COHERENT STRUCTURES AND PATTERN FORMATION IN VLASOV-MAXWELL-POISSON SYSTEMS

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Abstract

We present the applications of methods from nonlinear local harmonic analysis for calculations in nonlinear collective dynamics described by different forms of Vlasov-Maxwell-Poisson equations. Our approach is based on methods provided the possibility to work with well-localized in phase space bases, which gives the most sparse representation for the general type of operators and good convergence properties. The consideration is based on a number of anzatzes, which reduce initial problems to a number of dynamical systems and on variational-wavelet approach to polynomial/rational approximations for nonlinear dynamics. This approach allows us to construct the solutions via nonlinear high-localized eigenmodes expansions in the base of compactly supported wavelet bases and control contribution from each scale of underlying multiscales. Numerical modelling demonstrates formation of coherent structures and stable patterns.

1 INTRODUCTION

In this paper we consider the applications of numerical--analytical technique based on the methods of local nonlinear harmonic analysis or wavelet analysis to nonlinear beam/accelerator physics problems which can be characterized by collective type behaviour and described by some forms of Vlasov-Maxwell-Poisson equations [1]. Such approach may be useful in all models in which it is possible and reasonable to reduce all complicated problems related with statistical distributions to the problems described by systems of nonlinear ordinary/partial differential equations with or without some (functional)constraints. Wavelet analysis is a set of mathematical methods, which gives the possibility to work with well-localized bases in functional spaces and gives the maximum sparse forms for the general type of operators (differential, integral, pseudodifferential) in such bases. Our approach is based on the variational-wavelet approach from [2]-[13], which allows us to consider polynomial and rational type of nonlinearities. The solution has the multiscale/multiresolution decomposition via nonlinear high-localized eigenmodes, which corresponds to the full multiresolution expansion in all underlying time/space scales. The same is correct for the contribution to power spectral density (energy spectrum): we can take into account contributions from each level/scale of resolution. In all these models numerical modelling demonstrates the appearance of coherent high-localized structures and stable patterns formation. Starting from Vlasov-Maxwell-Poisson equations in part 2, we consider the approach based on variational-wavelet formulation in part 3. We give the explicit representation for all dynamical variables in the base of compactly supported wavelets or nonlinear eigenmodes. Our solutions are parametrized by solutions of a number of reduced algebraical problems one from which is nonlinear with the same degree of nonlinearity as initial differential problem and the others are the linear problems which correspond to the particular method of calculations inside concrete wavelet scheme. In part 4 we consider numerical modelling based on our analytical approach.

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2 COLLECTIVE MODELS VIA VLASOV-MAXWELL-POISSON EQUATIONS

Analysis based on the non-linear Vlasov-Maxwell-Poisson equations leads to more clear understanding collective effects and nonlinear beam dynamics of high intensity beam propagation in periodic-focusing and uniform-focusing transport systems. We consider the following form of equations (ref. [1] for setup and designation):

$$\Big\{\frac{\partial}{\partial s}+p_x\frac{\partial}{\partial x}+p_y\frac{\partial}{\partial y}-\Big[k_x(s)x+\frac{\partial\psi}{\partial x}\Big]\frac{\partial}{\partial p_x}-\Big[k_y(s)y+\frac{\partial\psi}{\partial y}\Big]\frac{\partial}{\partial p_y}\Big\}f_b(x,y,p_x,p_y,s)=0, \quad (1)$$

$$\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\psi=-\frac{2\pi K_b}{N_b}\int \mathrm{d}p_x\mathrm{d}p_y f_b, \quad (2)$$

$$\int \mathrm{d}x\mathrm{d}y\mathrm{d}p_x\mathrm{d}p_y f_b = N_b \quad (3)$$

The corresponding Hamiltonian for transverse single-particle motion is given by

$$H(x,y,p_x,p_y,s)=\frac{1}{2}(p_x^2+p_y^2)+\frac{1}{2}[k_x(s)x^2+k_y(s)y^2]+H_1(x,y,p_x,p_y,s)+\psi(x,y,s), \quad (4)$$

where H_1 is nonlinear (polynomial/rational) part of the full Hamiltonian. In case of Vlasov-Maxwell-Poisson system we may transform (1) into invariant form

$$\frac{\partial f_b}{\partial s}+[f_b,H]=0. \quad (5)$$

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3 MULTISCALE REPRESENTATIONS

