Longitudinal Instabilities of Bunched Beams in the ISR

P. Bramham, S. Hansen, A. Hofmann, E. Peschardt
CERN, Geneva, Switzerland

Summary

Microwave instabilities occur in bunched beams in the ISR leading to a dilution of the phase space density and limiting the longitudinal density of the stacked beams. According to D. Boussard this instability can be described as a coasting beam instability inside bunches. Experimental investigations of this microwave instability support this theory and give a high frequency impedance \( Z_L \) of \( 14 \) ohms. Injecting large currents in bunches of large area increases the threshold of this instability. The larger currents can produce coupled bunch mode instabilities which can be cured by a higher harmonic cavity.

1. Introduction

In the ISR the beams are accumulated by stacking in longitudinal phase space. The injected bunches are matched to the RF voltage and accelerated towards their final orbit. Then the RF voltage is reduced and finally turned off and the beam debunches. During this process microwave instabilities occur which are similar to the bunch widening (turbulence) observed in electron rings. They lead to a blow-up of the bunch area and hence to a dilution of the phase space density. This effect limits the longitudinal density of the stacked beams and affects the performance of the ISR. This microwave instability can be described as a coasting beam instability inside bunches.

2. Microwave Instabilities (Bunch Widening)

2.1 Theoretical Description

Bunch widening or turbulence has been treated by several authors. We use here an approach first described by D. Boussard and apply his results to our measurements. A longitudinal coasting beam instability can occur inside bunches if the frequency of the instability is so high that the corresponding wavelength is short compared to the bunch; and if the growth rate (calculated for the coasting beam) is large compared to the phase oscillation frequency \( \omega_0 \). In this case the coasting beam stability criterion can be applied to the instantaneous values of momentum spread and current.

\[
\frac{1}{2} \left( \frac{\delta p}{p} \right)^2 < \frac{I}{\delta v} ~ \text{inst.} \tag{1}
\]

with \( I = \text{current}, \delta p = \text{momentum spread}, \) and \( \frac{1}{2} \left( \frac{\delta p}{p} \right)^2 \) is the average momentum spread squared at half height. The effective RF voltage \( V^* \) seen by a particle in the bunch is given by

\[
V^* = \frac{n^2 I_{Q0}}{2 M A^2} \left( \frac{\delta p}{p} \right)^2 \tag{2}
\]

where \( n = \text{number of bunches}, I_{Q0} = \text{average current of all bunches}, V^* = \text{effective RF voltage seen by a particle}, \delta p = \text{momentum spread}, \frac{1}{2} \left( \frac{\delta p}{p} \right)^2 = \text{average momentum spread squared}, and M = \text{number of bunches}.

For a given \( \frac{1}{2} \left( \frac{\delta p}{p} \right)^2 \), the quantity \( I/\delta v \) has a threshold value above which a microwave instability will occur. For proton beams of phase space area \( A \) of the bunch is conserved and the above quantity is smaller for short bunches (for electrons \( \delta p \) is fixed by synchrotron radiation and the above quantity becomes smaller for long bunches). We consider here protons with bunches in large stationary buckets having a parabolic "line" density with respect to the RF phase angle \( \phi \).

2.2.1 Blow-up of the bunch area. In one approach we measured the bunch length \( \delta L \) as a function of injected current. The current was controlled by a vertical scraper which should not affect the longitudinal particle distribution. The effective RF voltage \( V^* \) was then determined using an inductance of \( 10 \) \( \mu \text{H} \) and correcting for space charge effects. This inductance is based on recent measurements and is slightly smaller than the value quoted earlier. The quantity \( I/\delta v \) and the bunch area \( A \) are calculated from (3) and (4). Fig. 1 shows an example of such a measurement. The top curve shows the increase of bunch length with current.
expected for the potential well distortion. Above a certain threshold excessive bunch lengthening occurs. The bunch area A (centre curve) is at first independent of current but above the threshold a blow-up occurs. The last curve shows the quantity $I/(\Delta \beta)^2$ as a function of current. It increases as expected up to the threshold and decreases later due to an excessive blow-up of the bunch area (overshoot). If the initial (injected) value of $I/(\Delta \beta)^2$ is larger than the threshold value, its final value will be given by the Dory relation

$$I/(\Delta \beta)^2_{\text{initial}} \cdot I/(\Delta \beta)^2_{\text{final}} = (I/(\Delta \beta)^2)_{\text{threshold}}$$

which is indicated in Fig. 1 and agrees well with the measurements. From several such measurements we determined the threshold value of $I/(\Delta \beta)^2$ and calculated the impedances with the result $|Z_L|/n = 13.5 \, \text{ohms}$. In a second approach we injected a fixed current and reduced the RF voltage adiabatically. According to (3) the quantity $I/(\Delta \beta)^2$ increases in the process and can reach the threshold value. Such a measurement is shown in Fig. 2. These measurements were analyzed in the same way as the ones described before with the result $|Z_L|/n = 13.4 \, \text{ohms}$.

2.2.2 Observation of high frequency signals. During debunching $I/(\Delta \beta)^2$ is growing steadily (5). High frequency signals can be observed directly in time domain or in frequency domain through a filter. The observed thresholds agree with the expectation but the results are probably less accurate since we cannot correct for the potential well. High frequency signals are also observed during voltage reduction (Fig. 3). At a certain time a signal appears suddenly. From the effective voltage at that time and the initial bunch we again determined the impedance $|Z_L|/n \approx 13 \, \text{ohms}$.

Finally, Fig. 4 shows high frequency signals appearing at injection for different frequencies and injected currents. The bunch area A is at first independent of current but above the threshold it decreases later due to an excessive blow-up of the bunch area (overshoot). High frequency signals are also observed during voltage reduction (Fig. 3). At a certain time a signal appears suddenly. From the effective voltage at that time and the initial bunch we again determined the impedance $|Z_L|/n \approx 13 \, \text{ohms}$.

2.3 Results

Measurements of the microwave instability carried out under quite different conditions show that the instability seems to occur over a large frequency span. The average value of the high frequency (0.3-1.8 GHz) impedance divided by mode number is for the ISR

$$|Z_L|/n \approx 14 \, \text{ohms}.$$
the wavelength of this higher harmonic oscillation. In order to have clean conditions we reduced the resonant frequency at the Landau cavity to the 4th harmonic of the RF and operated close to $I + kN \approx 0$. This gives a large spread and leads to bunches with a flat top, see Fig. 5. Experiments to stack large injected currents with this cavity are under way and look promising.

**Acknowledgement**

We thank G. Döme, H.G. Hereward, K. Hüblner, W. Schnell and B. Zotter (CERN), and G.A. Voss (DESY) for stimulating discussions.

**References**

5. W. Hardt, private communication.
8. C. Fischer et al., CERN-ISR-OP/77-18.