OVERVIEW OF IMPEDANCE AND SINGLE-BEAM INSTABILITY MECHANISMS

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Abstract

The transverse resistive-wall impedance is discussed in the particular case of the LHC collimators, which reveal a new physical regime. Single-bunch and coupled-bunch instability mechanisms are then reviewed in both longitudinal and transverse planes. Stabilization by Landau damping, feedbacks, or linear coupling between the transverse planes is also treated.

INTRODUCTION

As the beam intensity increases, the electromagnetic fields self-generated by the beam, particularly the fields generated by the beam interacting with its surroundings, will perturb the external fields prescribed by the accelerator design, which is made considering the beam as a collection of noninteracting single particles [1,2]. These electromagnetic fields are called wake fields since they remain usually behind the (ultra-relativistic) exciting particles. These wake fields can influence the motion of trailing particles, in the longitudinal and in one or both transverse directions, leading to energy loss, beam instabilities, or producing undesirable secondary effects such as excessive heating of sensitive components at or near the chamber wall. Therefore, in addition to the "single-particle phenomena", "collective effects" become important. In fact, for a collective instability to occur, the beam must not be ultra-relativistic, or its environment must not be a perfectly conducting smooth pipe. This is never the case in practice due to the complexity of the vacuum vessel, which is always composed of expansion bellows, connection flanges, pumping ports, accelerating RF cavities, monitors, kickers, collimators (highly resistive graphite will be used in the LHC to withstand the high temperatures generated by the impact of high-energy protons), etc. Therefore, in practice the elements of the vacuum chamber should be designed to minimise the self-generated electromagnetic fields. For example, chambers with different cross-sections should be connected with tapered transitions; bellows need to be separated from the beam by shielding; plates should be grounded or terminated to avoid reflections, etc.

Two approaches are usually used to deal with collective instabilities. One starts from the single-particle equation while the other solves the Vlasov equation, which is nothing else but an expression for the Liouville conservation of phase-space density seen by a stationary observer. In the second approach, the motion of the beam is described by a superposition of modes, rather than a collection of individual particles.

The first formalism was used by Courant and Sessler to describe the transverse coupled-bunch instabilities, extending the theory developed by Laslett, Neil and Sessler for continuous beams. Courant and Sessler studied the case of rigid (point-like) bunches, i.e. bunches oscillating as rigid units, and they showed that the transverse electromagnetic coupling of bunches of particles with each other can lead (due to the effect of imperfectly conducting vacuum chamber walls) to a coherent instability. The physical basis of the instability is that in a resistive vacuum tank, fields due to a particle decay only very slowly in time after the particle has left (long-range interaction). The decay can be so slow that when a bunch returns after one (or more) revolutions it is subject to its own residual wake field which, depending upon its phase relative to the wake field, can lead to damped or anti-damped transverse motion. For M equi-populated equi-spaced bunches, M coupled-bunch mode numbers exist ($n = 0, 1, ..., M-1$), characterized by the integer number of waves of the coherent motion around the ring. The bunch-to-bunch phase shift $\Delta\phi$ is related to the coupled-bunch mode number n by $\Delta\phi = 2\pi n / M$. Pellegrini and, independently, Sands then showed that short-range wake fields (i.e. fields that provide an interaction between the particles of a bunch but have a negligible effect on subsequent passages of the bunch or of other bunches in the beam) together with the internal circulation of the particles in a bunch can cause internal coherent modes within the bunch to become unstable. The important point here is that the betatron phase advance per unit of time (or betatron frequency) of a particle depends on its instantaneous momentum deviation (from the ideal momentum) in first order through the chromaticity and the slippage factor. The betatron phase varies linearly along the bunch (from the head) and attains its maximum value at the tail. The total betatron phase shift between head and tail is the physical origin of the head tail instability. A new (within-bunch) mode number $m = ..., -1, 0, 1, ...$, also called head-tail mode number, was introduced. This mode describes the number of betatron wavelengths (with sign) per synchrotron period. It can be obtained by superimposing several traces of the directly observable average displacement along the bunch at a particular pick-up. The number of nodes gives the mode number $|m|$.

