GLOBAL-BETA MEASUREMENT AND CORRECTION
AT THE KEKB RINGS

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Abstract

The global-beta correction is one of the optics corrections such as dispersion and xy-coupling corrections to realize a model lattice [1]. In the KEKB B-Factory, it is performed to regularize the ring optics for achieving high luminosity. The global-beta function is measured by reconstructing responses of orbits generated by 6 different steering magnets per plane. The correction of the beta function and the phase advance is performed by using the fudge factors of power supplies of quadrupole magnets. This correction scheme has been successfully. In the typical case, the rms of the beta function beat and the betatron tune difference are corrected within 5% and 0.0005, respectively. In the luminosity run, we can operate the low energy ring (LER) with the horizontal betatron tune very close to half-integer (45.5050). In this paper, we will report the calculation of the closed orbit distortions (CODs) generated by the steering magnets per plane. The correction of the beta function and the phase advance is performed by using the Newton-Raphson method with the steering kick. From the first order perturbation theory, the normalized orbit distortion is given as

\[ \chi_s' = \frac{\sqrt{\beta_X(s)}}{2 \sin \pi \nu} \int_C \Delta \chi(s') \sqrt{\beta_X(s')} \cos (|\phi_X(s) - \phi_X(s')| - \pi \nu) ds', \] (1)

where \( \Delta \chi(s') \) is the distribution of the \( \chi \) direction kick and \( \int_C \) means the integration over one revolution of the ring. The direction symbol \( \chi \) represents either \( x \) or \( y \) direction.

In the KEKB ring, we can observe both the betatron tune \( \nu_X \) and the COD \( \chi(s) \) at the beam position monitors (BPMs). Assuming that the steering does not overlap with the BPMs, the COD at \( i \)-th BPM generated by \( j \)-th steering is obtained as

\[ \Delta \chi_i^j = \frac{\Delta \theta_j}{2 \sin \pi \nu} \sqrt{\beta_i^M} \sqrt{\beta_j^S} \cos (|\phi_i^M - \phi_j^S| - \pi \nu). \] (2)

For convenience, we introduce following notations:

\[ f_j = \frac{\Delta \theta_j}{2 \sin \pi \nu}, \quad S_{ij} = \text{sign} (\phi_i^M - \phi_j^S), \] (3)

\[ X_i^l = \sqrt{\beta_i^M} \cos \phi_i^M, \quad Y_i^l = \sqrt{\beta_i^M} \sin \phi_i^M \quad (l = M, S). \] (4)

Using these notations, Eq. 2 is rewritten as follows:

\[ \Delta \chi_i^j = (\cos \pi \nu f_j X_i^S - S_{ij} \sin \pi \nu f_j Y_i^S) X_i^M \\
+ (\cos \pi \nu f_j Y_i^S + S_{ij} \sin \pi \nu f_j X_i^S) Y_i^M \] (5)

\[ = (\cos \pi \nu X_j^M + S_{ij} \sin \pi \nu Y_j^M) f_j X_i^S \\
+ (\cos \pi \nu Y_j^M - S_{ij} \sin \pi \nu X_j^M) f_j Y_i^S. \] (6)

To obtain the beta function, we fit Eq. 5-6 to the set of the measured closed orbits by using the least square method. The residual error sum of squares looks like the quadratic form of either \( \{X_i^M, Y_i^M\} \) or \( \{X_i^S, Y_i^S\} \) coordinates. Thus, the residual error is easily improved by interlacing two types of the linear fit as follows. At the first step, the temporary \( \{X_i^M, Y_i^M\} \) is given by using the model optics adjusted with the measured betatron tune. At the second step, \( \{X_j^S, Y_j^S\} \) is calculated from given \( \{X_i^M, Y_i^M\} \) by using the least square fit against Eq. 6. At the third step, improved \( \{X_i^M, Y_i^M\} \) is obtained from calculated \( \{X_j^S, Y_j^S\} \) by using the least square fit against Eq. 5. Both second and third step are iterated until the residual error sum is converged. To stabilize the iteration method, the normalization of both \( \beta(s) \) and \( \phi(s) \) is required, because the fitting residual of Eq. 2 is only determined by both the beta function product and the betatron phase advance difference. In order to normalize \( \{X_i, Y_i\} \) after each fitting step to keep absolute norm: \( \sum_i (|X_i| + |Y_i|) \).