We obtain our multiscale/multiresolution representations (formulae (11) below) for solutions of equations (1)-(5) via variational-wavelet approach for the following formal systems of equations (with corresponding obvious constraints on distribution function),which are the general form of these equations. Let L be an arbitrary (non) linear differential/integral operator with matrix dimension d, which acts on some set of functions $\Psi \equiv \Psi(s,x) = \Big(\Psi^1(s,x),\dots,\Psi^d(s,x)\Big)$, $s,x \in \Omega \subset \mathbf{R}^{n+1}$ from $L^2(\Omega)$:

$$L\Psi \equiv L(R(s,x),s,x)\Psi(s,x) = 0, \qquad (6)$$

(x are the generalized space coordinates or phase space coordinates, s is "time" coordinate). After some anzatzes [13],[14] the main reduced problem may be formulated as the system of ordinary differential equations

$$Q_i(f)\frac{\mathrm{d}f_i}{\mathrm{d}s} = P_i(f,s), \quad f=(f_1,...,f_n), \qquad (7)$$
$$i=1,\dots,n, \quad \max_i deg\, P_i = p, \quad \max_i deg\, Q_i = q$$

or a set of such systems corresponding to each independent coordinate in phase space. They have the fixed initial (or boundary) conditions $f_i(0)$, where P_i, Q_i are not more than polynomial functions of dynamical variables f_j and have arbitrary dependence on time. As result we have the following reduced algebraical system of equations on the set of unknown coefficients λ_i^k of localized eigenmode expansion (formula (9) below):

$$L(Q_{ij},\lambda,\alpha_I) = M(P_{ij},\lambda,\beta_J), \qquad (8)$$

where operators L and M are algebraization of RHS and LHS of initial problem (7) and λ are unknowns of reduced system of algebraical equations (RSAE) (8). After solution of RSAE (8) we determine the coefficients of wavelet expansion and therefore obtain the solution of our initial problem. It should be noted if we consider only truncated expansion with N terms then we have from (8) the system of $N\times n$ algebraical equations with degree $\ell = max\{p,q\}$ and the degree of this algebraical system coincides with degree of initial differential system. So, we have the solution of the initial nonlinear (rational) problem in the form

$$f_i(s) = f_i(0) + \sum_{k=1}^N \lambda_i^k f_k(s), \qquad (9)$$

where coefficients λ_i^k are the roots of the corresponding reduced algebraical (polynomial) problem RSAE (8). Consequently, we have a parametrization of solution of initial problem by the solution of reduced algebraical problem (8). The obtained solutions are given in the form (9), where $f_k(t)$ are basis functions obtained via multiresolution expansions and represented by some compactly supported wavelets. Because affine group of translation and dilations is inside the approach, this method resembles the action of a microscope. We have contribution to final result from each scale of resolution from the whole infinite scale of spaces. More exactly, the closed subspace $V_j (j \in \mathbf{Z})$ corresponds to level j of resolution, or to scale j. We consider a multiresolution analysis of $L^2(\mathbf{R}^n)$ (of course, we may consider any different functional space) which is a sequence of increasing closed subspaces V_j:

$$...V_{-2}\subset V_{-1}\subset V_0\subset V_1\subset V_2\subset ...$$

satisfying the following properties: let W_j be the orthonormal complement of V_j with respect to V_{j+1} : $V_{j+1} = V_j \bigoplus W_j$, then

$$L^2(\mathbf{R}) = \overline{V_0 \bigoplus_{j=0}^{\infty} W_j}, \qquad (10)$$

As a result the solution of equations (1)-(5) has the following multiscale/multiresolution decomposition via nonlinear high-localized eigenmodes

