Introduction

The TRIUMF cyclotron accelerates more than 100 mA of negative hydrogen ions to 520 MeV. Recently we have studied several schemes to take the beam from TRIUMF and accelerate it to energies above 6 GeV to produce copious beams of secondary particles K, p and \( \bar{p} \). We have considered post accelerators based on synchrotrons and also on a cyclotron or series of cyclotrons. Cyclotron post accelerators have the advantage that their time structure is completely compatible with the TRIUMF beam. The reference design consists of two isochronous superconducting ring cyclotrons. The first would accelerate the protons to 3.5 GeV, the second to 9, 12 or 15 GeV. In this report we consider the optics of two proposed versions of this second stage (see Table I): a 30-sector 9 GeV and a 42-sector 12 GeV cyclotron. A plan view of the 12 GeV machine is shown in Fig. 1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>30-sector 9 GeV</th>
<th>42-sector 12 GeV</th>
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</thead>
<tbody>
<tr>
<td>Injection energy</td>
<td>3.5 GeV</td>
<td>3.5 GeV</td>
</tr>
<tr>
<td>Extraction energy</td>
<td>9 GeV</td>
<td>12 GeV</td>
</tr>
<tr>
<td>Sectors</td>
<td>30</td>
<td>42</td>
</tr>
<tr>
<td>Radius (max)</td>
<td>40.5 m</td>
<td>41.1 m</td>
</tr>
<tr>
<td>Radius (min)</td>
<td>20.2 m</td>
<td>40.5 m</td>
</tr>
<tr>
<td>Energy gain per turn</td>
<td>25 MeV</td>
<td>50 MeV</td>
</tr>
<tr>
<td>RF frequency</td>
<td>69.5 MHz</td>
<td>115 MHz</td>
</tr>
<tr>
<td>Beam time width</td>
<td>0.5 ns</td>
<td>0.3 ns</td>
</tr>
<tr>
<td>Peak sector field</td>
<td>5 T</td>
<td>5 T</td>
</tr>
<tr>
<td>Radial beam size at extraction</td>
<td>1.2 mm</td>
<td>1.2 mm</td>
</tr>
</tbody>
</table>

In an isochronous cyclotron, the radial focusing frequency \( \psi_{r} \) is approximately equal to the relativistic parameter \( \gamma \) (Fig. 2). Hence, any integer and half-integer imperfection resonances and some non-linear intrinsic resonances are traversed. The axial focusing frequency, \( \psi_{z} \), depends upon sector shape and work in progress to minimize the number of axial and coupled resonances. It appears, however, that there will be at least one axial integer resonance and that the \( \psi_{z} = 2\psi_{r} \) coupling resonance is unavoidable. Some thought must also be given to the placement of RF cavities so that resonances are not driven by the accelerating field.

We have been able to derive order of magnitude estimates for the construction tolerances of such cyclotrons. These tolerances depend also upon the rf system. In particular, a large energy gain per turn is desirable so that resonances are quickly traversed. Also, harmonic cavities will be necessary to flat-top the wave form, prevent single turn operation and avoid precessional mixing.

Integral Resonances

The chief effect of an integral resonance is to introduce a similar coherent amplitude for all particles in the phase space region. This may be represented as a vector in r-\( \phi_{F} \) space. Studies with the accelerated orbit code GOBLIN have shown that for the 30-sector machine, a coherent amplitude of 5 mm would be introduced by a 1 G 3th harmonic at 4.2 GeV or by a 2 G 12th harmonic at 8.7 GeV. This is in excellent agreement with the values expected from simple theory, namely that the presence at \( \psi_{n} = m \) of an nth harmonic field error \( B_{n} \) (in c.u.) results in a coherent oscillation of amplitude (also in c.u.)

\[
A = \frac{\psi_{r}}{2\pi} B_{n}
\]

where \( \psi_{r} \) is the change in \( \psi_{r} \) per turn.

The beam traverses nine such integral resonances in the 12 GeV machine. It is expected that second harmonic cavities would be used to flat-top the wave form over the 12° of phase expected to contain 100 uA and thus yield single turn conditions for the 240 turns expected.

![Fig. 2. The radial focusing frequency as a function of energy for the 42 sector cyclotron. The relativistic parameter \( \gamma \) is shown for comparison.](image-url)
In principle, the total coherent amplitude may be compensated at any point, e.g. at a resonance before extraction. Should the flat-topping not be complete, say because of deviations from isochronism, then precessional mixing may transform a coherent amplitude into an effective incoherent amplitude.

