

# DOGBONE GEOMETRY FOR RECIRCULATING ACCELERATORS\*

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## Abstract

Most scenarios for accelerating muons require recirculating acceleration. A racetrack shape for the accelerator requires particles with lower energy in early passes to traverse almost the same length of arc as particles with the highest energy. This extra arc length may lead to excess decays and excess cost. Changing the geometry to a “dogbone” shape, where there is a single linac and the beam turns completely around at the end of the linac, returning to the same end of the linac from which it exited, addresses this problem. In this design, the arc lengths can be proportional to the particle’s momentum. This paper proposes an approximate cost model for a recirculating accelerator, attempts to make cost-optimized designs for both racetrack and dogbone geometries, and demonstrates that the dogbone geometry does appear to be more cost effective.

## 1 THE RACETRACK AND DOGBONE GEOMETRIES

A recirculating accelerator accelerates bunches by passing particles through the same linac several times by constructing arcs through which the beam returns to the linac. This is a very effective way of accelerating muons since there is minimal synchrotron radiation from the muons at lower energies, and thus they can be bent, and yet the muons decay and thus must be accelerated very rapidly. There is generally a separate arc for each pass through the linac, since the bunches have widely varying energies due to the large amount of acceleration per pass through the linac, and the circulation time is too short for the magnetic fields to be increased with increasing bunch energy.

Due to the very high cost per unit length of a linac, it is generally not cost effective to accelerate muons to the maximum desired energy with a single linac. However, as one increases the number of turns, more arcs are required, and the arc cost rapidly dominates the recirculating accelerator cost as one adds more arcs. One could generally decrease the arcs costs if one could reduce the arc length without significantly increasing the cost of the magnets in that arc.

Figure 1 shows the racetrack and dogbone [1] geometries for a recirculating accelerator. In the racetrack geometry, the lower energy arcs are forced to be roughly the same length as the higher energy arcs due to the distance the must traverse from one linac to the other. That distance is determined by the length of the highest energy arc. The dogbone

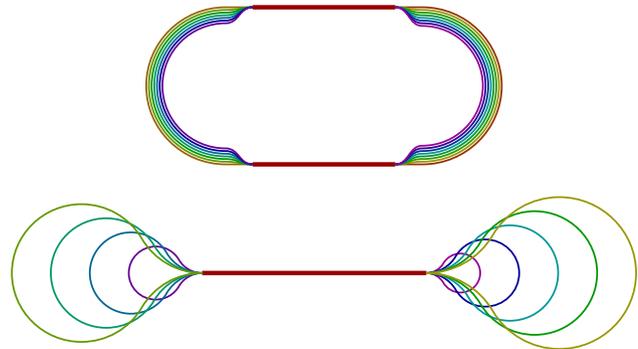


Figure 1: Racetrack (above) and dogbone (below) geometries.

geometry tries to solve this problem by eliminating that distance between the linacs: in fact there is only one linac. The bunch simply returns to that same linac, entering the linac at the same end from which it exited. The minimum arc length for a given bending radius is achieved if there is  $420^\circ$  of bending in that arc. Since the minimum bend radius should be roughly proportional to the bunch’s momentum, the lower energy arcs can be significantly smaller than the higher energy arcs.

One expects the dogbone geometry to be less costly than the racetrack geometry by the following argument: Start with a given racetrack geometry recirculating accelerator. Take its two linacs and make it into a single long linac for the dogbone geometry. The linac costs of these two machines are the same. To accelerate to the same energy, two  $180^\circ$  arcs in the racetrack geometry become one  $420^\circ$  arc in the dogbone. However, the average arc length is around half the maximum arc length. Thus, the total length of arc in the dogbone is around  $7/12$  of the total arc length in the racetrack, and the dogbone should thus cost less. Furthermore, the decays should be substantially reduced since the total length traveled by the muons has been substantially reduced, and in particular reduced at the lowest energies. In another scenario, take only one of the two linacs in the racetrack, and use it in the dogbone. Each  $180^\circ$  arc from the racetrack will become  $420^\circ$  of arc in the dogbone, but as before the average arc length in the dogbone is half the maximum arc length. Thus the dogbone has around  $7/6$  of the arc length of the racetrack, but half the linac, and thus should be less expensive. Decays should be nearly the same in this case.

