A METHOD FOR CALCULATING NEAR-OPTIMUM ION-EXTRACTOR PROFILES*

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Abstract

A process and program have been developed to interactively calculate the near-optimum electrode profiles for high-perveance ion-extraction systems. A Mathcad™ program determines electrode profiles for high-current (high-perveance) high-quality beams. The program input starts with key parameters: plasma density, estimated mix of ions, extraction voltage, total current, plus desired output beam size and divergence. The computations simulate a spherically convergent extraction system that simultaneously minimizes the aberrations from the exit aperture while directly compensating for both the exit aperture de-focusing lens, and internal space charge in the beam. The program outputs cylindrical (r,z) coordinates of the emission and extractor electrodes, plus displays the beam perveance and output beam size and divergence. This technique was used successfully in multiple projects over the past 25 years. Electrode shapes used in past hardware tests are examined with the successive over relaxation (SOR) code PBGUNS in an accompanying paper.

Background And Need

The ability to extract a high-quality (low-emittance) high-current beam of ions or electrons from an emitting surface or a plasma is critical to nearly all devices that utilize energetic charged-particle beams. Included in this category are all electron-beam RF power sources and all particle accelerators.

There is often a need for higher currents and higher beam quality. This improved performance is most often noted by a combination of increased power, smaller size, improved efficiency, lower activation, simpler cooling, and/or simplified setup and operation. Achieving and maintaining high quality in any beam is an on-going challenge, one that becomes much more difficult as the beam current and total power are increased.

PROCESS

The extractor geometry to be discussed is that of the ‘diode’ extractor shown in Figure 1. Goal of the process is to simulate the ‘perfect’ inward flow of ions (or electrons) between two concentric spheres. This concept was investigated by Langmuir & Blodgett [1] and discussed by Pierce [2]. A very similar process may be used for ‘triode’ or other extractor geometries. But the author has emphasized diode extractors only because they have demonstrated superior performance in the projects he has pursued.

\begin{equation}
\Phi_{r} = \frac{-2}{r} \Phi'_{r} - \frac{r_{b}^{2} J_{0}}{\varepsilon_{0}} \sqrt{\frac{m}{2e}} \left( V - \Phi_{r} \right)^{0.5} \frac{1}{r^{2}}
\end{equation}

Langmuir and Blodgett [1] are credited with showing that the radial potential solution to this DE may be nicely approximated by:

\begin{equation}
V(r) = V_{a} \left( \frac{-\alpha}{-\alpha_{a}} \right)^{4/3}
\end{equation}
with \( \gamma = \ln \left( \frac{r}{r_b} \right) \) \( (3) \)

where

\[ \alpha = \gamma - 0.3 \gamma^2 + 0.075 \gamma^3 - 0.001432 \gamma^4 + 0.002161 \gamma^5 - 0.0002679 \gamma^6 + ... \] \( (4) \)

The most-serious imperfection in this process is the extractor aperture through which the charged particles exit the two-electrode system. The exit aperture creates a divergent lens, and internal space charge causes further divergence in the exiting beam. Fortunately the convergent effect of the initial two-electrode extractor can effectively compensate for the first-order effects of both the exit lens and space charge. There remains a small aberration from the edge effects that fortunately normally impacts only the outer 1—2% of the beam.

The Paraxial exit lens effect is given by:

\[ \frac{1}{f} = \frac{\Delta r'}{r} = \frac{\Delta E}{4V_0} \]

\[ \Delta r' = \frac{r \Delta E}{4V_0} \] \( (5) \)

There is also a first-order space-charge term, given by:

\[ \Delta r' = \frac{\Delta z C P_e}{r} \] \( (6) \)

where \( \Delta z \) is the distance over which the full space charge is effective, \( P_e \) is the beam perveance (in \( \mu \)Pervs), and C is a constant equal to 15.2 mrad/\( \mu \)Perv. When combined, we have a total exit divergence effect:

\[ \Delta r' = \frac{r \Delta E}{4V_0} + 0.0152 \frac{\Delta z}{r} P_e \] \( (7) \)

**Design Process Details**

During program execution, only six parameters need be input: 1) total beam current, 2) effective ion mass, 3) extraction voltage, 4) current density at emission aperture, 5) space-charge neutralization length & 6) convergence half-angle of the diode extractor. The program first computes the beam perveance, resultant beam divergence (after exit lens) and peak field within the extractor gap. The operator then may make corrections to ensure that output beam divergence and peak fields are acceptable. All other calculations are automatic, with the program output including graphic and tabular representations of the emission and extractor electrode profiles.

The potential distribution along the beam edge is calculated by eqn 2 above and a number of points are fitted with a summation of four Legendre Functions. The output of this process establishes the values for the coefficients for these functions. We have shown that the following four Legendre functions (Eqn 8) provide a good fit to the potential, create physically realizable profiles, and avoid instabilities.

\[ V(r, \theta) = a_0 + a_1 \cos \theta + b_2 \left( \frac{3 \cos^2 \theta - 1}{2} \right) + b_3 \left( \frac{5 \cos^3 \theta - 3 \cos \theta}{2} \right) \] \( (8) \)

A non-linear equation solver gives values for the four coefficients \( a_0, a_1, b_2 \) & \( b_3 \). The MINERR function in Mathcad™ works well for this, but previous versions have used BASIC code and Mathematica™, with similar success.

The remaining step is the determination of the equipotentials for the two electrodes (which we call the emission and extraction electrodes).

A sample of calculated electrode shapes is shown in figure 3, fabricated as figures of rotation (fig 4).
For good practical reasons, our team has almost always completed a PIC code simulation of the geometry before committing a design to hardware. The PIC simulation provides two benefits:

- Because the Poisson equation is a form of elliptic differential equation, this process of fitting a potential along an edge and estimating the profiles farther out is potentially unstable, and does not yield unique profiles. Use of the PIC simulation code ensures that the chosen profiles are workable.
- The solutions obtained with this method are based on several approximations, most significantly that of making a first-order correction for the exit lens effect. As a result, the PIC codes often indicate reduced beam aberrations by a slight reduction (≈5%) in the extractor gap. Also in practice, we might make small adjustments in the plasma current density to effect a better shape for the plasma sheath.

Below in figure 5 we show the PBGUNS PIC simulation of the analytic solution obtained earlier. This simulation was done by Joe Sherman and is explained in detail in a companion paper [3].

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**SUMMARY**

This basic process of achieving a workable spherically convergent extractor has been used successfully by the author over the past 25 years*. The first published reference was in 1981 [4]. This early work was done for a high-current H$_2^+$ injector for the FMIT program, and required a beam emittance of nearly 0.7 µPerv. Later, we used similar techniques for a very high-brightness H$^+$ extractor for the neutral particle beam program and somewhat more recently, a very successful proton extractor for the Low-Energy Demonstration Accelerator (LED) [5]. In all those cases and others, initial use of this design code greatly expedited the process of converging to a successful extractor design.

The process described here differs from that often used by others in that this technique does not rely entirely on either the use of FEM (finite element codes) or analog measurements (e.g., electrolyte tanks) to generate the desired field profile along the beam edge. The use of this process will save much time, as the fast-running code permits converging to a near-ideal profile prior to committing to the time-intensive PIC simulations.

**REFERENCES**


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* This method was developed by the author, largely as a hobby, but in direct support of injector development projects while the author was employed by the Los Alamos National Laboratory. Funding for the projects during that 25-year span came primarily from the US Department of Energy.