The work of Courant and Sessler, or Pellegrini and Sands, was done for particular impedances and oscillation modes. Using the Vlasov formalism, Sacherer unified the two previous approaches, introducing a third mode number $q = ..., -1, 0, 1, ...$, called radial mode number,

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which comes from the distribution of synchrotron oscillation amplitudes. The advantage of this formalism is that it is valid for generic impedances and any high order head-tail modes. This approach starts from a distribution of particles (split into two different parts, a stationary distribution and a perturbation), on which Liouville theorem is applied. After linearization of the Vlasov equation, one ends up with Sacherer's integral equation or Laclare's eigenvalue problem to be solved. Because there are two degrees of freedom (phase and amplitude), the general solution for the single-bunch motion is a twofold infinity of coherent modes of oscillation ($m, q = ..., -1, 0, 1, ...$). For protons a parabolic density distribution is generally assumed, and the corresponding oscillation modes are sinusoidal. For electrons, the distribution is usually Gaussian, and the oscillation modes are described in this case by Hermite polynomials. In reality, the oscillation modes depend both on the distribution function and the impedance, and can only be found numerically by solving the (infinite) eigenvalue problem. However, the mode frequencies are not very sensitive to the accuracy of the eigenfunctions. Similar results are obtained for the longitudinal plane.

The part of this paper discussing the beam instability mechanisms is a summary of Ref. [3] where more details are given (analytical predictions, benchmarking with some instability codes, and experimental results). Many references, which I will not quote again here for space considerations, are given there.

IMPEDANCE

The Fourier transform of the wake field is called the impedance. The idea of representing the accelerator environment by an impedance was introduced by Sessler and Vaccaro [4]. As the conductivity, permittivity and permeability of a material depend in general on frequency, it is usually better (or easier) to treat the problem in the frequency domain.

Both longitudinal and transverse resistive-wall impedances were already calculated forty years ago by Laslett, Neil and Sessler. However, a new physical regime is revealed by the LHC collimators. Small aperture paired with large wall thickness ask for a different physical picture of the transverse resistive-wall effect than the classical one. The first unstable betatron line in the LHC is around 8 kHz, where the skin depth for graphite (whose measured isotropic DC resistivity is 10 µΩm) is 1.8 cm. It is smaller than the collimator thickness of 2.5 cm. Hence one could think that the resistive thick-wall formula would be about right. In fact it is not, as will be shown below. The resistive impedance is about two orders of magnitude lower at this frequency.

A number of papers have been published recently on this subject, and are discussed in Ref. [5]. Zotter's new result for the transverse resistive-wall impedance deduced from field matching [6] is compared to the result from Burov-Lebedev [7] in Fig. 1. A very good agreement is obtained between the two, without and with copper coating. In fact Zotter's formalism unifies the approach of Burov-Lebedev for "low frequencies" (they made the approximation $\omega << c / b$) and the approach of Bane for high frequencies [8], and it is also valid for any beam velocity (see Fig. 2). Note that for a flat chamber, Yokoya's factors have to be used in both transverse planes, i.e. $\pi^2 / 12 \approx 0.8$ in vertical and $\pi^2 / 24 \approx 0.4$ in horizontal for a vertically flat chamber [9].

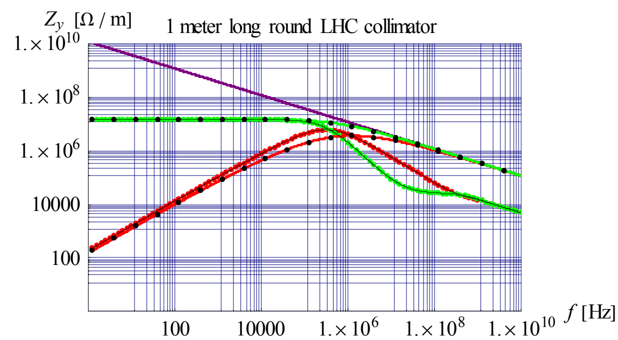

Figure 1: Comparison between Zotter's and Burov-Lebedev's formalisms in the case of a 1m long round (flat in reality) LHC graphite collimator with a half gap of 2 mm. Burov-Lebedev's plots are in black: dots without and lines with copper coating (5 µm, with a resistivity of 17 nΩm). The thickness of graphite is assumed to be infinite here. The real part of the impedance tends to zero at low frequency, while the imaginary part tends to a constant value. The classical "thick-wall" formula is plotted in purple (equal real and imaginary parts).

