SENSITIVITY STUDY FOR EVALUATING THE EXTRACTED BEAM PARAMETERS OF THE LLUMC PROTON THERAPY SYNCHROTRON

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Abstract

The MINOS nonlinear constrained optimization program, working in concert with the beam optics code TRANSPORT, has been shown in recent work to provide a fast, efficient and reliable procedure for determining the parameters of the beam extracted from the Loma Linda University Medical Center (LLUMC) proton therapy synchrotron. MINOS and TRANSPORT work together as Modules of the Particle Beam Optics Laboratory (PBO Lab\(^TM\)) software. The software was used to determine the parameters of the beam extracted from the synchrotron accelerator that best fit the extensive beam profile data used to monitor the LLUMC proton therapy beamlines. Additional constraints have been utilized with the procedure, and the parametric dependence upon certain parameters has been examined, in order to evaluate the sensitivity of the best fit extracted beam parameters to various assumptions. In this paper two examples of these sensitivity studies are reported. The sensitivity of the results to the momentum spread assumed for the beam is explored by carrying out parametric optimizations. Second, the character of the optimizer solutions under the imposition of additional constraints upon the emittance values is examined. The methods used for the calculations are outlined and selected results are presented.

INTRODUCTION

PBO Lab is a set of software application modules that support beamline design [1], personnel training [2], and accelerator operations [3]. The PBO Lab Basic Package provides a common graphic interface for constructing and editing sophisticated iconic object-based computer models of accelerators and beamlines. The broad range of applications that may be addressed, together with the ease-of-use provided by the intuitive graphic interface, has made PBO Lab a popular package and it is now used by more than ninety laboratories in over twenty countries [4].

The PBO Lab Optimization Module [5] has recently been used in conjunction with the TRANSPORT optics Application Module [6] to determine parameters describing the extracted beam of the LLUMC synchrotron that best fits measured beam size data in the proton therapy beamlines [7]. The procedure involves a least squares fit of the difference between the measured beam sizes and the corresponding sizes predicted by TRANSPORT. The PBO Lab Optimization Module is utilized to minimize the standard deviation (root mean square) of that difference.

The extracted beam parameters, which form the initial beam description for each TRANSPORT calculation, constitute the optimizer variables. This automated procedure has resulted in savings of many days and occasionally months [7] compared to the previous method used to determine the best fit extracted beam parameters for the synchrotron utilizing TRANSPORT alone [8].

The Optimization Module extends the capabilities of PBO Lab to address beamline optimization problems that are more complex than can be handled by traditional optics codes such as TRANSPORT [9]. The Optimization Module integrates the constrained nonlinear optimization programs MINOS [10] and NPSOL [11] into the PBO Lab framework. NPSOL and MINOS were developed in the Stanford University Operations Research Department, but use different approaches to solving constrained nonlinear optimization problems. Both programs are included in the Optimization Module to provide users with alternate approaches to solving a particular problem.

The procedure for setting up the LLUMC extracted beam problem with the PBO Lab Optimization Module has been summarized previously [12]. The initial setup of a PBO Lab optimization problem involves three steps. The first step selects the model parameters (“Optimizer Variables”) to be varied by the Optimization Module codes (NPSOL and/or MINOS). For the LLUMC problem discussed here, the Optimizer Variables are the initial beam parameters at the start of the transfer lines, that is, the parameters for the beam at the extraction septum. The second step is to specify which TRANSPORT outputs (“Stored Parameters”) are to be used in the problem. The \(x\) and \(y\) beam sizes at the locations of the beam profile monitors were assigned as TRANSPORT Stored Parameters, making them available to the Optimization Module. The third step is to use the Optimizer Variables and Stored Parameters to construct an Objective Function to be minimized, as well as any linear and nonlinear constraints to be satisfied simultaneously. The Objective Function used in the prior studies [12] was the sum of the squares of the difference between the TRANSPORT computed beam sizes and those measured by the profile monitors. The PBO Lab windows utilized to complete these 3 steps were illustrated in Figure 1 of reference [12]. Note that no source code programming is required. This paper illustrates the additional use of the Optimization Module for carrying out selected sensitivity studies for the extracted beam parameters.
SENSITIVITY STUDIES

The primary example used for the sensitivity studies described here is the same one discussed in reference [12]. The extracted beam parameters are determined which best fit the profile data for the beamline into gantry room 2 (G2) at 250 MeV [8]. However, the Objective Function used for this work was modified in two ways. Denoting the sum of the squared differences between the measured and computed beam sizes by $\sum |\Delta i|^2$, then the standard deviation $\sigma$ for a computed fit to the profile data is:

$$\sigma = \left[ \frac{1}{N} \sum |\Delta i|^2 \right]^{1/2} .$$  

(1)

$N$ is the number of data points included in the summation. In the 250 MeV G2 beamline $N=24$, corresponding to x and y beam sizes for 12 profile monitors. For this work, the Objective Function was taken to be $\sigma$ directly, rather than $\sum |\Delta i|^2$ as used in [12]. Second, the values of the measured beam sizes were input to the calculation as Constant Parameters of the Optimization Module, rather than as numerical data in the Objective Function. The use of Constant Parameters permits the values to be specified in PBO Lab as Import Parameters. Import Parameters can be loaded from a text file using an External Data Interface Tool [13]. The measured data can be updated in the text file (e.g. by a diagnostic program) and then imported into the calculation. This facilitates incorporating changes such as those associated with different beamline tunes.

