Abstract

Beam-beam effects in the ring-ring version of eRHIC was studied with a self-consistent beam-beam simulation by using the particle-in-cell method. Beam-beam limits of the two beams were examined as the thresholds of the onset of the coherent beam-beam instability. For the proposed luminosity, the bunch intensities optimized in consideration of the beam-beam effect were discussed. As the beam-beam interaction in the eRHIC is traditionally considered as a strong-weak case, this represents an example of the coherent beam-beam effect in the strong-weak case of beam-beam interactions.

INTRODUCTION

In the proposed ring-ring version of eRHIC, an electron ring will be constructed at BNL that will provide collisions between a polarized 5-10 GeV electron (e) beam and an ion beam from one of the RHIC rings. The design goals of this ring-ring version of eRHIC are to achieve high luminosity of \(10^{31}-10^{32}\) cm\(^{-2}\)s\(^{-1}\) for \(e-p\) or \(e-Au\) collisions and to have high longitudinal polarization (>70\%) at the interaction point (IP) [1]. In order to achieve such a luminosity, large bunch intensity and small beta-functions at the IP have to be employed. Such measures result in large beam-beam parameters. The beam-beam effect, especially the coherent beam-beam effect, is therefore one of important issues to the eRHIC [2]. Moreover, the proposed configuration of unequal circumference of the electron and ion rings could further enhance the coherent beam-beam effect. The beam-beam effect of the eRHIC has therefore been studied with a self-consistent beam-beam simulation of the \(e-p\) collision by using the particle-in-cell method. Beam-beam limits of the two beams were examined as the thresholds of the onset of coherent beam-beam instability.

SIMULATION MODEL

In this study, the linear lattices of the e and p rings with one IP were used and the e and p beam energy are 10 GeV and 250 GeV, respectively. The collision at IP is head-on with flat beams. The beam-beam interaction was represented by a kick in transverse phase space and the kick was calculated by using particle-in-cell method as described in Ref. [3]. The computational parameters were carefully studied to ensure the numerical convergence. For the e beam, the quantum excitation and synchrotron damping were treated as kicks in each turn during the tracking. The damping time is 7.2 ms that corresponds to 571 turns of the e beam in both horizontal and vertical directions. Since the circumference of the electron ring is one third of that of the RHIC ring, each e bunch circulates three turns and collides with three different p bunches during each revolution of the p beam. One e bunch and three p bunches were used in the tracking. Each e or p bunch was represented by \(5 \times 10^5\) macro-particles in the simulation. The initial bunch particle distributions in the normalized transverse phase space were Gaussians truncated at 4\(\sigma\) where \(\sigma\) is the standard deviation. For the e or p beam, \(\sigma = \sigma_z/\sqrt{\beta_{z,e}}\) or \(\sigma_z/\sqrt{\beta_{p,z}}\) where \(z = x\) and \(y\) for the horizontal and vertical directions, respectively. The transverse beam size of two beams are matched at IP with the beam sizes \((\sigma_x, \sigma_y) = (0.1, 0.05)\) mm. The beta functions at IP are \((\beta_{e,x}, \beta_{e,y}) = (0.19, 0.27)\) m for the e beam and \((\beta_{p,x}, \beta_{p,y}) = (1.08, 0.27)\) m for the p beam. Without the beam-beam interaction, the initial bunch distributions match exactly with the lattices. The betatron tunes are \((\nu_{e,x}, \nu_{e,y}) = (26.104, 22.146)\) for the e beam and \((\nu_{p,x}, \nu_{p,y}) = (28.19, 29.18)\) for the p beam.

