, thus either reducing the maximum achievable luminosity, or forcing operation back to larger $\epsilon_p$ and therefore larger total number of antiprotons.

\*Source Requirements and Phase Space Considerations

Assuming, by necessity without proof, a luminosity lifetime of at least 20 hours in the collider, we see that a $\bar{p}$ flux of $3 \times 10^{10}$ s$^{-1}$ is adequate allowing a filling time of $\sim 11$ hours for $1.2 \times 10^{13}$ antiprotons or $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$. We envisage a debuncher/accumulator complex similar in concept to that of the FNAL $p$-source. At a proton momentum $p = 120$ GeV/c and a $\bar{p}$ momentum $p_{\bar{p}} = 10$ GeV/c we expect the following $\bar{p}$ flux $\phi$:  

\[
\phi = 0.2 \epsilon \frac{\Delta \phi}{\sigma_p} \frac{N_p}{\tau_p} \quad \text{for} \ 2 \times 10^5 \text{m} < \epsilon < 4 \times 10^{-5} \text{m} \quad \text{(3)}
\]

where $\epsilon$ is the unnormalized $\bar{p}$ emittance, $\sigma_p$ the $p$ momentum spread at the target, $N_p$ the number of protons hitting the target and $\tau_p$ the repetition period of the process. $N_p$ is limited by target heating, $\epsilon$ and $\sigma_p$, $p_{\bar{p}}$ by considerations regarding lattice and transport system design. We obtain the desired $\bar{p}$ flux using $\epsilon = 2 \times 10^{-5}$ m (unnormalized), $\sigma_p = 0.04$, $N_p = 6 \times 10^{12}$ and $\tau_p 1$s. To reach the required emittance, $\epsilon_p = 5 \times 10^{-5}$, the $\bar{p}$ emittance must be cooled by a factor 20.

To estimate the required longitudinal compression assumptions must be made about the momentum spread in the final collider configuration as well as about the accumulator circumference $CA$, and the effectiveness of the debuncher in reducing the momentum spread of the $\bar{p}$'s. Denoting by $\psi[pv^{-1}]$ the longitudinal $\bar{p}$ density in the accumulator we write:

\[
\psi_{\text{in}} = \frac{\phi \tau_p}{\epsilon_p} \quad \text{and} \quad \psi_{\text{out}} = \frac{CA}{\epsilon_p} \frac{N_p}{\tau_p} \quad \text{(4a,b)}
\]

If after debunching, we have $(\Delta \phi/\sigma_p) = 0.25$ then $\psi_{\text{in}} = 12$ ev$^{-1}$ Use $CA = 800$ m and $\sigma_p$ (full width) $= 6 \times 10^{-4}$ for the 250 bunches of $\sim 1$ m length at 20 TeV to obtain $\psi_{\text{out}} = 4300$ ev$^{-1}$, indicating an increase in longitudinal phase space density of $\sim 350$ that must be achieved by stochastic cooling.

$\bar{p}$ Production and Accumulation Sequence

We postulate the existence of the following rings:

- Booster 1: $E_{\text{max}} = 0.2$ TeV, $B_{\text{max}} = 1.4$ T, $C = 4$ km
- Booster 2: $E_{\text{max}} = 1.4$ TeV, $B_{\text{max}} = 8$ T, $C = 10$ km
- Main Ring: $E_{\text{max}} = 20$ TeV, $B_{\text{max}} = 8$ T, $C = 60$ km