This iteration method should be converged because of monotone decreasing of the residual error sum of squares, but the convergence speed is very slow. The minimum point obtained as the limit of our iteration method is the fixed point of the mapping defined by one iteration loop. In the practical use, we use the Newton-Raphson method with the numerical differential to find the fixed point of the mapping from \( \{X_j^S, Y_j^S\} \) to \( \{X_i^S, Y_i^S\} \), because the length of the steering side vector is shorter than the length of the BPM side vector.

GLOBAL-BETA CORRECTION

Assuming that the measured optics distortion is caused by the gradient field error of the quadrupole field, we try to correct the optics distortion by tuning the quadrupole field using the amplitude fudge factor (AF) of the quadrupole.
magnet power supply. Although the major source of the optical deformation is the alignment offsets of the sextupoles, they can be correctable by the nearby quadrupole settings. The beta function distortion by the gradient field error is given as

$$\frac{\Delta \beta_x(s)}{\beta_x(s)} = \frac{1}{2 \sin 2 \pi \chi} \int_C \frac{\Delta K_x(s')/\beta_x(s') \cos(2(\phi_x(s) - \phi_x(s')) - 2\pi \nu_x)}{ds'}$$

where \(\Delta K_x(s')\) is an error distribution of gradient field in the \(x\) plane. In the term of the quadrupole field strength \((K_1 = \frac{1}{2\pi} \frac{\partial^2 B_y}{\partial x^2})\), \(\Delta K_x(s')\) and \(\Delta K_y(s')\) are described as \(\Delta K_1(s')\) and \(\Delta K_3(s')\), respectively. The phase modulation and the tune shift are given by following integral forms

$$\Delta \phi_x(s) = \int_0^s \left( \frac{1}{\beta_x(s') + \Delta \beta_x(s')} - \frac{1}{\beta_x(s')} \right) ds'$$

$$= \frac{1}{2 \sin 2 \pi \chi} \int_C \frac{\Delta K_x(s')/\beta_x(s')}{ds'}$$

\(\{2 \sin 2 \pi \chi \sin^2 \phi_x(s) - \phi_x(s'), 0\} + \sin \phi_x(s) \cos(2\phi_x(s') - \phi_x(s) - 2\pi \nu_x)\}$$

$$\Delta \nu_x = \frac{1}{4\pi} \int_C \Delta K_x(s')/\beta_x(s') ds'.$$

The perturbation of the gradient field is described by using the amplitude fudge factor for \(i\)-th quadrupole family \((AF^i)\) as follows

$$\Delta K_1^i(s) = K_1^i(s) \Delta AF^i, \quad \Delta AF^i = AF^i - 1,$$

where \(K_1^i(s)\) is the designed field distribution of \(i\)-th quadrupole. Integrating Eq. 7-9 under Eq. 10, the response matrix from the quadrupole family \(\Delta AF^i\) to the optics functions \(\Delta \beta(s)/\beta(s), \Delta \phi(s)\) and \(\Delta \nu\) is obtained. In order to determine the correction fudge factor, we minimize the following residual error sum of squares constructed by the beta function, phase advance and betatron tune of the model optics and the measurement

$$\epsilon^2 = \left(\frac{\pi}{\sin 2\pi \nu}\right)^2 \left| \Delta \nu - (\nu^{\text{measured}} - \nu^{\text{model}}) \right|^2$$

$$+ \sum \left( \frac{\Delta \beta(s)}{\beta(s)} - \frac{\beta^{\text{measured}}(s) - \beta^{\text{model}}(s)}{\beta^{\text{model}}(s)} \right)^2$$

$$+ \left| \Delta \phi(s) - (\phi^{\text{measured}}(s) - \phi^{\text{model}}(s)) \right|^2,$$

where \(\sum\) means the sum over the measured points. The fudge factor \(\Delta AF^i\) minimizing Eq. 11 is obtained by the Singular Value Decomposition(SVD) of the response matrix generated from Eq. 7-10. The obtained fudge factor shows the correction factor for fitting the model optics to the measured one. Thus, the optics error can be compensated by updating the fudge factor of the quadrupole magnet power supply using following procedure

$$AF^{i}_{\text{new}} = \frac{AF^i_{\text{old}}}{1 + \Delta AF^i},$$

In Eq. 11, the equal weighting is applied to the individual residual error squares. If the variance of the measured value is known, the individual residual error square in Eq. 11 should be multiplied by the inverse square of the standard deviation. In the actual use, the extra weighting factor is applied in order to correct preferentially the specific optics parameter (ex. betatron tune, beta function at interaction region).

