A MAGNETIC FIELD MODEL FOR THE UNDULATOR IN HLS*

He Zhang#, Lin Wang, NSRL, USTC, Hefei, Anhui, P.R.China
Yongjun Li, DESY, Hamburg

Abstract
It is important to understand the influence of wigglers and undulators on the beam dynamics in design and optimization of a storage ring, especially when the storage ring runs on a low emittance mode. We present an analytic model of the undulator field in HLS, which can be used in the tracking study to evaluate the effects of it. Coefficients needed by the model are generated by fitting to the results of a numerical field calculation. Fringe fields are included in this model. Then we use three different methods to track particles through the undulator, and compare the results.

INTRODUCTION
Two insertion devices are installed in the HLS storage ring, a superconductive wiggler and a planar pure permanent-magnet undulator. They work well when the storage ring runs on the general purpose light source (GPLS) mode. The operation of an optical krystron is also on plan.

Now we are trying to find a low emittance mode lattice that is suitable to insertion devices. In this condition, the nonlinear components of the insertion devices magnetic field maybe one of the main limitations of the dynamic aperture[1]. So the tracking study of the particles through the insertion devices is necessary to get the transfer maps, which is a prerequisite for the study of particle dynamics of the whole ring. We use three methods, Runge-Kutta integration, symplectic integration, and generating function to do tracking and to construct transfer maps. An analytic field model is necessary or convenient to use these methods.

In this article, we build a field model of the undulator in the HLS storage ring, and get the transfer map of it. The similar methods can also be used to treat the wiggler and the optical krystron in HLS.

FIELD MODEL FOR THE UNDULATOR IN HLS
The undulaor (UD1) in HLS is designed to produce tunable, quasi-monochromatic, partially coherent radiation, the frequency of which is in the range from VUV to soft X-ray. Some parameters of the UD1 are given in Table 1.

The three-dimensional magnetic field data of UD1 versus position is calculated by RADIA[2]. To get the magnetic field model, we treat the periodic field and the fringe fields separately. The periodic field is described as the following form:

$$B_x = -\sum_i \frac{k_{xi}}{k_{yi}} B_i \sin(k_{xi}x) \sinh(k_{yi}y) \cos(k_i z)$$
$$B_y = \sum_i B_i \cos(k_{xi}x) \cosh(k_{yi}y) \cos(k_i z)$$
$$B_z = -\sum_i \frac{k_i}{k_{yi}} B_i \cos(k_{xi}x) \sinh(k_{yi}y) \sin(k_i z)$$

where $k_{yi}^2 = k_{xi}^2 + k_i^2$, $k_i = i \cdot 2\pi / \lambda$ is the wave number of $i$th harmonic along longitudinal direction, $B_i$ is its peak magnetic field amplitude, $\lambda = 92$ mm is the period length of the undulator. These expressions of magnetic field satisfy Maxwell’s equations.

By choosing the gauge $A_z = 0$, the corresponding vector potential $A = (A_x, A_y, 0)$ is given by the following form:

$$A_x = \sum_i \frac{1}{k_i} B_i \cos(k_{xi}x) \cosh(k_{yi}y) \sin(k_i z)$$
$$A_y = \sum_i \frac{k_{yi}}{k_{xi} \cdot k_i} \sin(k_{xi}x) \sinh(k_{yi}y) \sin(k_i z)$$

Because of the symmetry of the periodic magnetic field, only the odd harmonics are used to fit the magnetic data. Given a calculation of the field at a set of points, the problem is to find a sum of terms, which minimize the variance between field calculation data and fit data. The fit results are listte in Table 2. Field calculation results and fit curves of $B_y$ as a function of $x$, $y$ and $z$ are shown in Figure 1. Only 3 terms are used for the fit. The peak field is about 0.41 T and the RMS difference between calculation data and fit data is 8.5 Gauss, which gives an RMS to peak field ratio of 0.17%.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$B_i$ (T)</th>
<th>$k_{xi}$ (mm)</th>
<th>$k_{yi}$ (mm)</th>
<th>$k_i$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.7615e-5</td>
<td>6.7879e-2</td>
<td>2.1584e-1</td>
<td>2.0489e-1</td>
</tr>
<tr>
<td>5</td>
<td>7.6398e-4</td>
<td>5.0851e-3</td>
<td>3.4152e-1</td>
<td>3.4148e-1</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the undulator in HLS

<table>
<thead>
<tr>
<th>Magnet type</th>
<th>Permanant magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period length</td>
<td>0.092 m</td>
</tr>
<tr>
<td>Period number</td>
<td>29</td>
</tr>
<tr>
<td>Range of the magnet gap</td>
<td>0.036 ~ 0.096 m</td>
</tr>
<tr>
<td>Peak field</td>
<td>0.456 ~ 0.06 T</td>
</tr>
<tr>
<td>Total length</td>
<td>2.67 m</td>
</tr>
</tbody>
</table>

