MEASURING AND UNDERSTANDING THE MOMENTUM APERTURE
IN A STORAGE RING

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Abstract

The momentum aperture of a storage ring is a very important parameter that strongly influences the performance, especially the beam lifetime. For the special case of synchrotron light sources with small emittance like the Advanced Light Source (ALS), the momentum aperture depends strongly on the transverse dynamics. It is very sensitive to machine conditions such as the tunes, chromaticities, lattice symmetry, and spurious coupling, since depending on those conditions the Touschek scattered particles explore different resonance regions in the phase space. In light sources, the momentum aperture usually also depends strongly on the vertical physical aperture. Applying frequency analysis techniques in simulations and for turn-by-turn orbit measurement data provides a very powerful tool to measure and understand limitations of the dynamic momentum aperture. The techniques presented are applicable to other light sources, as well as damping rings and many types of colliders.

INTRODUCTION

Storage rings are used for a variety of science and technology applications - for example as synchrotron light sources or as colliders for particle physics. In these storage rings, bunched particle beams circulate for many hours. The motion of a particle can be described in terms of transverse (betatron) and longitudinal (synchrotron) motions with respect to the reference particle. Some particles may be lost due to various aperture limitations. The momentum aperture is defined as the maximum momentum deviation that a particle can have without becoming unstable and being lost. The momentum aperture is determined by the complex 6-dimensional dynamics of the particle. Because of the complexity of the dynamics, in the past there have been unexplained discrepancies between the predicted and measured momentum aperture.

In many cases the momentum aperture is the dominating factor determining the beam lifetime. Long lifetimes are desirable to users of synchrotron light sources since they increase the integrated photon flux, reduce the number of refills necessary, and improve the stability by reducing thermal effects. Even in cases where top-up (or quasi-continuous) injection is being used, long beam lifetimes reduce radiation levels and enable a lower injection frequency, minimizing the impact of injection transients to users. In those storage rings where the dominant lifetime process is Touschek scattering, the lifetime has a stronger than quadratic dependence on the momentum aperture. The Touschek lifetime at the ALS [1] of 9 hours for example is much shorter than the vacuum lifetime of 60 hours, so the ALS would benefit greatly from a larger momentum aperture. Therefore it was important for the ALS to fully understand what limits the momentum aperture. Such knowledge can also help to improve the performance of existing light sources as well as to help predict and optimize the performance of future storage rings.

Figure 1 illustrates how physical, dynamic and RF-apertures affect the momentum aperture in a storage ring for Touschek scattered particles. Because particles undergoing scattering at a nonzero dispersion start horizontal betatron oscillations, induced oscillation amplitudes are shown as well (thin blue lines). In the straight section, where there is no dispersion, the induced amplitude is zero. This means the particle will only change its energy but not start a betatron oscillation. In the arc section with a finite dispersion the induced amplitude in the simplest case shows a linear behavior. For the ALS the induced amplitude is (nearly) constant in the arcs because $H_{x}$ is (nearly) constant in the arcs.

Figure 1: Contributions of the different apertures to the total momentum aperture as a function of the relative momentum deviation $\delta$. The thin blue lines represent the induced amplitudes, for an arc or a straight section of the ring. Dashed-dotted red lines stand for the physical apertures, and the vertical green lines show the RF-momentum aperture. This figure as well as the following ones is plotted at the injection point (\(\beta_{x}(s_{0}) = 12\ m\)).

The thick lines represent the various apertures. The vertical green lines show the RF-momentum aperture with $\varepsilon_{RF} = 3\%$, corresponding to the present available RF-
voltage (for 1.5 GeV). The dashed-dotted red line shows the physical apertures \( x_{\text{phys}} \). The solid red line is a sketch of the dynamic aperture. As discussed later, the size and the shape of the dynamic aperture strongly depends upon the machine conditions (i.e. tunes, chromaticities, coupling, symmetry [2]).

The momentum aperture is defined as the smallest crossing point of the induced amplitude and the smallest of the apertures. As the induced amplitude varies around the ring so does the momentum aperture.

**PARTICLE LOSS MECHANISM**

The particle dynamics [3, 4] and momentum aperture [5] have been extensively studied at the ALS. A schematic of the process leading to particle loss after Touschek scattering is shown in Fig. 2. Due to Touschek scattering a particle receives a certain energy offset (here 3%). If the scattering happens at a position of the ring with dispersion, this energy change will also induce a transverse oscillation (red circle). Due to the tuneshift with energy (chromaticities) and tuneshift with betatron amplitude, the betatron tunes (i.e. the number of transverse oscillations in one revolution) of the particle change as well (right part of the figure). Afterwards, the particle undergoes energy oscillations and slowly damps back to the nominal orbit (green circle). Because of the chromaticities and the tuneshift with amplitude, the tunes get modulated during this process and eventually the particle might encounter a resonance or an area of high diffusion and might be lost.

