LINEAR DAMPING SYSTEMS FOR THE INTERNATIONAL LINEAR COLLIDER∗

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Abstract

The International Linear Collider (ILC) requires very low transverse emittance beams in order to realize the specified high luminosity. These beams are conventionally produced using radiation damping in specially designed damping rings. A linear damping system, consisting of alternating wigglers and accelerating structures arranged in a straight line, can be considered to replace, or to augment, conventional damping rings. In this paper, the basic features, feasibility, advantages, and disadvantages, of such systems, as applied to the International Linear Collider, will be discussed.

MOTIVATION

Circular radiation damping systems for the ILC have a number of challenges related to the dynamic aperture of the rings, and to beam stability over the relatively long damping period. The length of the bunch train requires a compression process, necessitating the use of very fast rise and fall time kickers, or a very large circumference damping ring.

These problems can be neatly circumvented by the use of a single-pass linear damping system[1, 2], placed between the pre-linacs following the particle sources and the main linacs. The length of this system is independent of the length of the bunch train, and could allow for a re-optimization of the bunch train length, and other collider parameters.

SCHEME AND METHOD OF ANALYSIS

The basic building block of the linear damper is a section, comprised of a subsection containing a wiggler, in which the beam radiates energy, followed by an acceleration subsection, in which the energy is restored. The acceleration subsection is taken to have a gradient $G$ and a length $L_a$. We designate the wiggler subsection length by $L_w$, and the wiggler period by $\lambda_w$. The wiggler field is assumed to have a sinusoidal variation along the beam axis,

$$B_0(s) = B_0 \cos \frac{2\pi s}{\lambda_w}.$$  

We assume that there are separated-function quadrupoles between each wiggler and acceleration subsection, which provide overall transverse focusing.

The standard approach for the analysis of radiation damping is employed to develop equations for the damping of the energy spread and the emittance in both planes.

We neglect the vertical emittance generated by the vertical opening angle of the synchrotron radiation, and we assume that the only dispersion is in the horizontal plane, and is generated by the wiggler. We assume that $\lambda_u \ll \rho_i$, in which $\rho_i$ is the radius of curvature of the initial energy beam in the wigglers peak field $B_0$.

RESULTS

Vertical Emittance and Damping Length

The beam centroid energy at the entrance to the wiggler subsection is $E_{0,i}$, and at the exit of the wiggler is $E_{0,f}$. The requirement that the acceleration subsection make up the energy lost in the wiggler subsection leads to the following relation between $L_a$ and $L_w$:

$$L_a = \frac{\gamma_{0,i} \Gamma L_w}{g (1 + \Gamma L_w)},$$

in which

$$\Gamma = \frac{\Lambda \gamma_{0,i}}{2}, \quad \Lambda = \frac{2r_e \gamma_{0,i}^2}{3p_i^2},$$

$g = G/mc^2$, and $\gamma_{0,i}(f) = E_{0,i}(f)/mc^2$.

If the initial normalized vertical rms emittance is $(\epsilon_0 \epsilon_v)_i$, then the final normalized vertical rms emittance at the end of a single section is

$$(\gamma_0 \epsilon_v)_f = \frac{\gamma_{0,f}}{\gamma_{0,i}} (\gamma_0 \epsilon_v)_i = \frac{(\gamma_0 \epsilon_v)_i}{(1 + \Gamma L_w)}.$$  

Let the complete system have $M$ sections. Then, if $\Gamma L_w \ll 1$, the normalized vertical rms emittance after $M$ sections will be

$$(\gamma_0 \epsilon_v)_f = (\gamma_0 \epsilon_v)_i \exp \left( -\frac{L}{\lambda_t} \right),$$

in which $L = M \left( L_w + L_a \right)$, and the damping length is

$$\lambda_t = \frac{2}{\Lambda \gamma_{0,i}} + \frac{\gamma_{0,i}}{g}.$$  

As an example, we show in Fig. 1 the damping length as a function of beam energy, for 3 different values of the peak wiggler field. We have chosen $G = 35$ MV/m for the accelerating gradient, in anticipation of an application to the ILC discussed below. The damping length minimizes at an energy

$$\gamma_{0,i,\min} = \frac{5.562 \times 10^4}{B_0[T]} \Rightarrow \lambda_{t,\min} \text{[km]} = \frac{111.25}{B_0[T] \sqrt{g}}.$$  

Fig. 2 shows the minimum damping length, and Fig. 3 shows the beam energy at which the damping length minimizes, both as a function of peak wiggler field.

