OBSERVATION OF SUPERPOSITION OF WAKE FIELDS GENERATED BY ELECTRON BUNCHES IN A DIELECTRIC-LINED WAVEGUIDE*

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Abstract
We report results from an experiment, done at the Accelerator Test Facility, Brookhaven National Laboratory, which demonstrates the successful superposition of wake fields excited by 50MeV bunches which travel ~50cm along the axis of a cylindrical waveguide which is lined with alumina. Wake fields from two short (5-6ps) 0.15–0.35nC bunches are superimposed and the energy losses of each bunch are measured as the separation between the bunches is varied so as to encompass approximately one wake field period (~21cm). A spectrum of 40 TM_{0n} eigenmodes is excited by the bunch. A substantial retarding wake field (2.65MV/m·nC for just the first bunch) is developed because of the short bunches and the narrow vacuum channel diameter (3mm) through which they move. The energy loss of the second bunch exhibits a narrow resonance with a 4mm (13.5ps) footprint. This experiment may be compared with a related experiment reported by a group at the Argonne National Laboratory [1]. In their case, the wake field is excited by relatively long bunches (rms length ~9mm) in a ∅10mm channel, consists of ~10 eigenmodes, and has an E_{z} – field with a broad footprint (2σ_{wake} ~22mm) but a low amplitude (decelerating wake field after one bunch ~32kV/m·nC). In contrast, the experiment conducted at ATF on which we report here generates a wake field having ~40 modes and a decelerating field after one bunch ~980kV/m·nC. The high axial E_{z} – field with a narrow footprint (2σ_{wake} ~4mm) is achieved because of narrow bunches (rms length ~1.8mm) and a narrow channel (∅3mm). Our observation technique is different from that used in the Argonne experiment, where the measured energy spectrum of drive bunches was compared with a computed energy spectrum while the bunch spacing was fixed and equal to the wake field period. In our case, the bunch spacing is changed, and the difference in energy losses between the second and first bunches is observed, which allows one to verify agreement between theory and experiment when the bunch spacing differs from the wake field period.

INTRODUCTION
Wake field accelerators are generally attractive because no external source of energy is used in the structure itself. In 1999, some new refinements were proposed to boost the acceleration gradient in dielectric-lined waveguides [2]. Comparatively short drive bunches were suggested, so that excitation of a large number of high-amplitude TM_{0n} eigenmodes would form a high-amplitude wake field. The waveguide design [3] would be chosen so that the wake field is nearly periodic, with the same period as that of the train of drive bunches. If several drive bunches can be used to excite a DWA, the E_{z} – field at the test bunch (a bunch that undergoes acceleration) location will have a much higher amplitude than that which would be set up by only one bunch. In this acceleration scheme, all drive bunches must radiate coherently so that every consecutive drive bunch enhances the wake field produced by the drive bunches which have preceded it; in this case a constructive superposition of wake fields generated by the drive bunches occurs. Depending on

THEORY OVERVIEW
We use a cylindrical dielectric structure to test the physical principles. The theory has been thoroughly developed in recent years [4-7]. The wake field is assumed to be excited by passage of a train of N driving bunches moving with the velocity v=cβ along the axis (z-direction) in an infinite structure. There is vacuum in the region r < A, a dielectric material with dielectric constant ε everywhere between A < r < R, and a metal in the region r > R, where A is the inner radius and R is the outer radius. In cylindrical geometry, the wake field can be described as a superposition of orthonormal wave functions which separate into TE and TM classes for
Every drive bunch loses a very specific amount of energy per unit of length. For the first bunch (with the rms length $\Delta z = 2\sqrt{\epsilon z^2 - z^2}$ and charge $Q_1$), the energy loss is

$$W(1)/Q_1 = (4\pi e_0 A^2)^{-1} \sum_{n} \Omega(n, \Delta z),$$

where the coefficient $\Omega(n, \Delta z)$ weakly depends on the bunch shape [8]. (Here $\omega_n$ is the $n$th eigen-frequency that satisfies the dispersion relationship given in [2].) The bunch energy loss is directly proportional to the bunch charge. For the second bunch, the energy loss is

$$W(2)/Q_2 = \frac{W(1)}{Q_1} \left(1 + 2 \frac{Q_1}{Q_2} \sum_{n} \Omega(n, \Delta z) \cos(\omega_n S_{12}/c\beta)\right),$$

where $W(1)/Q_1$ is given above, $S_{12}$ is the distance between bunches (bunch spacing), and we assume that both bunches have the same rms-length. The behaviour of $W(2)/Q_2$ vs. the bunch spacing is shown in Fig. 1. It always has a resonance-like character in the vicinity of $JL/2$ (where $L$ is the wake-field period and $J$ is any integer), which can be used as an indication of the constructive wake field superposition happening when $S_{12} \rightarrow L$ [see Fig. 2].