Figure 1 illustrates the PBO Lab window used to define and edit the Objective Function. Both the numerical value and Constant Parameter implementations of the measured data are shown. Note that multiple Objective Functions may be defined – to switch among Objective Functions one simply makes a new selection under the Use column.

Sensitivity to Momentum Spread

The MINOS program in the PBO Lab Optimization Module was used in [12] to find the minimum in the standard deviation $\sigma$ of equation (1). By exploring a number of different starting values for the Optimizer Variables (labeled ID=1-4, U and L in [12]) it was shown that the solutions found were global, within the constraint bounds imposed upon those Optimizer Variables. However, the details of the solutions could depend in principle upon other assumptions. Part of the sensitivity studies reported on here involved examining assumptions on the momentum spread and momentum correlation terms (dispersion terms) of the extracted beam.

To examine the sensitivity to the assumed rms momentum spread $\delta$ of the extracted beam, the optimization was repeated for values of $\delta$ between 0 and 0.015%. Figure 2 illustrates the dependence of $\sigma$ and the x phase plane rms beam parameters (x envelope, $x'$ envelope and the x-x' correlation coefficient $r_{12}$) on $\delta$ for the solutions found by the Optimization Module.

![Figure 2: Dependence of the Optimizer Solutions on the assumed value of the momentum spread $\delta$.](image)

The value of the standard deviation $\sigma$ shows a broad minimum for values of $\delta \leq 0.01\%$. This indicates that the optimizer solutions are of comparable quality for any assumed value less than about 0.01%. There is some trade off between $x'$ and $r_{12}$ in the optimized solution for momentum spreads in this region. The values reported previously [8,12] correspond to $\delta=0.0075\%$.

Sensitivity to Emittance Constraints

Additional linear and nonlinear constraints can be added to any problem formulated with the Optimization Module. The use of nonlinear constraints can be especially helpful for solving certain types of problems [5]. Here we use additional nonlinear constraints to study the sensitivity of the optimizer solutions to imposed emittance values. The approach is to specify lower and upper constraints on the rms emittances in the x and y phase planes. The lower and upper constraints are separated by only a narrow margin about a central (imposed) value. The Optimization Module is then used to find solutions that satisfy these additional constraints while simultaneously minimizing...
the Objective Function. The rms emittance for the x phase plane, \( \varepsilon_x \), can be computed from the rms semi-axes \( x \) and \( x' \), together with the correlation parameter \( r_{12} \):

\[
\varepsilon_x = x x' [(1 - r_{12}^2)]^{1/2}.
\]

A similar expression is applicable for \( \varepsilon_y \). Constraints were imposed on \( \varepsilon_x \) and \( \varepsilon_y \) by specifying that these emittances should be within a small range about given target values, denoted here as \( \varepsilon_x^T \) and \( \varepsilon_y^T \). The constraints were set up using inputs available on the Nonlinear Constraints tab panel of the window shown in Figure 1, and were of the form:

\[
1 - \Delta x < \frac{\varepsilon_x}{\varepsilon_x^T} < 1 + \Delta x, \quad (3)
\]

and

\[
1 - \Delta y < \frac{\varepsilon_y}{\varepsilon_y^T} < 1 + \Delta y. \quad (4)
\]

The same Objective Function was used for the optimization, but now the solutions are constrained by (3) and (4). Figure 3 illustrates results for this constrained optimization for difference values of the target emittance \( \varepsilon_y^T \). The value of \( \varepsilon_y^T \) was 0.15 \( \pi \)-cm-mrad, while both \( \Delta x \) and \( \Delta y \) were set to fairly small values (0.0001).

Figure 3: Optimizer Solutions for different values of the constraint imposed upon the emittance \( \varepsilon_x \).

The standard deviation \( \sigma \) displays a broad minimum in the optimization solution for values of \( \varepsilon_x \) between 0.01 and 0.04 \( \pi \)-cm-mrad. This characteristic has been used to find emittance-constrained optimum beam parameters as a smooth function of the extracted beam energy [7].

**SUMMARY**

The PBO Lab Optimization Module has proven to be a useful tool for obtaining fast, accurate and reliable sets of initial beam parameters for the LLUMC proton therapy transfer lines. Sensitivity studies have been described that further explored the descriptions obtained for the beam at the extraction septum. These studies were easy to carry out using the PBO Lab Optimization Module, but would have been quite tedious (if not infeasible) without it.

**REFERENCES**