Booster 1 will deliver a train of highly bunched protons, $6 \times 10^{12}$ every second, on target. $3 \times 10^8$
\[ \hat{\beta}'s \text{ are then injected into the debuncher where their} \]
\[ \text{momentum spread is reduced to } 0.25\% \text{ by RF-bunch rotation} \]
\[ \text{and adiabatic debunching. In the debuncher the} \]
\[ \text{transverse emittance is also reduced to } 3 \mu \text{m} \text{ (30 nm) by} \]
\[ \text{stochastic cooling. The } \hat{\beta}'s \text{ are then transferred to the} \]
\[ \text{accumulator where the required phase space density. These particles} \]
\[ \text{will be extracted, accelerated to } 200 \text{ GeV in} \]
\[ \text{all} \text{ the required} \text{ phase space density. These particles} \]
\[ \text{are then transferred to the accumulator, freeing the} \]
\[ \text{debuncher for the next batch. In the accumulator the} \]
\[ \text{\hat{\beta}'s are stochastically stacked while the transverse} \]
\[ \text{emittance is reduced to } 1 \mu \text{m \text{ (10 pm normalized).} \]
\[ \text{But they greatly facilitate the accumulator design} \]
\[ \text{and their design is, at least conceptually, straightforward. A certain R} \]
\[ \text{and D effort is certainly required to design the electrodes, and the} \]
\[ \text{lattice design of the debuncher must take into account that} \]
\[ \text{the PU and K arrays (each } > 1 \text{ m long) must be} \]
\[ \text{broken up into sub-arrays of only a few m length,} \]
\[ \text{located in fairly low \& sections in order to keep} \]
\[ \text{their apertures small enough for the envisaged frequency band.} \]

**Stochastic Cooling Systems**

The most critical of all the cooling systems used in the outlined \( \hat{\beta} \)-source are the transverse system in the debuncher and the stochastic stacking system in the accumulator. Furthermore, high demands on bandwidth are made by any system contemplated to cool \( > 10^9 \hat{\beta}'s \) in booster 2, if this should be desired. We examine briefly some of the characteristics of these systems.

For cooling systems with sufficiently linear electrodes transverse cooling is well described by:

\[ \epsilon(t) = e^{-\varepsilon t} (\epsilon(0) - \epsilon(\infty)) + \epsilon(\infty) \]  
\[ \text{where } \epsilon(\infty) \text{ is the asymptotic value determined by} \]
\[ \text{the thermal noise characteristics of the system.} \]
\[ \text{Strictly speaking the cooling rate } s, \text{ and therefore} \]
\[ \epsilon, \text{ are functions of the revolution frequency of the} \]
\[ \text{particles. A conservative estimate is obtained by} \]
\[ \text{calculating } s = s(\omega_c) \text{ for a distribution } f(\omega) \]
\[ \text{symmetric about } \omega_c. \text{ The cooling rate } s, \text{ including} \]
\[ \text{the effects of signal suppression, is then given by} \]
\[ \text{a simple integral that has been evaluated in reference} \]
\[ \text{4 from which we obtain} \]
\[ s = \frac{\pi N f(\omega_c)}{n} \]  
\[ \text{where } n = (f_{\max} - f_{\min})/f_0, \text{ } G_0 = \frac{4}{\xi} \]  
\[ \text{is defined as} \]
\[ \xi = \frac{\pi N f(\omega_c)}{n} \]  

Using a parabolic momentum distribution of full width \( \Delta p/\Delta \beta \), \( \xi \) becomes 0.75N(\( f_{\max} \)/\( f_{\min} \))/1 and for \( N = 3 \times 10^3 \) \( f_{\max} = 6 \text{ GHz, } f_{\min} = 4 \text{ GHz, } \text{and } n = 4 \times 10^{-3} \) we find the following values: \( \xi = 5.6 \times 10^{-3} \text{ sec}^{-1}, n = 1.07 \times 10^{-6} \text{ sec}^{-1} \) and \( s = 1.9 \text{ sec}^{-1}. \)

This gives a sevenfold reduction in emittance in one second, provided \( \epsilon(\infty) \) is low enough. \( G_0 \) is completely determined by the coupling characteristics and number of pick-up (PU) and kicker (K) electrodes and the net electronic gain.\text{ Assuming loop couplers made up by 70 \text{\&} striplines in pushpull configuration for both PU and K, we arrive, based on standard expressions, at the approximate system parameters summarized in Table I. \)