BEAM-BEAM LIMIT OF THE ERHIC

In the ZDR design [1] of the eRHIC, the bunch intensity of the e beam \((N_{e0})\) and the p beam \((N_{p0})\) are \(N_{e0} = N_{p0} = 1.0 \times 10^{11}\), which corresponds to beam-beam parameters of \((\xi_{e,x}, \xi_{e,y}) = (0.029, 0.08)(ebeam)\) and \((\xi_{p,x}, \xi_{p,y}) = (0.0065, 0.003)\) for the e and p beam, respectively. With the design parameters and under the normal operation condition in which each e bunch collides with three p bunches during one revolution of the p beam, no beam-beam instability has been observed in this study and the beam-beam interaction only results in a limited (\(\sim 10\%)\) reduction of the luminosity. In Figs. 1 and 2, the evolution of four-dimensional emittances \((\epsilon_{x} + \epsilon_{y})\) of both the beams were calculated with different \(N_{p}\) while \(N_{e} = N_{e0}\) (Fig. 1) or calculated with different \(N_{e}\) while \(N_{p} = N_{p0}\) (Fig. 2), where \(N_{p}\) and \(N_{e}\) are the p and e bunch intensity used in the simulation. When \(N_{p} \sim N_{p0}\) and \(N_{e} < 2.5N_{e0}\) (case a and b in Figs. 1 and 2), the emittance growth of the p beam is insignificant and the e beam reaches a quasi-stationary state after a fast short-term emittance growth that occurs within a few damping time of the e beam. The fast short-term emittance growth of the e beam is mainly due to the nonlinear beam filamentation in a severely distorted phase space because of the strong beam-beam perturbation. The limited emittance growth of the e beam results only in a small (\(< 10\%)\) luminosity reduction. When \(N_{p} \geq 1.25N_{p0}\) or \(N_{e} \geq 2.5N_{e0}\), on the other hand, a significant emittance growth of the p beam and dynamical
emittance variation of the e beam are due to the onset of the coherent beam-beam instability [3]. The threshold of the onset of the coherent beam-beam instability is therefore around $1.25N_{p0}$ for the p bunch intensity and $2.5N_{e0}$ for the e bunch intensity in the eRHIC. These thresholds in bunch intensity corresponds to the critical beam-beam parameter of $\xi_c \sim 0.1$ for the e beam and $\xi_c \sim 0.015$ for the p beam. A study of beam-beam tune spreads of the two beams indicates that the onset of the coherent beam-beam instability in the eRHIC is related to the 6th-order resonances as shown in Fig. 3. When the beam-beam parameters are below $\xi_c$, the e beam crosses individual (isolated) 6th-order resonance lines (see Fig. 3a) while the p beam is free of even order (beam-beam) resonances of the 10th-order or lower. The isolated 6th-order resonances of the e beam in this case are not strong enough to induce the instability. In the case that the beam-beam parameter of the e beam exceeds its $\xi_c$, the core of the e beam crosses an intersection of two 6th-order and several 8th- and 10th-order resonance lines (see Fig. 3b). For such the intersecting resonances, strongly enhanced chaotic diffusion can occur. The coherent beam-beam instability in Fig. 1 could therefore be due to these coupled resonances of the e beam. In the case that the beam-beam parameter of the p beam exceeds its $\xi_c$, the core of the p beam also crosses a group of 10th-order resonances (see Fig. 3d). The coherent beam-beam instability in this case could be due to the coupling between resonances of the two beams. Avoiding those beam-beam resonances, if possible, by changing working points could increase the threshold of the beam-beam instability.

It should be noted that the beam-beam parameters of the e beam is more than 10 times larger than that of the p beam in the eRHIC and, therefore, the beam-beam interaction is traditionally considered as a strong-weak case. The coherent beam-beam instability in the strong-weak case of beam-beam interactions was first studied numerically [4, 5] and later observed experimentally [6] on HERA. This is another example of strong coherent beam-beam effect in the strong-weak case of beam-beam interactions.

**COHERENT BEAM-BEAM INSTABILITY DUE TO MISSING COLLISIONS**

In the normal operation of the eRHIC, each e bunch will collide with three different p bunches during each revolution of the p beam. If there are missing p bunches, the corresponding e bunches could miss one or two collisions during every three revolutions of the e beam. Such the “modulated” beam-beam perturbation can induce additional resonances that can further enhance the coherent beam-beam effect. In both the cases of missing one or two collisions, higher-order coherent beam-beam instability with spontaneous beam-size oscillation has been observed at the nominal design bunch intensities and the onset of the beam-beam instability results in a severe reduction (> 60%) of the luminosity of the considered bunch. Moreover, the on-
Figure 3: Beam-beam tune spread of the $e$ beam for (a) case b in Fig. 1 as $N_e = N_{e0}$ and $N_p = N_{p0}$, (b) case d in Fig. 1 as $N_e = N_{e0}$ and $N_p = 1.5N_{p0}$, and (c) case e in Fig. 2 as $N_e = 3.5N_{e0}$ and $N_p = N_{p0}$. (d) The same as (c) but of the $p$ beam. Solid lines are the even-order resonances up to the 10th order and + indicates the lattice tune.

set of the coherent beam-beam instability due to the modulated beam-beam perturbation can occur at much lower bunch intensities. Figure 4 plots an example of the spontaneous beam-size oscillation of the $e$ beam due to the missing of one $p$ bunch. Note that the period of beam-size oscillation is three turns which is the same as the modulation period of the beam-beam perturbation. Due to the onset of the beam-beam instability, the closed orbits could also become unstable for beam centroids and the beams could develop a spontaneous chaotic off-center oscillation as shown in Fig. 5. For a comparison, a stable case of the coherent motion was also plotted in Fig. 5a. Due to the onset of the coherent beam-beam instability, the emittance growth of the $p$ beam is significantly enhanced as shown in Fig. 4b. To avoid an increase of background due to the increase of the tail particles, in the case of missing $p$ bunches the number of $e$ bunches has to be reduced accordingly.

**SUMMARY**

The study shows that under the design parameters no beam-beam instability has been observed and the beam-beam interaction results only in a very limited luminosity reduction. The proposed beam-beam parameter of the $e$ beam is, however, a little too close to the beam-beam threshold to be comfortable. On the other hand, for the dedicated single IP mode of the eRHIC, there is still a possibility to further increase the $e$ bunch intensity. In order to have a comfortable margin to the beam-beam limit and to meet or exceed the design luminosity, the option of reducing the $p$ bunch intensity and increasing the $e$ bunch intensity should be further studied.

**REFERENCES**