Table 2: Fit results of the periodic field

*Work supported by CAS Knowledge Innovation Project
#zhanghe@ustc.edu

Proceedings of 2005 Particle Accelerator Conference, Knoxville, Tennessee

0-7803-8859-3/05/$20.00 ©2005 IEEE 3994
The fringe fields are described as the following form:

\[ B_x = \sum_i \frac{k_i}{k_{yi}} \sin(k_{yi} x) \sinh(k_{yi} z)B_i(z) \]
\[ B_y = \sum_i \cos(k_{yi} x) \cosh(k_{yi} z)B_i(z) \]
\[ B_z = -\sum_i \frac{k_i}{k_{yi}} \cos(k_{yi} x) \sinh(k_{yi} z)B_i(z) \]

with \( B_i(z) = B_{yi} \cos k_{yi} z + B_{zi} \sin k_{yi} z \)

where \( k_{yi}^2 = k_{xi}^2 + k_{zi}^2 \), \( k_i \) and \( B_i \) have the same meaning as in equation (1) except that \( \lambda = 300 \text{ mm} \) here. These expressions also satisfy Maxwell’s equations.

By choosing the gauge \( A_z = 0 \), the corresponding vector potential \( A = (A_x, A_y, 0) \) is given by the following form:

\[ A_x = \sum_i \frac{1}{k_i} \cos(k_{yi} x) \cosh(k_{yi} z)A_i(z) \]
\[ A_y = \sum_i \frac{k_i}{k_{yi}} \sin(k_{yi} x) \sinh(k_{yi} z)A_i(z) \]

with \( A_i(z) = B_{yi} \sin(k_{yi} z) - B_{zi} \cos(k_{yi} z) \)

We use the similar method with what we use in the periodic field case to fit fringe fields. But in the case of fringe fields, both the odd and the even harmonics must be used to fit the data, and more terms are needed. In fact, we used 20 terms to make sure that the RMS difference between calculation data and fit data is 5.7 Gauss which gives an RMS to peak field ratio of 0.11%. Figure 2 shows field calculation results and fit curves of \( B_x \) as a function of \( x \) at \( y = 0, z = 2623 \text{ mm} \).

Since the field model has already been calculated, the Hamiltonian of charged particle motion in either period field or fringe fields in the paraxial approximation can be written as[3]:

\[ H(z) = \frac{(p_x - a_z)^2}{2(1 + \delta)} + \frac{(p_y - a_y)^2}{2(1 + \delta)} - a_z \]

where \( p_{xi} = P_{xi}/P_0 \) is the scaled transverse momenta, \( P_0 \) is the nominal mechanical momentum, \( a = qA/P_0c \) is the scaled vector potential, \( \delta = \Delta E / P_0c \) is the relative energy deviation.
Figure 3: (a) $x'$ as a function of $x_i$ using three different tracking methods in the case $x'_i = y'_i = 0$ and $y_i = 10\,\text{mm}$. (b) $y'$ as a function of $x_i$.

Figure 4 (a) shows the tracking results of $x'$ as a function of $y_i$. The results of GF and SI agree better than 0.001mrad, and the difference between RK and them is less than 0.011mrad. Figure 4 (b) shows $y'$ as a function of $y_i$, and Figure 4 (c) shows the difference of $y'$ using the three methods as the function of $y_i$. As it shows, the further $y_i$ leaves the axis, the larger the difference is. But the largest difference between SI and RK is less than 0.002mrad, difference between GF and RK less than 0.005mrad, and difference between SI and GF less than 0.007mrad.

If we only track particles through the periodic field, the tracking results of $x'$ using the three methods agree better than 0.001mrad. So the difference showed in Figure 3 (a) and Figure 4 (a) attributes to the existence of fringe fields. The reason why the tracking results of RK and SI do not agree well in the fringe fields needs more particular analysis later. The difference between GF and the other two methods showed in Figure 3 (b) is also reasonable. In the fitting data used to calculate the generating function coefficients, the maximum of $y_i$ is only 0.2mm. So it is natural that GF is no longer accurate when $y_i$ is much larger than 0.2mm, 10mm in this case. This fact is also consistent with Figure 4 (c), and it suggests that choosing proper data to fit the generating function coefficients is necessary. But why does GF agree well with SI and RK as Figure 4 shows, when $x_i = 20\,\text{mm}$, which is much larger than 0.2mm, the maximum of the fit data $x_i$? In Figure 1 and Figure 2, we can see that when $y_i$ changes 10 mm, $B_y$ changes 0.1T. Correspondingly when $x_i$ changes 20mm, $B_x$ changes only a little more than 0.01T. So the change of $x_i$ affects much little than the change of $y_i$.

CONCLUSIONS

The accurate undulator field model including fringe fields represented here makes possible the symplectic mapping which is necessary in long term tracking. In the three tracking methods, GF is chosen for later tracking simulation because of its speed and symplecticity. Similar methods can be used to treat the wiggler and the optical krytron in HLS, generating their models, building their generating functions, and thereby evaluating their dynamical effects by tracking simulation.

REFERENCES