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## 2 DESIGN AND OPTIMIZATION

To study the cost advantages of one recirculating accelerator design over another, one needs to have a semi-automated way of generating recirculating accelerator designs, and an approximate scheme for assigning costs to those designs.

In a recirculator, one generally wishes to maximize the acceleration that occurs while avoiding particle loss and blowup of the longitudinal emittance. This can be done by making the rf bucket area just large enough to hold the beam. In addition, one wishes to maximize the synchrotron tune of the machine so as to minimize the energy spread in the beam as well as reducing effects of beam loading and wakefields. Assuming the beam is injected matched to these criteria, these constraints plus the number of passes through the linacs are sufficient to determine the longitudinal parameters of the system such as rf phase, linac length, and arc momentum compaction [2]. If one takes into account the fact that the injected beam is not matched to this bucket, we can use the first two linac passes to rotate the bunch so that it is matched to this bucket. This will require an additional bucket area in the later matched turns due to nonlinear blowup, which must be determined empirically.

We thus have a method for finding a machine design as a function of the number of linac passes for any geometry of recirculating accelerator. The next task is to assign a cost to that design. The linac cost is assumed to be proportional to the linac length; this is taken to be  $C_L = 38$  per GeV for 200 MHz rf, based on [3]. The arc cost is assumed to be proportional to the arc length (for a given “style” of magnetic lattice) and proportional to the relative momentum spread. Arc length can be given in units angle-GeV, where the energy really relates to the average bend radius of the arc. The arc cost is taken to be  $C_A = 0.18$  per half arc per GeV per percent momentum spread, again based on [3].

The cost is simply computed as a function of the number of turns, and the minimal cost configuration can be chosen. It is clear that there will be such a cost minimum: as one goes to more turns, the cost should increase linearly with the number of turns, while the linac cost becomes negligible. As one reduced the number of turns, the linac costs increases inversely with the number of turns, as the arc costs go toward zero. One thus has a function which goes to infinity as the number of turns goes to zero, and goes to infinity again as the number of turns goes to infinity. Such a function must have a local minimum. In fact that minimum will occur when the cost of the linac is approximately equal to the cost of the arcs. The reason is that the dependence of energy spread and linac phase on the number of turns is very weak; thus, the cost is approximately of the form

$$\frac{k_L}{n} + k_A n, \quad (1)$$

where the left term is the linac cost and the right term is the arc cost, with  $n$  being the number of turns. The minimum

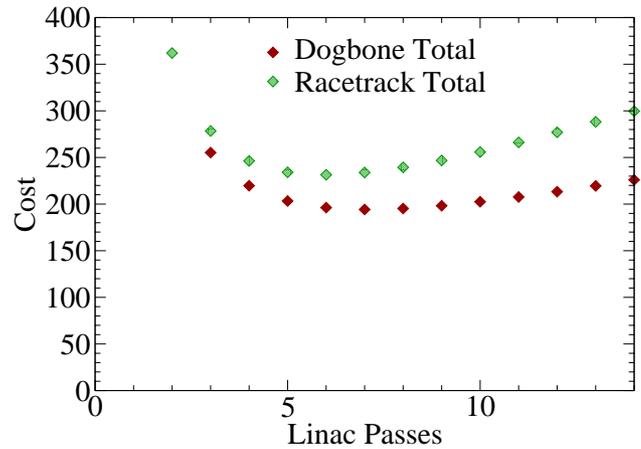


Figure 2: Total system cost as a function of the number of passes through a given linac.

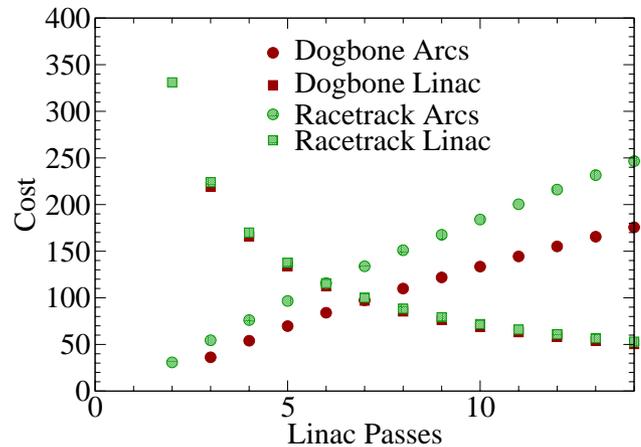


Figure 3: Arc and linac costs as a function of the number of passes through a given linac.

of this function with respect to  $n$  occurs when  $k_L/n = k_A n$ .