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>4 to 8 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of PU's = No. of K's</td>
<td>512 loop pairs</td>
</tr>
<tr>
<td>Amplifier gain</td>
<td>6.5 \times 10^5 \text{ [-136 dB] }</td>
</tr>
<tr>
<td>Total Power</td>
<td>- 1.8 kW \text{ [cryogenic PU's, 4 dB NF preamp] }</td>
</tr>
<tr>
<td>Cooling rate, s</td>
<td>= 1.9 s^{-1}</td>
</tr>
<tr>
<td>( \epsilon(\infty) )</td>
<td>= 3.5 \times 10^{-7} \text{ m }</td>
</tr>
<tr>
<td>( \epsilon(t = 1) )</td>
<td>= 3.3 \times 10^{-6} \text{ m }</td>
</tr>
</tbody>
</table>

then follows \( \xi_0 = 20 \text{ MeV} \text{ with } a = 1.4, n = 4 \times 10^{-3} \) and some allowance for the fact that \( G \) will not be purely real. With these values for \( n \) and \( E \) we can accommodate 6.25 e-foldings of the density \( \psi(x) \) without Schottky band overlap at the top harmonic and the actually required 5.8 e-foldings will be achieved.
in a stack of < 120 MeV or \( \Delta p/p = 1.2\% \) width. Some important system parameters as derived from (9) and (8) are summarized in Table II.

### Table II: Accumulator Stack Tail System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{\text{max}}/f_{\text{min}} )</td>
<td>8 GHz, 4 GHz</td>
</tr>
<tr>
<td>( n_{\text{pu}} = n_{\text{k}} )</td>
<td>1024</td>
</tr>
<tr>
<td>( G_A )</td>
<td>( \approx 1.41 \times 10^5 ) [-122 dB]</td>
</tr>
<tr>
<td>( P_{\text{total}} )</td>
<td>-1.5 kW</td>
</tr>
<tr>
<td>( \alpha p/g )</td>
<td>( \approx 160 ) [i.e., ( \alpha p = 3 \text{m} ) for 15 mm PU gap]</td>
</tr>
<tr>
<td>( \chi )</td>
<td></td>
</tr>
<tr>
<td>Schottky/Noise</td>
<td>( -220 e^{-1/2} )</td>
</tr>
</tbody>
</table>

The signal-to-noise ratio is larger than 2 for the whole stack with exception of the last (highest density) 20 MeV. A low noise core cooling system with appropriately adjusted gain should make the thermal noise problem totally innocuous and provide some additional peaking of the distribution function \( \psi \).

If it were desired to operate the collider with smaller transverse emittance to reduce the circulating charge, that cooling could be carried out with \( 10^3 \) ps or more stored in booster 2 (\( \gamma = 25 \)). We calculate that a system with 256 PU's and 100 dB amplifier gain and a frequency band from 8 to 16 GHz would be able to cool \( 10^3 \) ps at 200 GeV from \( \epsilon_n = 10 \text{ m} \) to \( \epsilon_n = 1 \text{ m} \) in \( \sim 3000 \text{ sec} \).

### Conclusion

Stochastic cooling is capable of providing a flux of \( \sim 3 \times 10^8 \text{ S/second} \), adequate to fill a 20 TeV collider in \( \sim 12 \text{ hours} \) for operation at \( \mathcal{L} = 10^{32} \text{cm}^{-2} \). This represents an order of magnitude improvement over the FNAL-source design goal. It is made possible mainly by higher bandwidth (4 GHz vs. 1 GHz) and a lower \( \Delta \beta \max /\Delta \beta \max \) ratio (350 vs. \( \sim 10^4 \)). Systems operating in the 4-8 GHz band are necessary, a technology which goes beyond the 2-4 GHz systems presently under development, but appears within reach with a moderate R and D effort. Stochastic cooling in the 8-16 GHz range holds the promise of small transverse emittance (\( < 1 \text{ m} \) normalized), allowing even shorter filling times or the use of lower field, larger circumference colliders, should such devices prove more cost-effective.

### References