Figure 2: Left: Schematic of particle behavior after Touschek scattering. Initial particle position after being scattered (red dot) then oscillating in energy and amplitude (red line) and damping back to the closed orbit (green dot). Right: particle trajectory in tune space, resonances up to fifth order are shown.

Tracking particle trajectories using a realistic representation of the ALS lattice confirmed that the model of particle loss mentioned in the previous paragraph is correct. Fig. 3 shows the trajectory of a particle tracked for 10,000 turns including the effects of synchrotron radiation. On the top, one can see the transverse coordinates of the particle. On the bottom the betatron tunes are shown (calculated every 300 turns). At certain times (a)-(c) when the tunes cross resonances, growth of the oscillation amplitude is observed. On some of those occasions, the particle got very close to the physical aperture (±4 mm). Particles with slightly different initial conditions can be lost.

Figure 3: Tracking of particle with synchrotron oscillations and radiation damping. When the trajectory crosses a region with high diffusion (see labels (a) to (c)), the vertical oscillation amplitude increases and at (c) the particle gets very close to the physical aperture of 4 mm.

**MEASUREMENTS**

The main tools to understand the momentum aperture are two classes of measurements. One uses a scan of the rf-bucket height to quantitatively measure the momentum aperture but without insight into details of the dynamics determining it. The second one uses frequency analysis techniques and allows a fairly accurate quantitative measurement and provides a global picture of the underlying dynamics. Both methods clearly show that the major limitation to the momentum aperture for the ALS is the transverse beam dynamics, causing Touschek scattered particles to eventually reach large vertical amplitudes where they are lost on the vacuum chamber.

**RF-amplitude scan**

An easy way to study whether the momentum acceptance of a ring is solely determined by the voltage of the RF-cavities or whether there are dynamic effects limiting it, is to vary the RF-voltage (the bucket size and therefore the energy acceptance of the RF is proportional to the voltage). Because of the energy distribution of Touschek scattered particles one expects the lifetime to be a quadratic (to cubic) function of the RF-voltage, as long as there are no dynamic effects limiting the acceptance. The turnover point, at which a deviation from the quadratic behaviour occurs, provides information about the nonlinear dynamics.
effects limiting the dynamic acceptance. Fig. 4 shows three scans of the RF-voltage for different linear chromaticities of the ALS.

![Figure 4: Measurement of the beam lifetime as a function of the RF-momentum aperture for three different sets of chromaticities. One can see that the dynamic momentum aperture has a severe impact on the overall momentum aperture and therefore the Touschek lifetime.](image)

As one can clearly see, the dynamic momentum aperture and therefore the maximum achievable lifetime is very sensitive to the linear chromaticities. It is best for the nominal chromaticities, the ALS is operated at in multibunch operation (blue symbols), and significantly worse (red symbols) for a vertical chromaticity increased by 3 units. Unfortunately, instabilities require to increase the vertical chromaticity in two bunch operation, to provide enough Landau damping to damp vertical single bunch instabilities. Therefore it is of interest to understand the effect causing this difference (which is surprisingly large, considering that the strength of sextupoles and therefore the strength of nonlinearities in the lattice is only different by a few percent). The green symbols indicate a case where based on the studies described in the next paragraphs an improvement of about 25% was achieved for the high chromaticity case.

**Frequency Analysis**

To study and understand the effects limiting the dynamic momentum acceptance, some methods of Frequency Map Analysis [3] were adapted to study the evolution of off-energy particle trajectories. Frequency Map Analysis is a very effective tool to visualize the global dynamics for systems of 3 degrees of freedom and more. Originally it had been used to study the dynamics of particles with nominal energy.

For these studies two single-turn pinger magnets together with turn-by-turn beam position monitors (BPM) were used. To collect one data set, the RF-frequency was set to different values, corresponding to different circumferences of the electron trajectories and therefore different beam energies. This does not directly simulate the case of Touschek scattering, where the particle starts simultaneously with a transverse and a longitudinal kick. However, the same area in tune space is probed (compare Fig. 5), and the experiment is easier to carry out as well as the frequency analysis. We found experimentally that for the ALS all relevant details in the off-energy dynamics are probed in the case with fixed momentum offset. Newer measurements elsewhere have attempted to simulate the Touschek scattering process more closely [6]. For each setting of the RF-frequency the beam was then kicked with increasing horizontal amplitude (and a small vertical amplitude) and the beam oscillations and the fraction of beam loss recorded, until the beam was completely lost.