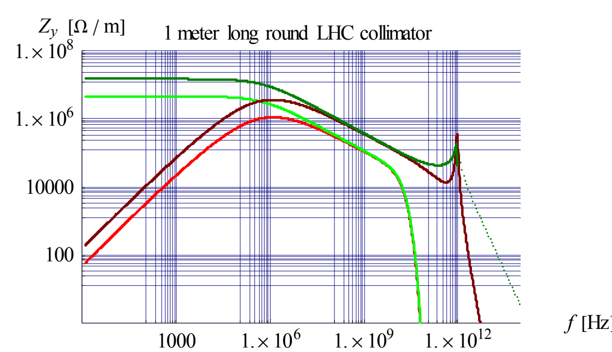

Figure 2: Zotter's results for the LHC, where $\gamma = 7462.69$ and $\beta = 1$, and for the CERN PSB, where $\gamma = 1.05$ and $\beta = 0.3$, to see the effect of a lower beam velocity. An AC conductivity is assumed here, $\sigma_{AC} = \sigma_{DC} / (1 + j\,\omega\,\tau)$, where $\tau \approx 0.8$ ps is the relaxation time. The high-frequency resonance near 1 THz is in perfect agreement with Bane's results [8] (the dots are used for the negative imaginary part of the impedance after the resonance). The curves for the lower beam velocity are in light green for the imaginary part of the impedance and in red for the real part.

TRANSVERSE INSTABILITIES

Low Intensity

At low intensity (i.e. below the intensity threshold discussed in the next section), the standing-wave patterns (head-tail modes) are treated independently. This leads to instabilities where the head and the tail of the bunch exchange their roles (due to synchrotron oscillation) several times during the rise-time of the instability. The complex transverse coherent betatron frequency shift of bunched-beam modes is given by Sacherer's formula for round pipes. For flat chambers a quadrupolar effect has to be added to obtain the real part of the coherent tune shift, which explains why the horizontal coherent tune shift is zero in vertically flat chambers. As an example, a head-tail instability with mode $|m| = 10$ is shown in Fig. 3.

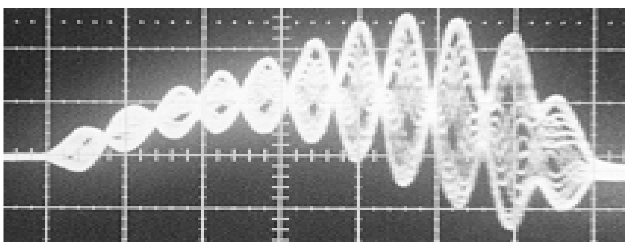

Figure 3: Signal from a radial beam position monitor during 20 consecutive turns observed in the CERN PS at 1.4 GeV kinetic energy in 1999. Time scale: 20 ns/div.

High Intensity

As the bunch intensity increases, the different head-tail modes can no longer be treated separately. In this regime, the wake fields couple the modes together and a wave pattern travelling along the bunch is created: this is the Transverse Mode Coupling Instability (TMCI). The TMCI for circular accelerators has been first described by Kohaupt in terms of coupling of Sacherer's head-tail modes. This extended to the transverse motion, the theory proposed by Sacherer to explain the longitudinal microwave instability through coupling of the longitudinal coherent bunch modes. The TMCI is the manifestation in synchrotrons of the Beam Break-Up (BBU) mechanism observed in linacs. The only difference comes from the synchrotron oscillation, which stabilizes the beam in synchrotrons below a threshold intensity by swapping the head and the tail continuously. In fact, several analytical formalisms exist for fast (compared to the synchrotron period) instabilities, but the same formula is obtained (within a factor smaller than two) from five, seemingly diverse, formalisms in the case of a Broad-Band (BB) resonator impedance ($Q_r = 1$): (i) Coasting-beam approach with peak values, (ii) Fast blow-up, (iii) BBU (for 0 chromaticity), (iv) Post head-tail, and (v) TMCI with 2 modes in the "long-bunch" regime (for 0 chromaticity). Two regimes are indeed possible for the TMCI according to whether the total bunch length is larger or smaller than the inverse of twice the resonance frequency of the impedance. The intensity threshold can be increased in the "long-bunch" regime by increasing the longitudinal emittance and/or the chromaticity. The coherent synchrobetatron resonances, important in large machines, are not discussed here.