## 3 RESULTS AND CONCLUSIONS

The optimization method was applied to a recirculating accelerator which accelerates muons from a momentum of 3 GeV/c to 20 GeV/c. The incoming ellipse which must be accepted has a half-length of 310 ps and an energy half-width of 169 MeV. These parameters are based on the acceleration system for a 20 GeV neutrino factory [4].

Figure 2 shows the total cost for the dogbone and racetrack geometries as a function of the number of passes through a given linac. For any given number of turns, the dogbone geometry is more cost-effective, and its minimum cost is lower than that for the racetrack.

Fig. 3 demonstrates the source of the cost difference: for a given number of linac passes, the arcs for the dogbone lattice cost significantly less than the arcs for the racetrack. The most important conclusion to be drawn from this is that if the low-energy arcs are not kept as small as possible, the

cost advantages of the dogbone configuration will not be realized.

For the dogbone, the optimal number of linac passes is 7, whereas it is 6 passes for the racetrack. The dogbone achieves a 11% cost savings over the racetrack.

One additional advantage of the dogbone configuration is that the switchyards become easier to design, and there will be fewer separate arcs to get tuned up. In a 6-pass racetrack, there are 11 separate arcs that the beam must pass through, 6 on one side and 5 on the other. In the 7-pass dogbone, however, there are only 6 separate arcs, 3 on each side. The switchyard in the dogbone is much easier to design due to the smaller number of arcs and the greater energy separation between the arcs.

The dogbone is not without its disadvantages. First of all, one can see from Fig. 1 that there are a large number of beamline crossings in the dogbone configuration. One cannot avoid these crossings and still keep the lower-energy arcs short. Thus, one must either introduce vertical bends or create a crossing between the vacuum chambers. This will almost certainly add additional costs.

Secondly, the energy spread at the end of the racetrack is 1.6%, while the energy spread at the end of the dogbone is 2.2%. This is caused by two factors: first of all, because the racetrack has more linac-arc pairs the synchrotron tune is effectively larger making the energy spread smaller. Further, the method used to do the phase space rotation (performing the entire rotation using the first two linacs and one arc) is not very good. It seems to work much better for the racetrack case than it does for the dogbone case, producing much more nonlinear blowup for the latter. A more adiabatic matching algorithm may give some improvements. The larger energy spread output from the dogbone will increase the cost of the storage ring and the transport line to it, and may make it more difficult to achieve the physics requirements of the muon beam.

Finally, the tunnelling costs have not been taken into consideration here. The dogbone arcs must be in separate tunnels, at least for part of their lengths, whereas all the arcs for the racetrack can be put in the same tunnel. However, the tunnel for the racetrack must be much wider to accommodate several arcs, and thus will be substantially more costly. In all likelihood, the dogbone tunnel costs will exceed those of the racetrack, but it is unlikely that this will be sufficient to make the dogbone more costly.

In summary, the dogbone geometry appears to be a more cost-effective geometry for recirculating acceleration, although there are still aspects of the design which need to be analyzed.

## 4 REFERENCES

- [1] J. S. Berg *et al.*, "Acceleration Stages for a Muon Collider," Proceedings of the 1999 Particle Accelerator Conference, New York (1999), pp. 3152–3154.
- [2] J. Scott Berg, "Acceleration for a High Energy Muon Collider," *CP530, Colliders and Collider Physics at the Highest Energies: HEMC'99 Workshop*, edited by B. J. King, AIP, Melville, NY, (2000), pp. 13–31.
- [3] N. Holtkamp and D. Finley, eds., *A Feasibility Study of a Neutrino Source Based on a Muon Storage Ring*, Fermilab-Pub-00/180-E (2000).
- [4] S. Ozaki *et al.* eds., "Feasibility Study-II of a Muon-Based Neutrino Source," BNL-52623 (2001).