$$\Psi(s,x) = \sum_{(i,j)\in Z^2} a_{ij}U^i(x)V^j(s), \qquad (11)$$
$$V^j(s) = V_N^{j,slow}(s) + \sum_{l\geq N} V_l^j(\omega_l^1 s), \quad \omega_l^1 \sim 2^l$$
$$U^i(x) = U_M^{i,slow}(x) + \sum_{m\geq M} U_m^i(\omega_m^2 x), \quad \omega_m^2 \sim 2^m,$$

which corresponds to the full multiresolution expansion in all underlying time/space scales. Formula (11) gives us expansion into the slow part $\Psi_{N,M}^{slow}$ and fast oscillating parts for arbitrary N, M. So, we may move from coarse scales of resolution to the finest one for obtaining more detailed information about our dynamical process. The first terms in the RHS of formulae (11) correspond on the global level of function space decomposition to resolution space and the second ones to detail space. In this way we give contribution to our full solution from each scale of resolution or each time/space scale or from each nonlinear eigenmode. This functional space decomposition corresponds to exact nonlinear eigenmode decompositions. It should be noted that such representations give the best possible localization properties in the corresponding (phase)space/time coordinates. In contrast with different approaches formulae (11) do not use perturbation technique or linearization procedures and represent dynamics via generalized nonlinear localized eigenmodes expansion. So, by using wavelet bases with their good (phase) space/time localization properties we can construct high-localized coherent structures in spatially-extended stochastic systems with collective behaviour.

4 MODELLING

Multiresolution/multiscale representations for the solutions of equations from part 2 in the high-localized bases/eigenmodes are demonstrated on Fig. 1–Fig. 3. This modelling demonstrates the appearance of stable patterns formation from high-localized coherent structures. On Fig. 1 we present contribution to the full expansion from level 1 of decomposition (11). Fig. 2, 3 show the representations for full solutions, constructed from the first 6 eigenmodes (6 levels in formula (11)). Figures 2, 3 show stable pattern formation based on high-localized coherent structures.

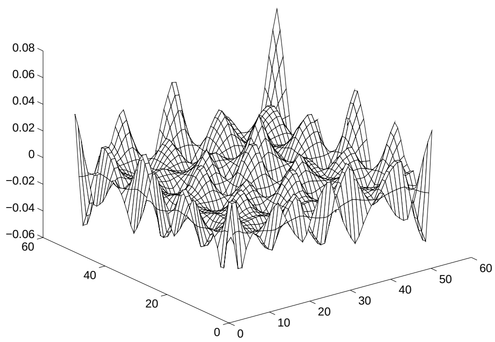

Figure 1: Eigenmode of level 1.

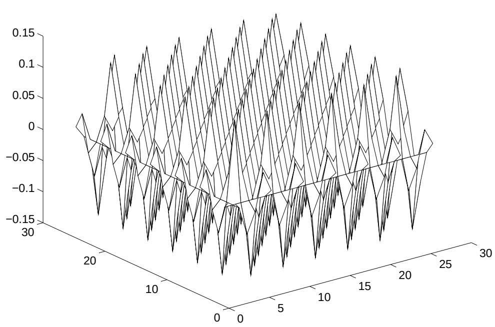

Figure 2: Appearance of coherent structure.

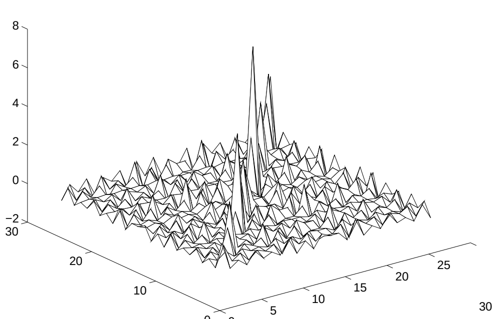

Figure 3: Stable pattern 1.

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