In an experiment, it is convenient to use the first bunch as a reference bunch, because its energy loss does not depend on the bunch spacing, and measure the difference $W(N) - W(1)$ vs. the bunch spacing. Because usually $Q_N \approx \cdots \approx Q_2 \approx Q_1$ and $\Delta z_N \approx \cdots \approx \Delta z_2 \approx \Delta z_1$, this difference must also be positive whenever the constructive superposition of fields occurs and must demonstrate the resonant-like behaviour when the bunch spacing $S_{12}$ is varied in the vicinity of the wake-field period, $L$.

**EXPERIMENTAL STUDIES**

This experiment has been run on beam-line #2 at ATF. Figure 3 shows the experimental setup.

HeNe-laser light is used to establish the apparatus axis. The optical beam is focused in the middle plane of the DWA, and has waist size at the entrance and exit of the DWA $w \approx 2.4/3$ ($A = 1.5\text{mm}$ is the inner radius of the vacuum channel), so that clear transmission occurs only if the light beam propagates along the DWA axis. The electron bunch path is aligned with the HeNe-light before the bunch enters the DWA. For the experiment at ATF, numerical simulations show that an electron bunch propagating near the apparatus axis is negligibly affected by the transverse wake fields [8]. Upon leaving the DWA, this bunch must have the same transverse size as a bunch that passes through the DWA in a free-space approximation. Diagnostics available at ATF permit measuring the emittances (horizontal and vertical) and initial Twiss parameters so that the transverse bunch sizes in the free-space approximation can be calculated at any point along the beam-line #2. The measurement of transverse size after the bunch leaves the DWA demonstrates excellent agreement between the measured and expected values (usually $\sigma_x \approx 250-300\mu\text{m}$) as soon as the bunch is aligned, and serves as a criterion to confirm that the bunch deviates negligibly from the apparatus axis. Diagnostic information, such as bunch charge and RMS-length, is obtained routinely from the permanent ATF facility hardware available to all users.

In the experiment, the first bunch is used as a reference bunch. A part of the transport line allows
measuring the initial energy difference between bunches, while the beam position monitor installed after the dipole #2 [see Fig.3] is used to measure the final energy difference. Thus, the difference in energy losses, $W(N) – W(1)$ caused by interaction with the wake field in the DWA is determined.

To produce several electron bunches, the single laser light pulse (FWHM ≈ 7 psec) delivered to the ATF photocathode gun with the repetition rate 1.5Hz is split into several pulses with the separation between them close to 700 psec (21 cm). The bunch spacing $S_{12}$ is varied by changing the laser pulse spacing. To connect $S_{12}$ with the laser pulse spacing, one constructs the phase-energy space mapping [9], and convolves it with the initial longitudinal bunch distribution at the gun cathode. The mapping depends upon the gun maximum electric field, the LINAC electric field that is set to achieve the final bunch energy of 50 MeV, the gun phase, and the corresponding LINAC phase which minimizes the energy spread. The mapping should be applied to bunches with a relatively low charge. The criterion of validity of the mapping is that the calculated bunch RMS-length (and/or FWHM) is, within the measurement accuracy (7-11% depending on the RMS-length), the same as the measured RMS-length [8].

Enough data have been collected already to demonstrate the resonant-like behaviour of $W(2) – W(1)$ vs. the bunch spacing.

![Figure 4](image-url)  
Figure 4: Measured difference in energy losses (marked by bars; normalized per 1 m) between the 2nd and 1st bunches, $W(2) – W(1)$, vs. the bunch spacing. Both bunches have the same rms-length. The bunch charges are shown in Fig.5. The solid line represents the best theoretical fit which occurs if the wake period is taken to be either $L = 700.28\text{psec} + \Delta L = 700.28\text{psec} + 3.6\text{psec}$ or $L = 700.28\text{psec} + \Delta L = 700.28\text{psec} + 3.7\text{psec}$ respectively. $\Delta L$ is determined with an accuracy ~10% (i.e. the wake field period is determined with an accuracy ± 0.38 psec ≈ ± 115 μm). The solid curve has a piece-wise character because the bunch charges change from one experimental point to another [see Fig. 5.a-b]. From the independent frequency measurement [3,10], one finds that the wake period for the DWA at ATF is $L = 700.28\text{psec} + \Delta L = 700.28\text{psec} + 4.08\text{psec}$, where $\Delta L$ is determined with an accuracy ±10%. That agrees with previously shown data within the measurement accuracy.

Thus we have found that the data presented are fully understood by the theory, and consequently, demonstrate that constructive superposition of wake fields occurs as expected.

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**REFERENCES**