For using the global-beta measurement described in the previous section, we must pay attention to the measurement uncertainty. Our measurement can not determine both the scaling factor of the beta function and the origin of the phase advance, because Eq. 2 is conserved by either the phase origin change or the beta function scale change keeping \(\beta_p^M \beta_p^S\). If the optics distortion is enough small, such uncertainty is canceled out by normalization using model optics. In another case, this uncertainty problem can be avoided by either introducing the fudge factor for the beta amplitude and the phase origin to Eq. 2 as the unknown quantity or constructing the residual error sum of squares from the relative value between the measurement positions.

**MEASUREMENT AND CORRECTION AT THE KEKB RINGS**

In our site, the global-beta measurement and correction method shown in the previous sections are implemented as an automation tool on SAD[2], which has the script interpreter including accelerator model, EPICS[3] channel access and GUI toolkit based on Tcl/Tk[4]. Using such automation tool, we correct the ring optics after every regular maintenance whose interval is normally two weeks. In the following subsections, we present the detail of our implementation and the performance of our correction system.

**Measurement at the KEKB Rings**

In the actual measurement at the KEKB rings, we use 6 steerings for the horizontal beta measurement and 6 steerings for the vertical beta measurement. For avoiding the degeneration of the measured COD set, the betatron phases of the steering magnets are spread out. We exclude the horizontal steering among the dispersive section for avoiding the momentum shift. As the KEKB proper condition, the 2.5\(\pi\) cell structure with non-interleaved chromaticity correction is used for arc cells[5]. In this structure, the sextupole pairs are placed without overlapping. The transfer matrix between sextupole pair is adjusted to \(-I^\prime\), which is a negative identical 4 by 4 transformation except the \(-I^\prime_{12}\) and \(-I^\prime_{34}\) components, in order to cancel out the quadrupole field generation by the closed orbit distortion propagated from outside of the sextupole pair. Therefore, we can not use the steering magnet between the sextupole pair for the measurement, because the single steering kick between the sextupole pair makes the serious tune shift and the optics distortion.

For keeping the amplitude of the COD, the steering kick
angle is scaled by the square root of the beta function at the steering magnet. The typical kick angle for the COD measurement is about $40\mu\text{rad}$ for 30m beta function. The typical COD amplitude at the arc section is about 0.5mm. The iteration of the Newton-Raphson method in the beta function fitting is normally converged until 4 iterations. The typical rms of the residual error of the COD fitting is about $1 \sim 2\mu\text{m}$ except the vertical measurement of the High Energy Ring (HER). The typical rms of the vertical fitting residual at the HER is about $4\mu\text{m}$.

**Correction at the KEKB Rings**

In our global-beta correction, the quadrupoles related with $-I'$ arc cell condition and the quadrupole at the injection point are excluded from the corrector quadrupoles. In the usual correction, we apply 10 times extra weighting for tune errors in Eq. 11 and 0.05 is used as the relative tolerance for truncating small singular values of the SVD of the response matrix. In the usual case such as a correction after regular maintenances, the beta correction is normally converged until 3 iterations. After corrections, the typical residual error of the beta function $\Delta\beta/\beta$ is about 5% in rms and the betatron tune shift exceeding $5 \times 10^{-3}$ is observed. At the measurement after two global-beta correction, the relative error of the beta function is converged into 5% in rms and the betatron tune shift is corrected into $5 \times 10^{-4}$. The interval between these two measurement is about 24 minutes that includes a XY coupling correction and a dispersion correction procedure.

**SUMMARY**

The global-beta measurement and correction presented in previous sections is used as a part of the optics correction of the KEKB ring startup process. It is very helpful to operate the KEKB rings nearby the half-integer resonance line.

**REFERENCES**