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horizontal rms emittance is shown as a function of peak wiggler field $B_0$ in Fig. 4. We have chosen $\beta = 2$ m and $\lambda_u = 11$ cm, in anticipation of an application to the ILC discussed below.

Figure 4: Normalized horizontal rms emittance $(\gamma_0 \epsilon_h)_f$ vs. peak wiggler field $B_0$, for $\beta = 2$ m, $\lambda_u = 11$ cm, and $G = 35$ MV/m.

**Energy Spread**

The square of the relative rms energy spread damps with a damping length given by $\lambda_t/4$. The equilibrium value of the relative rms energy spread is

$$\frac{\sigma_{\gamma,f}}{\gamma_{0,i}} = 0.009778 \times 10^{-3} \sqrt{B_0[T] \gamma_{0,i}}.$$  

If we operate at the value of $\gamma_{0,i}$ corresponding to the minimum for $\lambda_t$, then the relative rms energy spread is independent of the field and the energy, and is given by

$$\frac{\sigma_{\gamma,f}}{\gamma_{0,i}} = 2.306 \times 10^{-3} \sqrt{g}.$$  

**Radiated Power**

For an average current of $I$, the power radiated per unit distance is

$$\left\langle \frac{dP}{ds} \right\rangle [W/m] = 1.651 \times 10^{-4} I[A](B_0[T])^2 \gamma^2.$$  

**APPLICATIONS TO THE INTERNATIONAL LINEAR COLLIDER**

**Replacements for the Damping Rings**

For an application to the ILC to replace the electron damping ring, we would need to reduce the electron normalized vertical rms emittance from its value at the exit of the electron source, 40 $\mu$m, to the value required at the entrance to the main linac, 0.02 $\mu$m. This corresponds to $\ln(40/0.02) = 7.6$ damping lengths. For $B_0 = 10$ T wigglers, the energy corresponding to the minimum damping length is 23.5 GeV. The beam would thus need to be pre-accelerated to this energy. The damping length is 1.34 km, and 7.6 damping lengths correspond to 10.2 km.
Table 1: Linear damping examples for ILC applications. $\beta = 2$ m, and the radiated power is calculated for the nominal ILC current.

<table>
<thead>
<tr>
<th>Application</th>
<th>$B_0$ [T]</th>
<th>$E_0$ [GeV]</th>
<th>$N_{damp}$</th>
<th>$L$ [km]</th>
<th>$\lambda_u$ [cm]</th>
<th>$(\gamma_0\epsilon_h)_f$ [$\times 10^{-3}$]</th>
<th>$\sigma_{\gamma_f}$ [$\eta_{pk}$]</th>
<th>$\eta_{pk}$</th>
<th>$\langle dP/d\gamma \rangle$ [kW/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron damper</td>
<td>10</td>
<td>23.5</td>
<td>7.6</td>
<td>10.2</td>
<td>11</td>
<td>8.1</td>
<td>6.6</td>
<td>39</td>
<td>1.58</td>
</tr>
<tr>
<td>Positron damper</td>
<td>10</td>
<td>23.5</td>
<td>13.5</td>
<td>18.1</td>
<td>11</td>
<td>8.1</td>
<td>6.6</td>
<td>39</td>
<td>1.58</td>
</tr>
<tr>
<td>Positron predamper</td>
<td>15</td>
<td>5</td>
<td>5.9</td>
<td>9.1</td>
<td>13.5</td>
<td>41</td>
<td>3.7</td>
<td>275</td>
<td>0.160</td>
</tr>
<tr>
<td>Afterdamper</td>
<td>5</td>
<td>47</td>
<td>2.3</td>
<td>6.2</td>
<td>31</td>
<td>8.0</td>
<td>6.6</td>
<td>9.7</td>
<td>1.58</td>
</tr>
</tbody>
</table>