Another consideration is the size of the linear region in \( r-p \) space; for, if the coherent oscillations are at any time larger than this size, the phase space ellipse will be distorted and it will no longer be possible to completely compensate for the amplitude gain. We find that whereas the linear region is fairly large at low energies (>10 times the beam size), it is rather small at the highest energy. In particular, for extraction where the beam size in either of the cyclotrons is \( \approx 1 \) mm, the linear region is only \( \approx 2 \) mm in extent. This places stringent tolerances on the cyclotron design because the magnetic field error incurred by misplacing one sector by \( \Delta r \) is proportional to \( \beta v^2 \Delta r \). In particular, if \( v_p = n \) is the largest integer resonance before extraction and the sector positions are given by \( r_{n \pi} \cos n \theta \) then the largest tolerable value of \( r_p \) is given by

\[
\frac{1}{\pi} \sqrt{\frac{E_p}{E_0}} \Delta r
\]

where \( E_p \) is the energy gain per turn, \( E_0 \) is the particle rest energy and \( A \) is the largest tolerable coherent amplitude. To keep \( ACI \) mm in either cyclotron, the maximum allowable \( r_p = -0.005 \) mm. A positional harmonic of this amplitude will be expected to arise for example from randomly positioning the sectors within a tolerance of \( \pm 0.04 \) mm. It is probably not possible to achieve such a tolerance. Nonetheless, it may be practical to meet a positional tolerance of \( \pm 0.4 \) mm and to achieve the final tolerance by adjusting harmonic coils to a precision of \( \pm 0.1 \) G.

**Half-Integral Resonances**

Here we increase the linear amplitude in this way. In particular, dependent on the amplitude entering the resonance. The effect is to stretch the radial phase space and produce a subsequent mismatch between the phase space and cyclotron acceptance. Again we assume single turn conditions and no precessional mixing. GOBLIN runs show that a gradient \( 0.1 \) G/cm 15th harmonic introduced where \( v_p = 7.5 \) stretches the ellipse by 10% and a gradient \( 0.4 \) G/cm 31st harmonic introduced where \( v_p = 15.5 \) stretches the ellipse by 30%. Calculations based on equilibrium orbit transfer matrices yield amplitude gains of only 0.1% in either case. Furthermore, we find from GOBLIN runs that the amplitude gain is proportional to the size of the imperfection gradient of the magnetic field while the standard theory (e.g. Gordon\(^2\)) predicts that the amplitude gain is proportional to the square of the size of the imperfection gradient. The discrepancy occurs because the amplitude growth is caused by a non-adiabatic mismatch between the beam phase space ellipse and the stable 'static' ellipse which stretches considerably before and after the \( n/2 \) stop band. GOBLIN runs were also made for an isochronous but axially non-focusing cyclotron with no flutter (\( B=1-1^{1/2} / 12 \) in e.u.). These gave amplitude gains through \( n/2 \) imperfection resonances which agreed with the above results for the 42-sector machine. This indicates that the phenomenon of non-adiabatic amplitude gain is not peculiar just to the types of cyclotrons which we are considering.

To get a feel for the construction tolerances imposed by passage through half-integer resonances, we again imagine a cyclotron constructed of imperfectly placed perfect sectors. For the 42-sector cyclotron, there are \( \approx 20 \) half-integer resonances. We expect no correlation between the incoming stretched ellipse and the direction of stretching at each passage. We desire to

![Fig. 3. 7 turns before to 7 turns after \( v_p=30/3 \) (o) starting ellipse, (x) field with \( \beta v=0 \).](image)
or three such devices would enable the beam to enter an active magnetic channel. The conductors of these channels are arranged in such a way that the stray field in the inner, beam side of the device, is very small, <0.1% of the field in the channel. Room temperature devices\(^3\) have provided field changes of 0.35 T, proposed superconducting devices\(^4\) hope to achieve 2 T. Channels 1 m long would provide kicks of 0.008 rad and 0.046 rad respectively. To pass between the outer yokes the beam must emerge at an angle of 0.17 rad with respect to the equilibrium orbit. This may be difficult to achieve. It may be easier to direct the beam between two of the smaller inner yokes into the free space inside the ring.

It should be noted that the beam is unlikely to traverse the intrinsic and imperfection resonances and retain the pure elliptical shape of Fig. 4, and thus some loss must be anticipated in the extraction system. An extensive study will be necessary to minimize this loss and devise appropriate remote handling techniques.

Also evident from Fig. 4 is the non-linear stretching that takes place because the coherent oscillation is larger than the size of the linear regime. This itself is expected to cause the effective emittance of the extracted beam to be larger than the original emittance of \(2\pi \text{ mm-mrad}\).

**Conclusions**

It would appear that the tolerances set by field imperfections affecting the radial motion can be met, although empirical adjustments to cancel field errors would be necessary. This could most easily be done if one could observe the individual turns. It is also likely that we could extract the beam if the initial radial quality could be maintained. However, we are unlikely to cross the N/3 or N/4 intrinsic resonances without some filamentation. A 1% beam loss at extraction not only causes component activation but is also a 10 kW energy source in the neighbourhood of the superconducting magnet-coils. In Fig. 3 about 12% of phase space area of the partly compensated ellipse lies outside the area the matched ellipse would occupy in the absence of the resonances. The amount of beam lost would depend on the details of the magnet field and extraction geometry. The 42-sector machine has N/4 and N/3 resonances at 8.4 and 10.6 GeV. The turns remain separated to 9.4 GeV.

**References**