![Figure 5: Schematic of the measurement technique to study the off-momentum dynamics. The particle (red dot) damps down to the closed orbit (green line) without synchrotron oscillations. Left: configuration space with induced amplitude after a Touschek scattering (black lines). Right: tune space.](image)

Analysis of the measurement data using Frequency Map Analysis allows us to understand the details of the beam loss - identifying those resonances that limit the momentum aperture. One example using the nominal ALS lattice is shown in Fig. 6. The upper plot shows relative beam loss in the configuration space formed by energy offset and horizontal oscillation amplitude. One can clearly see the complicated structure of the boundary of the stable area. The lower plot shows the same data in frequency space. By recording the tunes of the particles after they have been kicked, one can clearly identify which resonance areas are causing the beam loss. By using this information, one can now understand (compare areas A-D in the plots) what caused particular loss regions. The knowledge gained as a result of these measurements allows us to adjust the machine parameters to improve the lifetime. In addition to the fundamental frequencies of the motion used for the frequency map analysis, the beam spectra also contain other resonance lines which can be analyzed in terms of resonance driving terms [7, 8].

The data sets for the high vertical chromaticity case is shown in Fig. 7. One can see that the area covered in tune space is very different and therefore different resonance regions determine the momentum aperture. The main differences to the case with nominal chromaticities are the loss areas A and C, which are caused by the vertical integer resonance and the coupling resonance, respectively. The difference in momentum aperture between the two cases is not caused by big difference in resonance strength but rather the different tune footprints. Moreover the measurement data provided the model independent guidance on how to
improve the momentum aperture by raising the horizontal chromaticity. This higher horizontal chromaticity shifts the crossover point with the coupling resonance to higher momentum deviation and therefore the loss area C, resulting in a bigger momentum aperture.

**SIMULATIONS**

Simulations based on the calibrated model of the ALS are in very good agreement with the measurements (see Fig. 8). The color code in the graphs indicates the diffusion rate (as defined by the change in transverse oscillation frequency with time) on a logarithmic scale. Blue areas indicate low diffusion, red areas high diffusion (10 billion times higher than dark blue) and areas without color indicate unstable regions (where particles are lost within 1000 turns). The simulations reveal the same high diffusion/loss areas as the measurements and allow a precise computation of expected momentum apertures for different lattices and machine upgrades in advance. They also reproduce all details of the difference in momentum aperture between the cases with three different chromaticities very well.

**IMPACT OF VERTICAL PHYSICAL APERTURE**

One important aspect of the dynamic momentum aperture in light sources is its strong dependence on the vertical physical aperture. In fact, in most light sources the majority of all particle losses occurs in the vertical plane. The main reason for this is that the aspect ratio of the physical apertures is usually very flat, since this allows to optimize the performance of undulators and wigglers. Therefore verti-
ical physical apertures are very important to understand the horizontal dynamic aperture and dynamic momentum aperture. This is a big change from the classical definition of the dynamic aperture, which used to be independent of any physical apertures.

Studying the interplay of vertical physical apertures and momentum aperture, we found that coupling played a big role in how sensitive the momentum aperture was to the physical aperture. One should emphasize that this effect is not directly related to the beamsize, since the vertical physical aperture in those cases is about 40-50 times the vertical beamsize. It is rather caused by the fact that all coupling resonances (low order and higher order) scale in strength with the overall coupling.

For the ALS, the sensitivity of the momentum aperture to the vertical physical aperture was reduced for a case with small coupling but vertical emittance increased using a vertical dispersion wave, compared to the case with large coupling (compare Fig. 9). Combined with other improvements in the lattice and a better correction of lattice symmetry errors this enabled the ALS to reduce the physical gap of insertion devices from 8-9 mm down to about 5 mm.

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Simulating the effects in tracking, we found good agreement between measurements and simulations (see Fig. 10): a correction of the coupling reduces the sensitivity of the dynamic momentum aperture to the vertical physical aperture [9]. The injection efficiency showed similar improvements when the coupling was corrected for small vertical gaps.

**SUMMARY**

Measurements have demonstrated that the dynamic momentum aperture in the ALS is the dominant effect determining the Touschek lifetime. Employing the method of frequency analysis provides a very powerful model independent diagnostic tool to visualize the global dynamics of the system, understand limitations and suggest improvement strategies. A significant improvement of the lifetime in two bunch operation was realized. For newer light sources with very elaborate sextupole configurations, this method will help to systematically optimize the momentum acceptance.

The agreement between measured data and simulation results is good, providing both confidence in the model and the possibility to accurately predict the performance of upgrades or the performance of new machines.

Detailed studies were carried out with respect to the influence of the vertical physical aperture on the momentum aperture. It was found that good correction of the coupling is essential to allow small gaps and that using global dispersion waves can provide control of the vertical emittance without the detrimental effects of coupling.

**REFERENCES**