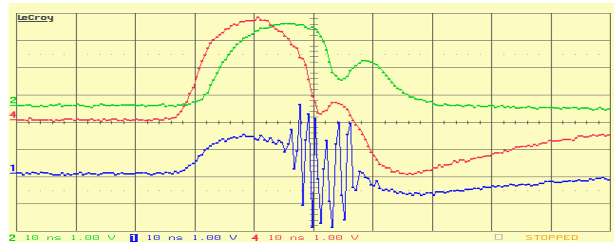

Figure 4: Fast instability observed in the CERN PS near transition (~6 GeV total energy) in 2000. Single-turn signals from a wide-band pick-up. From top to bottom: Σ, Δx, and Δy. Time scale: 10 ns/div. The head of the bunch is stable and only the tail is unstable in the vertical plane. The particles lost at the tail of the bunch can be seen from the hollow in the bunch profile.

LONGITUDINAL INSTABILITIES

Low Intensity

The same formalism as in the transverse plane can be used. An additional complication comes here from the Potential-Well Distortion (PWD) induced by the imaginary part of the longitudinal coupling impedance, which has to be taken into account and which makes the synchrotron frequency ($\omega_s = \omega_{s0} + \Delta\omega_s^i$), the bunch length and the momentum spread depend on the bunch intensity. Below the intensity threshold discussed in the next section, the bunch length is deduced from emittance (momentum spread) conservation for protons (leptons). In addition, there is also a synchronous phase shift, which is usually a small effect, due to the real part of the longitudinal coupling impedance. These two effects apply to the stationary distribution. Taking into account these effects, a new stationary distribution is defined. Around the new fixed point, the same method as in the transverse plane can be used. A perturbation is applied, and the longitudinal modes are deduced from the linearized Vlasov equation. The complex longitudinal coherent synchrotron frequency shift of bunched-beam modes is given by Sacherer's formula, and similar pictures as the one of Fig. 3 can be observed for the longitudinal profile. As far as frequencies are concerned, coherent and incoherent effects subtract. The coherent dipole (rigid bunch) is not affected by a constant reactive impedance since it carries the voltage distortion with it.

High Intensity

In the longitudinal plane, the microwave instability for coasting beams is well understood. It leads to a stability diagram, which is a graphical representation of the solution of the dispersion relation (taking into account the momentum spread) depicting curves of constant growth rates, and especially a threshold contour in the complex plane of the driving impedance. When the real part of the driving impedance is much greater than the modulus of the imaginary part, a simple approximation, known as the Keil-Schnell (or circle) stability criterion, may be used to estimate the threshold curve. For bunched beams, it has been proposed by Boussard to use the coasting-beam formalism with local values of bunch current and momentum spread. A first approach to explain this instability, without coasting-beam approximations, has been suggested by Sacherer through Longitudinal Mode-Coupling (LMC). The equivalence between LMC and microwave instabilities has been pointed out by Sacherer and Laclare in the case of BB driving resonator impedances, neglecting the PWD. Using the mode-coupling formalism for the case of a proton bunch interacting with a BB resonator impedance, and whose length is greater than the inverse of half the resonance frequency, a new formula has been derived taking into account the PWD due to both Space-Charge (SC) and BB resonator impedances. The result is depicted in Fig. 5.

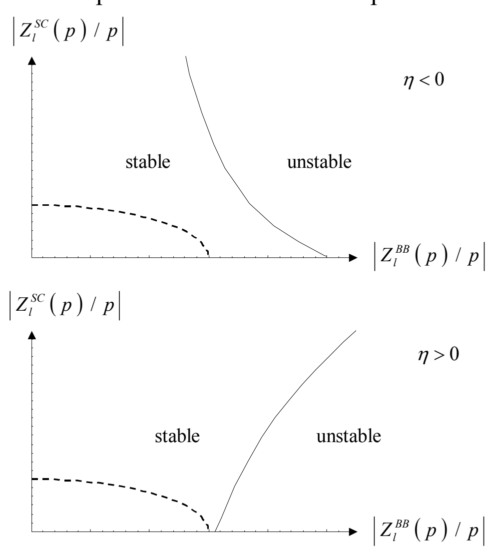

Figure 5: Stability diagram for the LMC instability below and above transition respectively for a proton bunch. The Keil-Schnell circle is represented by the dashed curve.