To replace the positron damping ring, we need to reduce the positron normalized vertical rms emittance from 14,000 to 0.02 $\mu$m, which requires $\ln(14000/0.02) = 13.45$ damping lengths. For $B_0 = 10$ T wiggles and a beam energy of 23.5 GeV, this is a length of 18.1 km.

The values of $\beta = 2$ m and $\lambda_u = 11$ cm, with $B_0 = 10$ T, give an equilibrium normalized horizontal rms emittance of about 8 $\mu$m, which is the design requirement. The equilibrium output rms energy spread is 0.66% at 23.5 GeV. This energy spread makes subsequent bunch compression rather difficult.

Bunch compression could perhaps be done as part of the pre-acceleration prior to entry into the linear damping structure. However, the short bunch in the high field damping wiggles may result in substantial coherent synchrotron radiation.

Another possibility is to build bunch compression sections into the linear damping system by using additional RF subsections with the proper phase to compress the bunch as it passes through the wiggles.

**Linear Positron Predamper**

We may consider a linear damper in place of a positron predamping ring. The final normalized vertical rms emittance required is 40 $\mu$m, so only $\ln(14000/40) = 5.9$ damping lengths are needed. However, the linear damper should be operated at 5 GeV, the energy of the positron damping ring. To get a reasonable damping length at this low energy, the wiggler field should be pushed as high as possible. For example, for $B = 15$ T and a beam energy for 5 GeV, the linear positron predamper would have a length of 9.1 km. The wiggler period can be relaxed to 13.5 cm (or $\beta$ could be increased), giving an equilibrium normalized horizontal rms emittance of about 40 $\mu$m.

**Afterdamper**

A third possibility is to consider an “afterdamper”. In this scheme, the vertical emittance requirements on the main damping rings are relaxed by a factor of 10. The beam is damped, but to a normalized vertical rms emittance of 0.2 $\mu$m rather than 0.02 $\mu$m, compressed, and then accelerated to 47 GeV. At this energy, the wiggler field corresponding to the minimum damping length is 5 T. The beam passes through $\ln(10) = 2.3$ damping lengths, a total of 6 km, and comes out with a normalized vertical rms emittance of 0.02 $\mu$m. The wiggler period may be increased to 31 cm, and the design normalized horizontal rms emittance is still achieved. (Alternatively, $\beta$ could be increased.) The relative rms energy spread is 0.66% at 47 GeV, which becomes 0.12% at 250 GeV. This is on the high side, but may still be acceptable. This scheme also suffers from the fact that the compressed beam passes through the wiggles.

**Summary and Issues**

The parameters of the linear damping systems discussed above are presented in Table 1.

There are a number of major issues in the implementation of these linear damping schemes. The technological challenge of building many kilometers of high-field, short period wiggles is daunting. Small beta functions and ultra-small dispersion functions are required throughout the wiggles (see Table 1). This will result in very tight tolerances on the alignment of the wiggles and the quadrupoles required for transverse focusing. The relatively large energy spread of the damped beam makes bunch compression very difficult to do subsequent to the linear dampers. Bunch compression prior to the dampers leads to high peak currents in the wiggles, which may present stability problems.

**CONCLUSION**

Linear damping systems in principle can provide radiation damping configurations which may complement or replace conventional damping rings.

Linear damping systems as applied to the ILC as replacements for the electron and positron damping rings seem more challenging than standard damping rings. A linear positron predamping system at 5 GeV would require a substantial length and a very high field wiggler. A linear afterdamper, providing a factor of 10 damping to allow the main damping ring performance requirements to be relaxed, could be feasible. More study is needed for this application.

**REFERENCES**