The threshold without space-charge is ~2 times higher below than above transition as also found by Ng [10]. For lepton bunches, which are usually much shorter than proton bunches, space charge is negligible and the intensity threshold increases with decreasing bunch length (in the "short-bunch" regime), which may explain why the classical instability threshold has been exceeded in some lepton machines. Experimentally, the most evident signature of the LMC instability is the intensity-dependent longitudinal beam emittance blow-up to remain just below the threshold.

STABILIZATION METHODS FOR THE LOW-INTENSITY CASES

Transverse Landau Damping

The Landau damping mechanism from octupoles leads also to a stability boundary diagram in the "complex tune shift plane". The transverse coherent tune shift given by Sacherer's formula (for a round pipe), a complex quantity, is then plotted in this diagram as a single point. If this point lies on the inside of the locus (the side which contains the origin), the beam is stable. The stability diagrams for the 2nd order, 15th order and Gaussian distribution functions [3] are plotted in Fig. 6 for the case of the LHC at top energy (7 TeV) with maximum available octupole strength. The transverse beam profiles corresponding to the 2nd order, 15th order and Gaussian distribution functions extend up to ~3.2σ, 6σ (which should be the case in the LHC due to the collimators setting), and infinity, respectively.

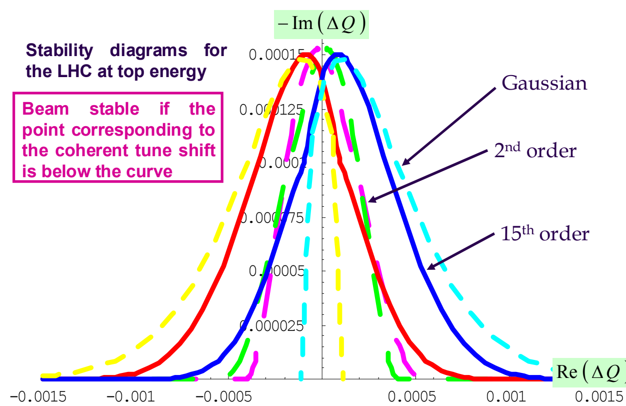

Figure 6: Stability diagrams (positive and negative detunings) for the LHC at top energy (7 TeV) with maximum available octupole strength, for the 2nd order (dashed curves), the 15th order (full curves), and the Gaussian (dotted curves) distribution.

The influence of space-charge nonlinearities on Landau damping has first been studied by Möhl and Schönauer for coasting and rigid bunched beams. Later Möhl extended these results to head-tail modes in bunched beams. The basic results of these studies are that in the absence of external nonlinearities, the space-charge nonlinearities have no effect on bean stability, as the incoherent space-charge tune spread moves with the beam. When octupoles are added, the incoherent space-charge tune spread is "mixed-in," and in this case the octupole strength required for stabilization can depend strongly on the sign of the excitation current of the lenses. Stability diagrams in the presence of both octupole and space-charge nonlinearities are discussed in Ref. [3].

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Longitudinal Landau Damping

By increasing the synchrotron frequency spread *S*, i.e. by increasing the bunch length, the coherent synchrotron frequency of the dipole mode ω_{c11}, which was equal to the low-intensity synchrotron frequency ω_{s0} without synchrotron frequency spread, moves closer and closer to the incoherent band (stable region). The two possible cases are represented in Fig. 7, which was obtained following Besnier's approach (who considered a parabolic distribution function, which introduces some pathologies in the stability diagram due to its sharp edge): the case of a capacitive impedance below transition or inductive impedance above transition corresponds to $U > 0$ (the coherent synchrotron frequency shift has been written $\Delta\omega_{1,1}^{l} = U - j\,V$) and $\Delta\omega_S^i < 0$ (and thus $\omega_s < \omega_{s0}$), and the case of a capacitive impedance above transition or inductive impedance below transition corresponds to $U < 0$ and $\Delta\omega_S^i > 0$ (and thus $\omega_s > \omega_{s0}$).

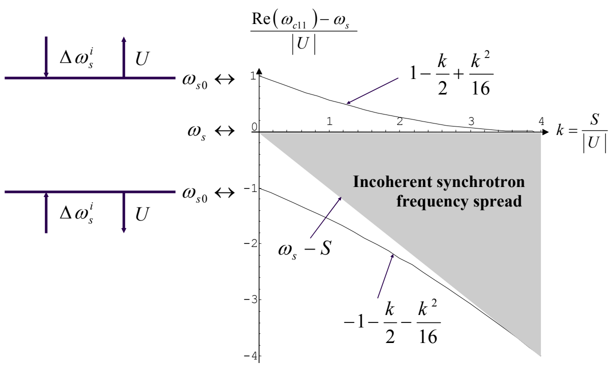

Figure 7: Evolution of the coherent synchrotron frequency for the dipole mode with respect to the incoherent frequency spread.

Motions $\propto e^{j\omega t}$ are considered, which means that the beam is unstable when $V > 0$ (V is called the instability growth rate). The usual case where the resistive part of the impedance is small compared to the imaginary part is assumed, i.e. $V << |U|$. Beam stability is obtained when ω_{c11} enters into the incoherent band. In both cases, the stability limit is reached for $k = 4$, i.e. $S = 4|U|$, which is Sacherer's stability criterion.

Feedbacks

An electronic feedback system is often used to damp coupled-bunch instabilities both in the longitudinal and transverse planes. Recently, it was found to help also for the (single-bunch) head-tail instability in the Tevatron.

Linear Coupling Between the Transverse Planes

In the absence of both linear coupling and frequency spread, and below the mode-coupling threshold, the stability condition for the *m*th head-tail mode is that the growth rate is negative in each transverse plane. In the presence of linear coupling, the necessary condition for stability of the *m*th mode is that the sum of the transverse instability growth rates is negative (note that if this is verified, it is verified for any intensity). If this is the case, then it is possible to stabilize this mode by increasing the skew gradient and/or by working closer to the coupling resonance $Q_h - Q_v = l$, where $Q_{h,v}$ are the transverse coherent tunes and l is any integer [see Fig. 8 (left)]. Note that the PS beam for LHC is stabilized by linear coupling only (i.e. with neither Landau octupoles nor transverse feedback). Furthermore, linear coupling can also have a beneficial effect on the TMCI.

In the presence of both octupoles and linear coupling between the transverse planes, the situation is more involved. To clearly see the effect of linear coupling on Landau damping let's consider the case of a beam without frequency spread in the horizontal plane and without wake field in the vertical one. It is seen from Fig. 8 (right), that below and above certain values of linear coupling strength, stabilization is impossible whereas for intermediate values stabilization is possible even with some tune split $|Q_h - Q_v - l|$. If the coupling is too small, there is not enough Landau damping transferred to the unstable plane. If the coupling is too large, the coherent frequencies fall outside the incoherent frequency spreads and thus prevent Landau damping. An optimum coupling has therefore to be found.

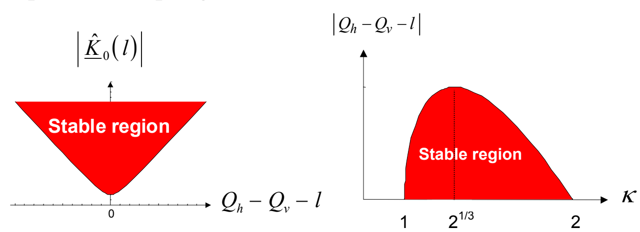

Figure 8: (Left) Shape of the stable region in the presence of linear coupling, but without frequency spread, when the sum of the transverse instability growth rates is negative; $|\hat{\underline{K}}_0(l)|$ is the *l*th Fourier coefficient of the skew gradient. (Right) Shape of the stable region in the presence of both linear coupling and frequency spread, for the case where the vertical spread is equal to two times the horizontal growth rate (for other values similar results are obtained); $\kappa \propto |\hat{\underline{K}}_0(l)|^2$.

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